T-FUZZY CONGRUENCES AND T-FUZZY FILTERS OF A BL-ALGEBRA

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ABSTRACT. In this note, we introduce the concept of a fuzzy filter of a BL-algebra, with respect to a t-norm briefly, T-fuzzy filters, and give some related results. In particular, we prove Representation Theorem in BL-algebras. Then we generalize the notion of a fuzzy congruence (in a BL-algebra) was defined by Lianzhen et al. to a new fuzzy congruence, specially with respect to a t-norm. We prove that there is a correspondence bijection between the set of all T-fuzzy filters of a BL-algebra and the set of all T-fuzzy congruences in that BL-algebra. Next, we show how T-fuzzy filters induce T-fuzzy congruences, and construct a new BL-algebras, called quotient BL-algebras, and give some homomorphism theorems.

1. Introduction

BL-algebras are the algebraic structure for Hájek basic logic [4]. Most familiar example of a BL-algebra is the unit interval [0,1] endowed with the structure induced by a continuous t-norm. In 1958, Chang [3] introduced the concept of an MV-algebra which is one of the most classes of BL-algebras. Turunen [10] introduced the notion of an implicative filter and a Boolean filter and proved that these notions are equivalent to BL-algebras. Boolean filters are an important class of filters, because the quotient BL-algebra induced by these filters are Boolean algebras. In [7] and [6], Lianzhen et al. introduced the concept of a fuzzy filter and fuzzy congruence in a BL-algebra and gave some related results. He proved that the fuzzy quotient algebras induced by fuzzy Boolean filters are Boolean algebras.

Now, in this paper, by considering [1], [6], [7], and the notion of a t-norm we define the notion of a fuzzy congruence with respect to a t-norm and give some related results.

2. Preliminaries

In this section, we give some fundamental definitions and results. For more details, we refer the readers to the references.

Definition 2.1. A BL-algebra is an algebra $(B, \vee, \wedge, *, \rightarrow, 0, 1)$ of type (2,2,2,2,0,0) such that
(BL1) $(B, \vee, \wedge, 0, 1)$ is a bounded lattice,
(BL2) $(B, *, 1)$ is a commutative monoid,