
بسم الله الرحمن الرحيم

CHAPTER 3

p–n Junction

3.6.2 Avalanche Multiplication

- The avalanche multiplication process is illustrated in Fig. 22b.

- The *p–n* junction, such as a p^+–*n* one-sided abrupt junction with a doping concentration of $N_D \cong 10^{17}\ cm^{-3}$ or less, is under reverse bias.

- A thermally generated electron in the depletion region (designated by 1) gains kinetic energy from the electric field.

- If the field is sufficiently high, the electron can gain enough kinetic energy that on collision with an atom, it can break the lattice bonds, creating an electron-hole pair (2 and 2′).
- The newly created electron and hole both acquire kinetic energy from the field and create additional electron-hole pairs (e.g., 3 and 3′).
- These in turn continue the process, creating other electron-hole pairs.
- This process is therefore called avalanche multiplication.

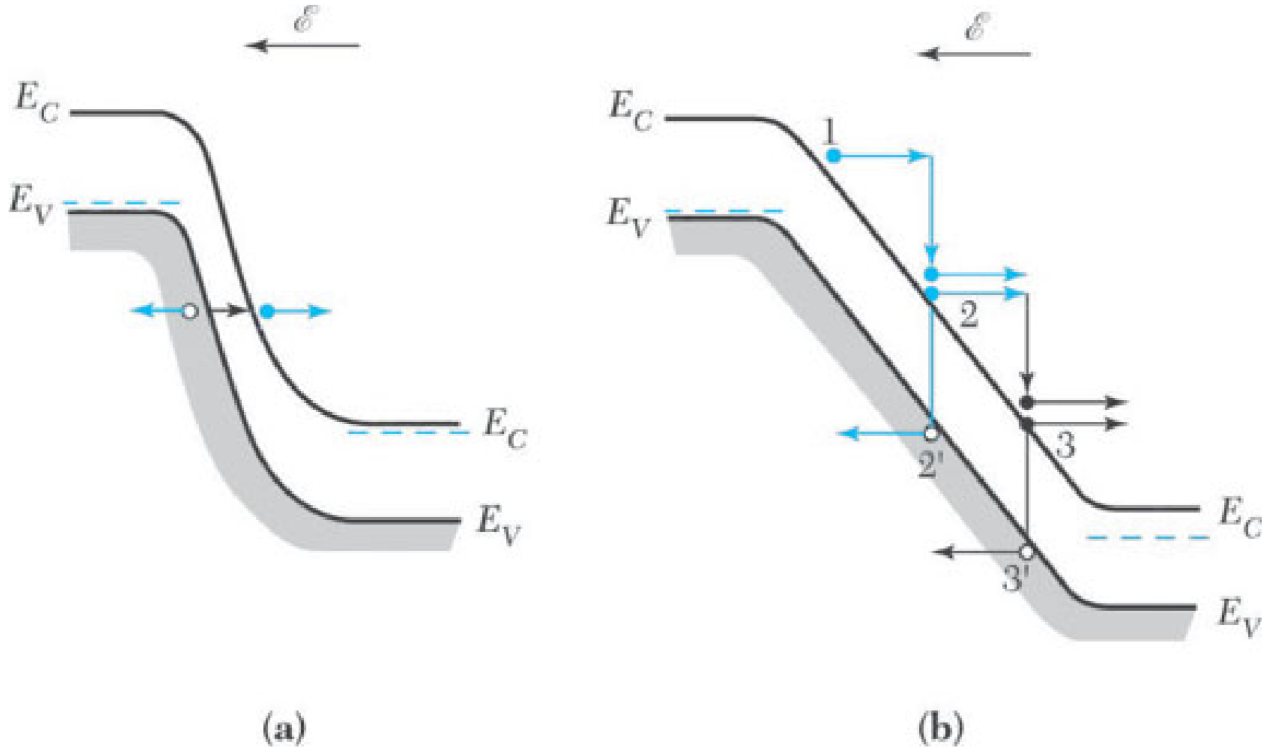


Fig. 22 **Energy band diagrams under junction-breakdown conditions. (*a*) Tunneling effect. (*b*) Avalanche multiplication.**

- To derive the breakdown condition, we assume that a current I_{no} is incident(وارد شدن) at the left-hand side of the depletion region of width W, as shown in Fig. 23.

- If the electric field in the depletion region is high enough to initiate(آغاز کردن) the avalanche multiplication process, the electron current I_n will increase with distance through the depletion region to reach a value $M_n I_{no}$ at W, where M_n, the multiplication factor, is defined as

$$M_n \equiv \frac{I_n(W)}{I_{no}}. \qquad (81)$$

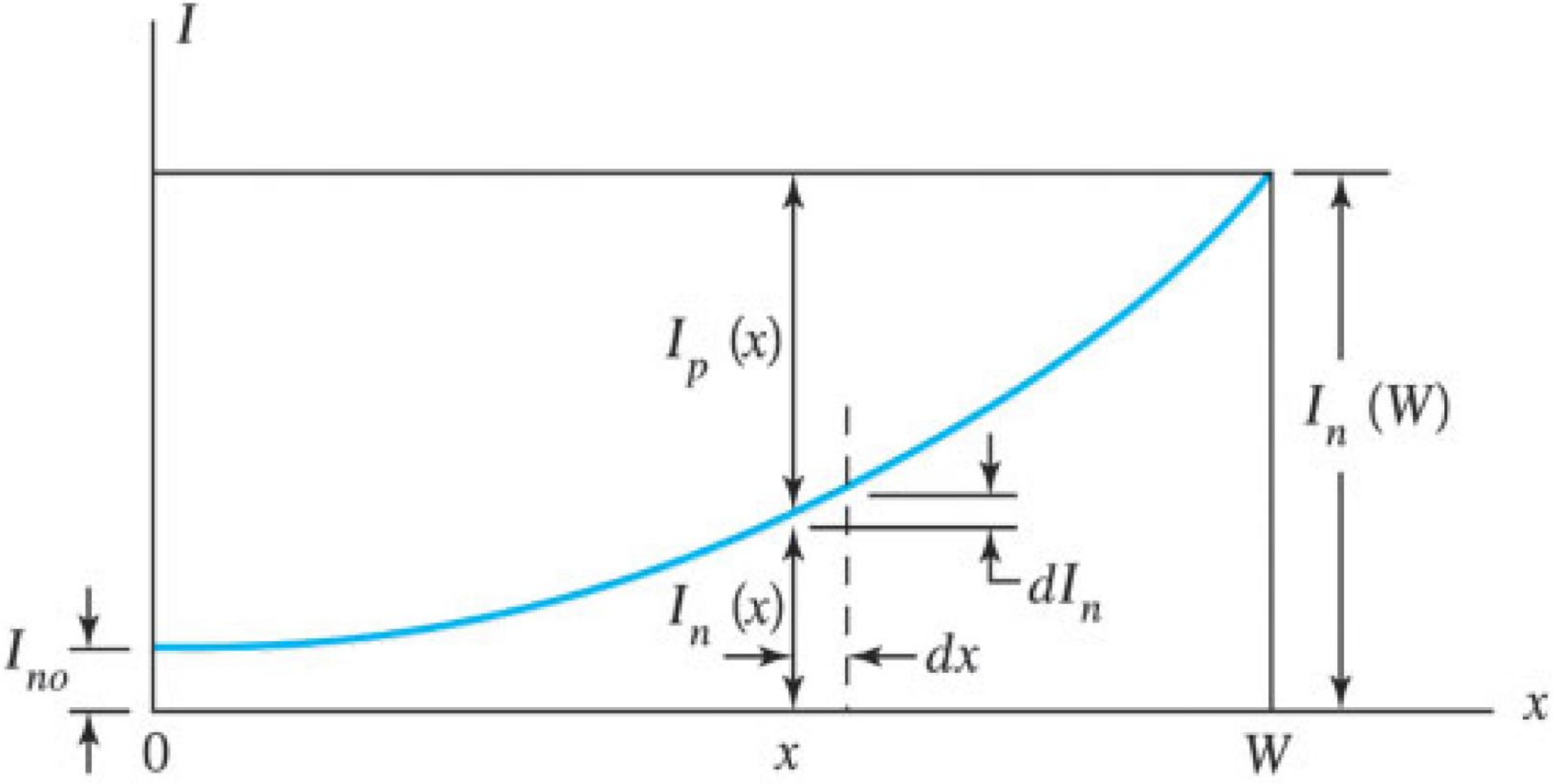


Fig. 23 **Depletion region in a *p*–*n* junction with multiplication of an incident current.**

- Similarly, the hole current I_p increases from $x = W$ to $x = 0$.

- The total current $I = (I_p + I_n)$ is constant at steady state.

- The incremental electron current at x equals the number of electron-hole pairs generated per second in the distance dx:

افزایش جریان الکترونی با وجود تولید زوج الکترون حفره ها بخاطر این
است که هر چه به سمت x=w می رویم غلظت الکترونها هم رو به افزایش
می گذارد چون به سمت ناحیه N حرکت می کنیم

↓

$$d\left(\frac{I_n}{q}\right) = \left(\frac{I_n}{q}\right)(\alpha_n dx) + \left(\frac{I_p}{q}\right)(\alpha_p dx) \qquad (82)$$

اثبات در اسلاید ۱۰

$$\frac{dI_n}{dx} + (\alpha_p - \alpha_n) I_n = \alpha_p I, \qquad (82a)$$

- where α_n and α_p are the electron and hole ionization rates, respectively.

- If we use the simplified assumption that $\alpha_n = \alpha_p = \alpha$, the solution of Eq. 82a is

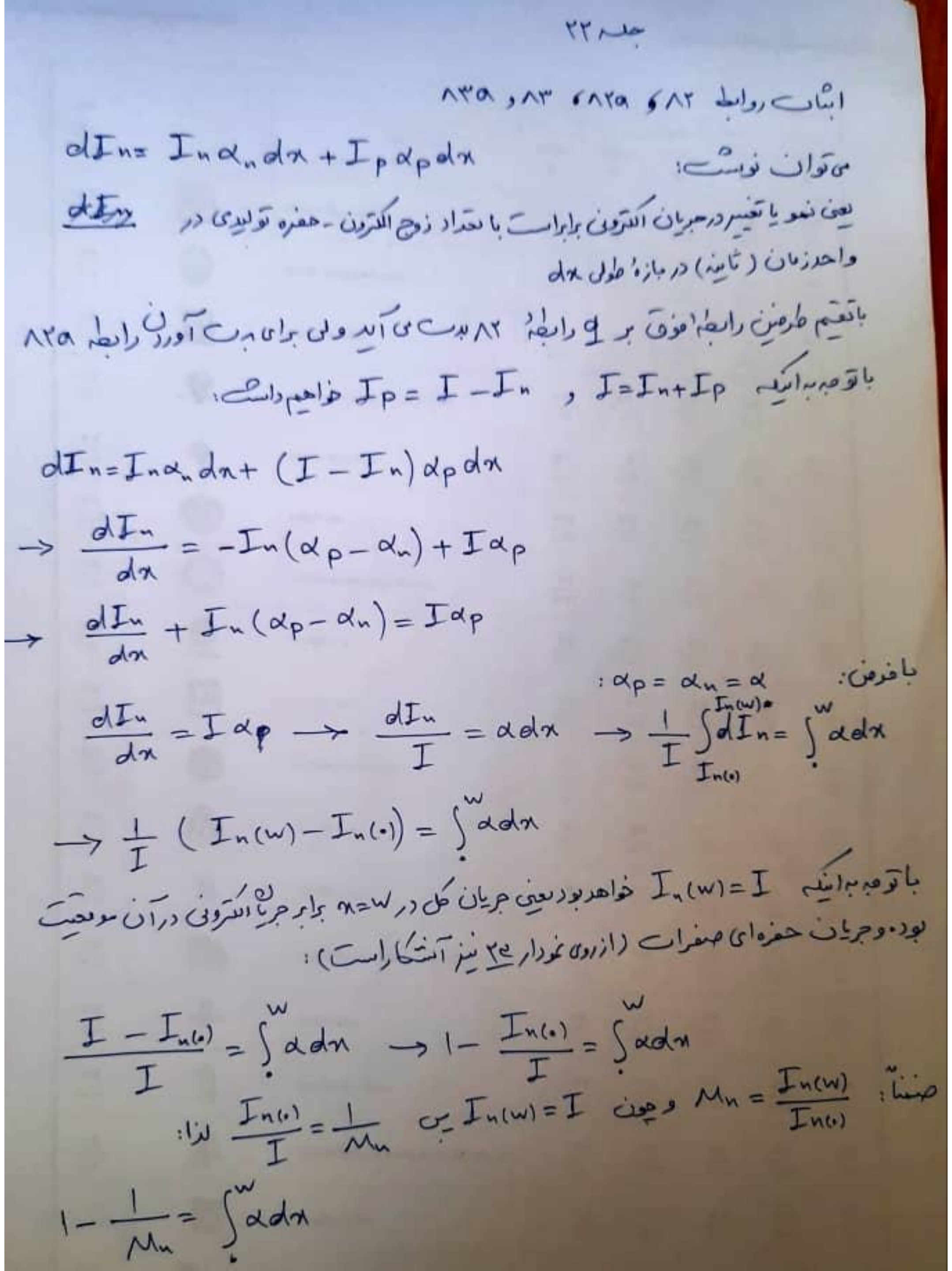
$$\frac{I_n(W) - I_n(0)}{I} = \int_0^W \alpha\, dx. \tag{83}$$

- From Eqs. 81 and 83, we have

اثبات در اسلاید بعدی

$$1 - \frac{1}{M_n} = \int_0^W \alpha\, dx. \tag{83a}$$

- The avalanche breakdown voltage is defined as the voltage at which M_n approaches infinity.



جلسه ۲۲

اثبات روابط ۸۲ و ۸۲a، ۸۳ و ۸۳a

می توان نوشت: $dI_n = I_n \alpha_n dx + I_p \alpha_p dx$

dI_n یعنی نمو یا تغییر در جریان الکترونی برابر است با تعداد زوج الکترون - حفره تولیدی در واحد زمان (ثانیه) در بازهٔ طول dx

با تقسیم طرفین رابطهٔ فوق بر q رابطهٔ ۸۲ بدست می آید ولی برای بدست آوردن رابطه ۸۲a با توجه به اینکه $I = I_n + I_p$ و $I_p = I - I_n$ خواهیم داشت:

$$dI_n = I_n \alpha_n dx + (I - I_n)\alpha_p dx$$

$$\rightarrow \frac{dI_n}{dx} = -I_n(\alpha_p - \alpha_n) + I\alpha_p$$

$$\rightarrow \frac{dI_n}{dx} + I_n(\alpha_p - \alpha_n) = I\alpha_p$$

با فرض: $\alpha_p = \alpha_n = \alpha$:

$$\frac{dI_n}{dx} = I\alpha_p \rightarrow \frac{dI_n}{I} = \alpha dx \rightarrow \frac{1}{I}\int_{I_n(0)}^{I_n(w)} dI_n = \int_0^w \alpha dx$$

$$\rightarrow \frac{1}{I}\left(I_n(w) - I_n(0)\right) = \int_0^w \alpha dx$$

با توجه به اینکه $I_n(w) = I$ خواهد بود یعنی جریان کل در $x=w$ برابر جریان الکترونی در آن موقعیت بوده و جریان حفره‌ای صفر است (از روی نمودار ۲۵ نیز آشکار است):

$$\frac{I - I_n(0)}{I} = \int_0^w \alpha dx \rightarrow 1 - \frac{I_n(0)}{I} = \int_0^w \alpha dx$$

ضمناً: $M_n = \frac{I_n(w)}{I_n(0)}$ و چون $I_n(w) = I$ پس $\frac{I_n(0)}{I} = \frac{1}{M_n}$ لذا:

$$1 - \frac{1}{M_n} = \int_0^w \alpha dx$$

- Hence, the breakdown condition is given by

$$\boxed{\int_0^W \alpha\, dx = 1\ .} \qquad (84)$$

- From both the breakdown condition described above and the field dependence of

شرط اول میدان بالا و شرط دوم رابطه ۸۴

the ionization rates, we may calculate the critical field (i.e., the maximum electric field at breakdown) at which the avalanche process takes place.

⬇

- Using measured α_n and α_p (Fig. 27 in Chapter 2), the critical field E_c is calculated for silicon and gallium arsenide one-sided abrupt junctions and shown in Fig. 24 as functions of the impurity concentration of the substrate.

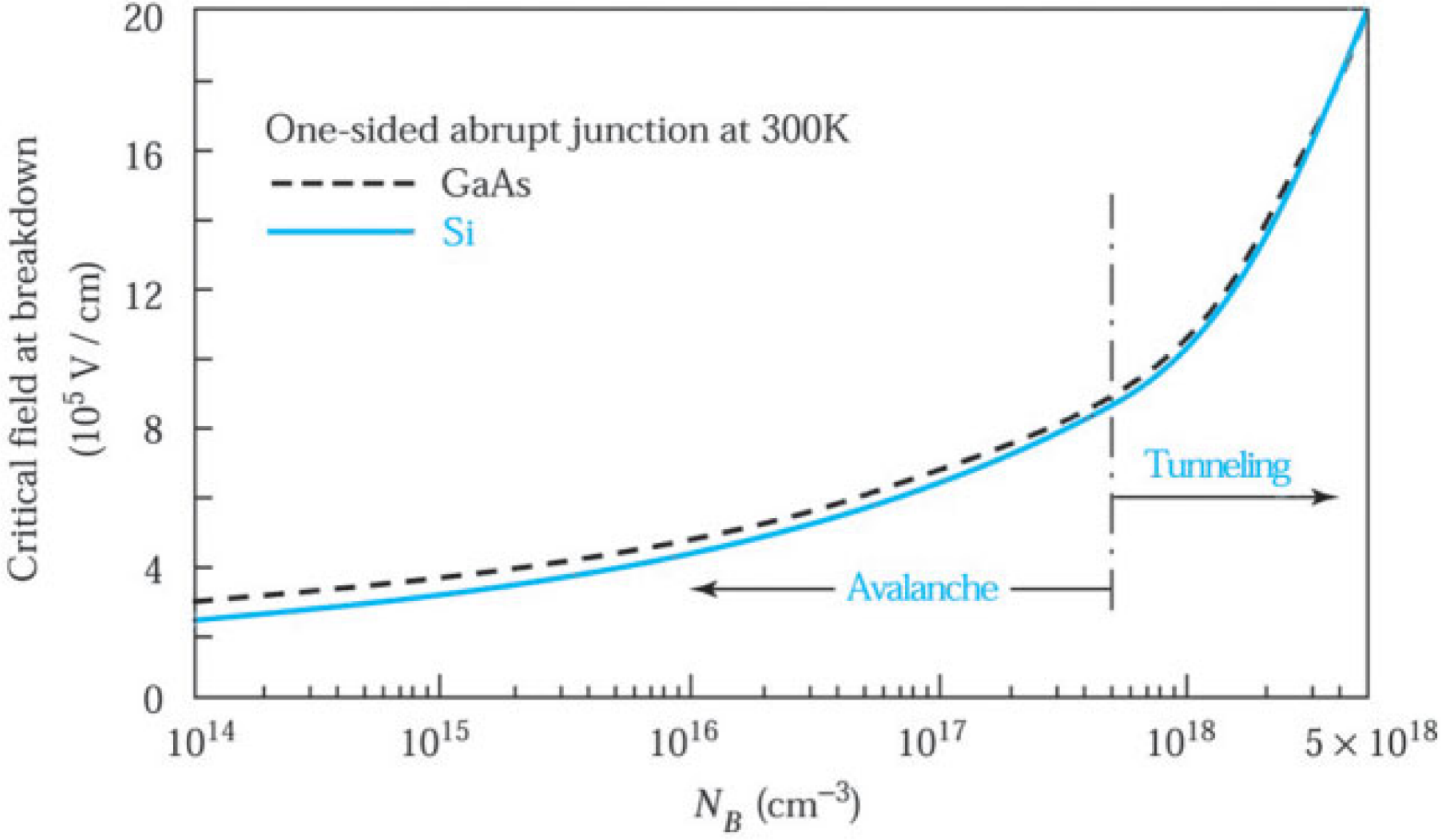


Fig. 24 **Critical field at breakdown versus background doping for Si and GaAs one-sided abrupt junctions.**[5]

- Also indicated is the critical field for the tunneling effect.
- It is evident that tunneling occurs only in semiconductors having high doping concentrations.
- With the critical field determined, we may calculate the breakdown voltages.
- As discussed previously, voltages in the depletion region are determined from the solution of Poisson’s equation:

$$\boxed{V_B\,(\text{breakdown voltage}) = \frac{\mathcal{E}_c W}{2} = \frac{\varepsilon_s \mathcal{E}_c^{2}}{2q}\left(N_B\right)^{-1}} \tag{85}$$

for one-sided abrupt junctions and

$$V_B = \frac{4\mathscr{E}_c^{3/2}}{3}\left(\frac{2\varepsilon_s}{q}\right)^{1/2}(a)^{-1/2} \qquad (86)$$

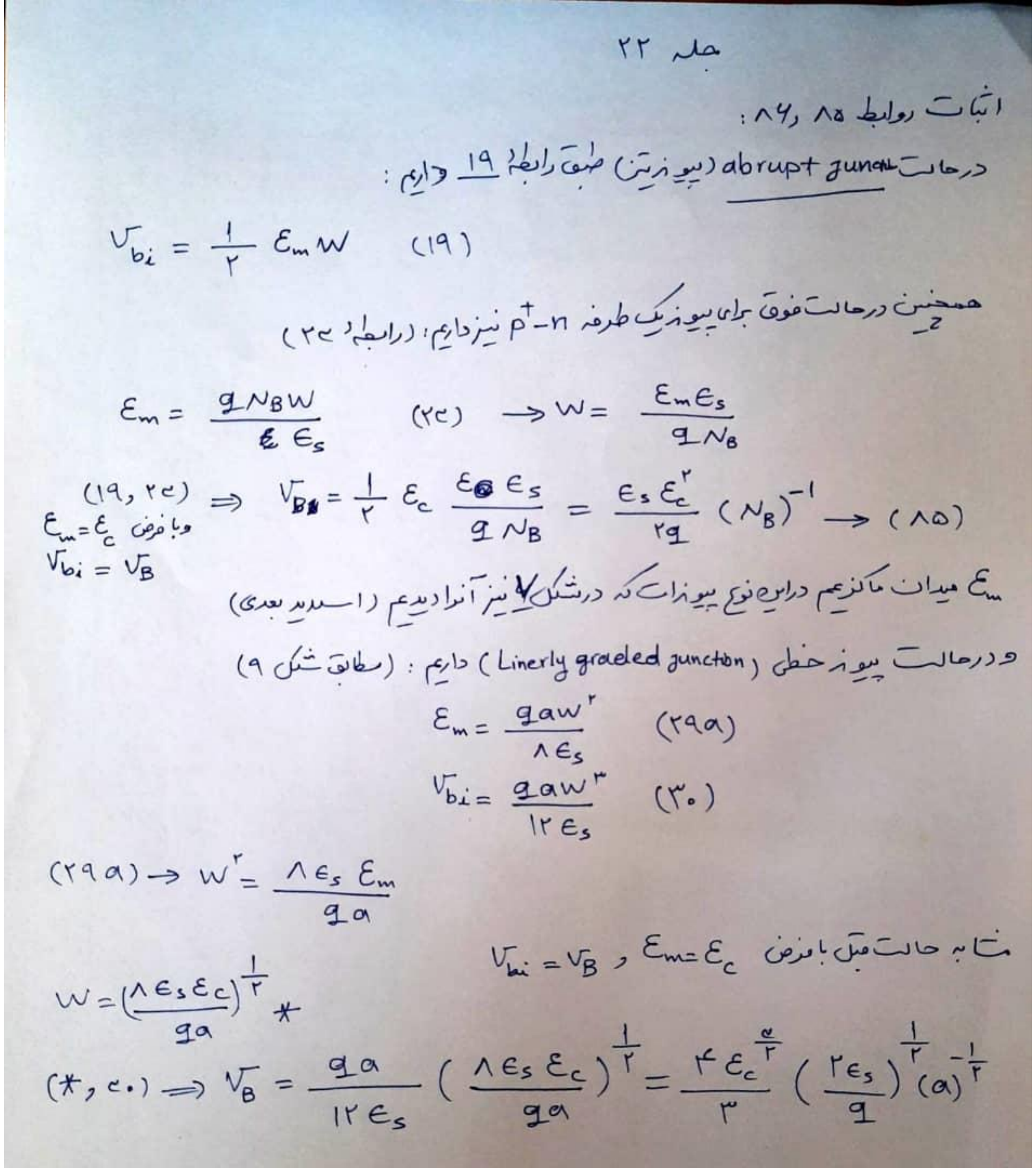
اثبات ۸۵ و ۸۶ در اسلاید بعدی

for linearly graded junctions.

where N_B is the background doping of the lightly doped side, ε_s is the semiconductor permittivity, and a is the impurity gradient.

- The breakdown voltage, as a first-order approximation, varies as N_B^{-1} for abrupt junctions and as $a^{-1/2}$ for linearly graded junctions.

بنظر می رسد در رابطه ۱۹ بدلیل اینکه ولتاژ معکوس اعمال می شود باید بجای V_{bi} کمیت $V_{bi}-V$ را قرار دهیم و در واقع V_B که پتانسیل شکست است، مقدار پتانسیل منتج است. همین بحث در مورد میدان الکتریکی نیز صادق است.



جلسه ۲۲

اثبات روابط ۸۵, ۸۶:

در حالت abrupt junction (پیوند تیز) طبق رابطه ۱۹ داریم:

$$V_{bi} = \frac{1}{۲}\mathcal{E}_m W \qquad (۱۹)$$

همچنین در حالت فوق برای پیوند یک طرفه p^+-n نیز داریم: (رابطه ۲۴)

$$\mathcal{E}_m = \frac{qN_BW}{\cancel{\varepsilon}\,\epsilon_s} \qquad (۲۴) \rightarrow W = \frac{\mathcal{E}_m\epsilon_s}{qN_B}$$

(۱۹, ۲۴)
و با فرض $\mathcal{E}_m=\mathcal{E}_c$
$V_{bi} = V_B$

$$\Rightarrow V_{B\cancel{i}} = \frac{1}{۲}\mathcal{E}_c\frac{\mathcal{E}_{\cancel{m}}\epsilon_s}{qN_B} = \frac{\epsilon_s\mathcal{E}_c^{۲}}{۲q}(N_B)^{-1} \rightarrow (۸۵)$$

$\mathcal{E}_m$ میدان ماکزیمم در این نوع پیوندات که در شکل ۷ نیز آن را دیدیم (اسلاید بعدی)

و در حالت پیوند خطی (Linerly graded junction) داریم: (مطابق شکل ۹)

$$\mathcal{E}_m = \frac{qaw^{۲}}{۸\epsilon_s} \qquad (۲۹a)$$

$$V_{bi} = \frac{qaw^{۳}}{۱۲\epsilon_s} \qquad (۳۰)$$

$$(۲۹a) \rightarrow w^{۲} = \frac{۸\epsilon_s\mathcal{E}_m}{qa}$$

مشابه حالت قبل با فرض $\mathcal{E}_m=\mathcal{E}_c$ و $V_{bi}=V_B$

$$W = \left(\frac{۸\epsilon_s\mathcal{E}_c}{qa}\right)^{\frac{1}{۲}} \quad *$$

$$(*, ۳۰) \Rightarrow V_B = \frac{qa}{۱۲\epsilon_s}\left(\frac{۸\epsilon_s\mathcal{E}_c}{qa}\right)^{\frac{1}{۲}} = \frac{۴\mathcal{E}_c^{\frac{۳}{۲}}}{۳}\left(\frac{۲\epsilon_s}{q}\right)^{\frac{1}{۲}}(a)^{-\frac{1}{۲}}$$

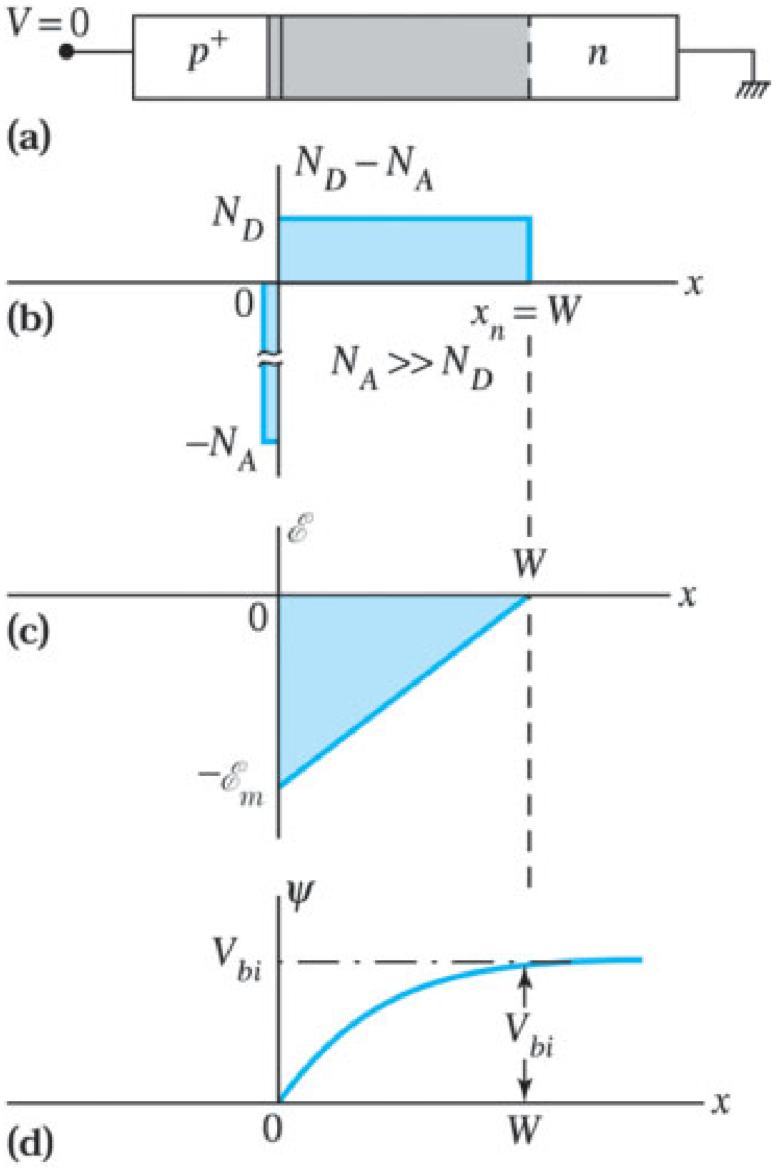


Fig. 7 (*a*) One-sided abrupt junction (with $N_A >> N_D$) in thermal equilibrium. (*b*) Space charge distribution. (*c*) Electric-field distribution. (*d*) Potential distribution with distance, where V_{bi} is the built-in potential.

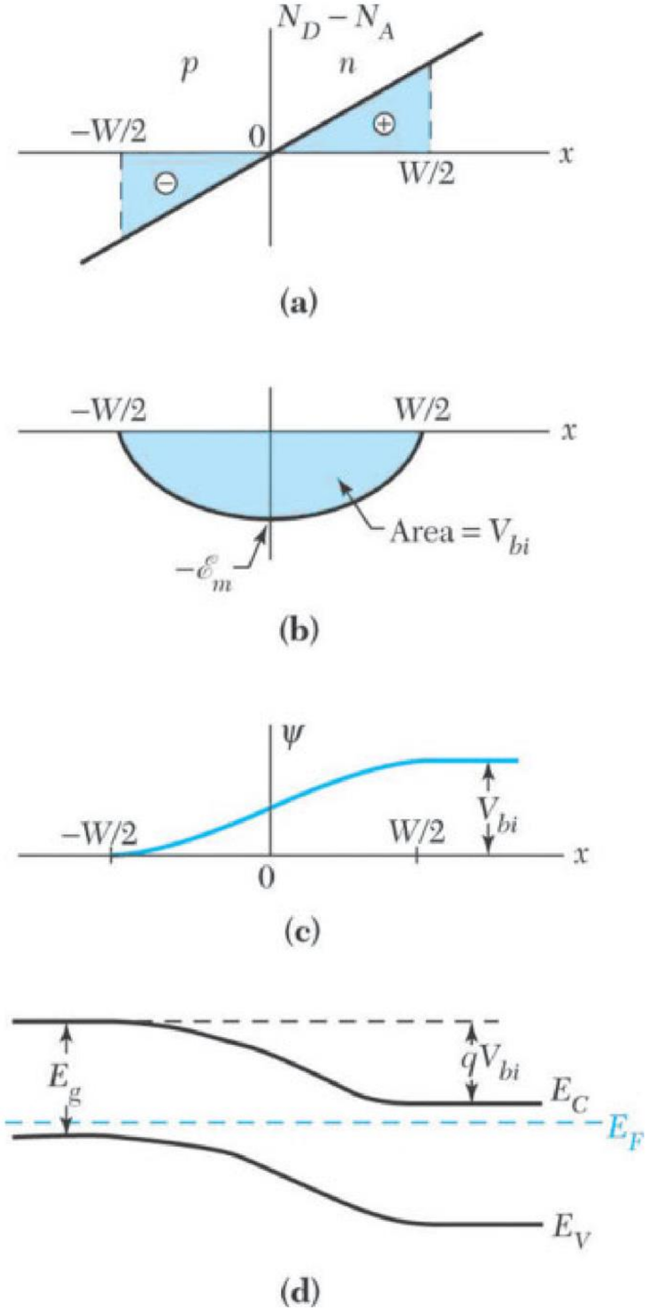


Fig. 9 **Linearly graded junction in thermal equilibrium. (*a*) Impurity distribution. (*b*) Electric field distribution. (*c*) Potential distribution. (*d*) Energy band diagram.**

- Figure 25 shows the calculated avalanche breakdown voltages for silicon and gallium arsenide junctions.
- The dash-dot line (to the right) at high dopings or high-impurity gradients indicates the onset of the tunneling effect.
- Gallium arsenide has higher breakdown voltages than silicon for a given N_B or a, mainly because of its larger bandgap.
- The larger the bandgap, the larger the critical field must be for sufficient kinetic energy to be gained between collisions.

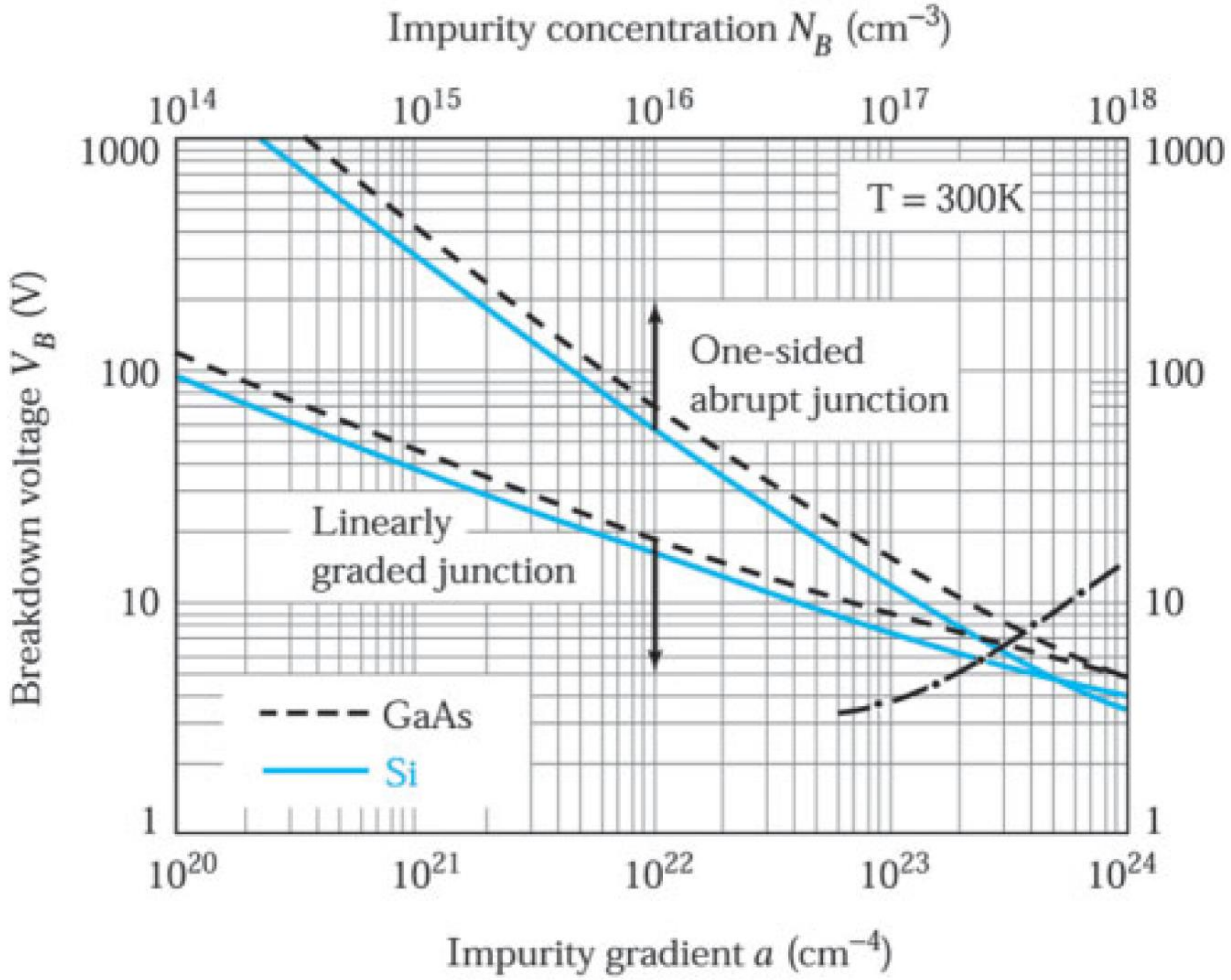


Fig. 25 **Avalanche breakdown voltage versus impurity concentration for a one-sided abrupt junction and avalanche breakdown voltage versus impurity gradient for a linearly graded junction in Si and GaAs. Dash-dot line indicates the onset of the tunneling mechanism.**[5]

- As Eqs. 85 and 86 demonstrate, the larger critical field, in turn, gives rise to higher breakdown voltage.

EXAMPLE 8

Calculate the breakdown voltage for a Si one-sided p^+–n abrupt junction with $N_D = 5 \times 10^{16}$ cm^{-3}.

SOLUTION From Fig. 24, we see that the critical field at breakdown for a Si one-sided abrupt junction is about 5.7×10^5 V/cm. Then from Eq. 85, we obtain...