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A study on dimensions of Fractal geometry in Iranian architecture

Halime Pudine[1] -*Department of Arts and Architecture, university of Sistan and Baluchestan*

Abstract
The subject of geometry and proportions has been regarded as an issue which has a close relationship with architecture. Since the entire universe, living beings and the human geometry can be seen clearly, that's why our unconscious essence has accustomed to these proportions whereby reflection of this mentality can be seen in the architect's hands and thought in the architecture. It can witness a close linkage between nature and architecture at different levels in Iranian architecture which the linkage at architecture geometry has been regarded as one of these levels. Iranian geometry using plant and geometric forms in Islamic buildings seeks to prove a special continuity in the life to plant and human world. This continuity in the world of geometry has been known with fractal geometry. The present research seeks to examine dimensions of fractal geometry in Iranian architecture and motifs. In following, an attempt is made to examine what the fractal means in Iranian architecture and clarify the motifs and decorations of Iranian architecture after defining the issue of fractal, fractal geometry and fractal in architecture.

Key words: *geometry, fractal geometry, Iranian architecture, Islamic architecture*

1. Corresponding Author, Tel: 09155403537, Email Address: hpudine@arts.usb.ac.ir

Introduction

In the long lost past, the humans who have lived in nature and accustomed to the nature had the architecture with order of nature. Since they have growing in nature, their subconscious mind has growing with order of nature. As a result, their artefact, architecture and designs have an order taken from natural beauties, i.e. an order which is called fractal in the present research. It should be noted that fractal geometry and architecture have existed since long lost past so that some wavy shapes in Baroque Churches, some uniform and similar decorations in Gothic architecture have represented manifestations of this geometry. But what is definite is the fractal geometry which is a non-Euclidean Geometry that has been inspired in some existing forms in nature. This issue was proposed by Benoit Mandelbrot in 1975, but it dates back to Iranian architecture for over one hundred years. Farness to nature and geometry based on nature and on the other hand farness to Iranian concepts and Imitation of Iranian forms have transformed to a major crisis especially for Iran. This has raised the necessity to examine and explore about revitalization of fractal geometry in the architecture. According to Nikos Salingaros, architecture and urban design are in a complicated status, because the rules learnt by the students are inconsistent with the organized structure of life forms. Fractal geometry of the traditional cities has been omitted deliberately due to imposing arbitrary stylistic rules. This issue has led to philosophical, mental and physical separation of the man from his surrounding environment. Use of fractal geometry in architecture can assist for revitalization of the relationship between man and nature which this type of geometry has more complicatedness than Euclidean geometry. Jencks believes that since the direction of evolution to complexity goes beyond and our project must be an imitation of the world, diversity must exist so far as the system grows in sake of size, energy and information and reaches to complexity, under which the architecture will grow. Despite Jencks's statement, complexity of architecture can enrich the complicated spirit of the man, mentioned that various scales at a fractal form causes to witness new information which is still interesting to us as soon as we approach to a form or building. The present research aims to provide a definition for fractal and the rules governing geometry and natural systems and display the extent to which they are used in Iranian architecture. For this, firstly it must recognize fractal geometry and examine various dimensions of this geometry by searching the nature and architecture and then evaluate role of fractal in Iranian architecture.

The research question

To achieve aim of research, the questions below are proposed: (1) whether the traditional architecture of Iran and Islamic architecture have inspired from nature and existing fractals in nature? (2) why the Iranian architects and artists have used fractal geometry in art of architecture? (3) how the fractal geometry has been used in Iranian architecture and motifs?

Research method

Descriptive-analytical method has been used as the research method in which an attempt has been made to examine all the facets of the subject by referral to the library documents and available documents. Overview of the related works by the researchers, overview of documents, data collection, data analysis and obtaining the results and response to the research questions have been mentioned as the process which have been mentioned in the present research.

Definition for fractal

Fractal has been taken from Latin word "frāctus" which means broken or fractured rock. Fractal is a geometric structure developed from the components which gains the early structure by magnifying any component to the certain ratio. In other words, fractal refers to a structure with the component same as the entire structure. Fractals refer to the shapes which are not regular despite Euclidean geom-

etry shapes. These shapes are irregular in the entire structure having the same irregularity at all the scales, deduced that the fractal shape is seen the same from the close and far area [1]. This implies that fractal geometry despite Euclidean geometry can be a better method to explain and create the phenomena such as nature. The term fractal was first used by mathematician Benoît Mandelbrot in 1975. Fractal was proposed for the first time by Dr Mandelbrot in a theory to resolve the problems in the universe, mentioned that the universe has replete with the natural phenomena which are the fractals in the universe which are unknown to us. When Mandelbrot conducted a study about the shores of England, he deduced that when the shores of England are measured at large scale, they will be greater than when they are measured at small scale.

Fractal geometry

In another approach, fractal geometry allows to describe the nature with all the unknowns. As the linear systems are the only state of non-linear systems, Euclidean geometry is the only tiny and simplified sub-set of the geometry of the real and natural world. In the same way, linear systems can detect only the controlled models of the real two-dimensional systems and three-dimensional volumes. Yet, when two-dimensional sheet of paper is slightly crumpled and a surface is obtained which is neither two-dimensional nor three-dimensional as it preoccupies the space, understanding Euclidean geometry falls apart from explanation and recognition. This geometry is the prevailing geometry in nature and the scholars at Chaos science have explored it by starting observation and deep thinking about natural phenomena. Edward Lorenz has dedicated his effort to recognize climate system and Mitchell kept seeing the river for hours [2]. According to Benoit Mandelbrot, the universe and all natural phenomena are somehow fractal. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line. By observing the existing shapes in nature, it is specified that Euclidean geometry does not enable to elaborate complicated shapes void of natural order. Euclidean geometry (volume of the perfect sphere, pyramids, cubes and cylinders) are not the best way of showing the natural elements. Clouds, mountains, shoreline and trees are all in contradiction with Euclidean volumes and are not only smooth but rugged, bringing about this irregularity at small scales which is one of the most important features of fractals [3]. This implies that fractal geometry despite Euclidean geometry is a better way to elaborate and create the phenomena such as nature. Any natural thing around us is essentially a fractal, because the smooth lines and plans exist only in the ideal world of mathematics. In addition to this theory, any system that can be perceived and analyzed can be a fractal. In mathematics, a fractal is a complicated geometric shape with similar details in its structure at any scale. The irregularity in fractal is to the same extent from far and near. Ultimately, we must know follows to compare fractal shapes with Euclidean shapes.

1- Euclidean shapes are produced using static functions, while fractal shapes arise with a dynamic process.

2- Fractal shapes have the property of self-resemblance which length of these shapes is infinite enclosed in a confined space.

3- Fractal shapes are more irregular to describe them with Euclidean geometry.

4- Fractal geometry has the structures with high capacity, while capacity of Euclidean shapes is limited containing repeated information.

5- Fractal geometry registers movement of shapes in space and displays the roughness of the world and the energy and dynamic changes in it.

Geometric properties of fractal

1-self-similarity: A geometric figure is self-similar if there is a point where every neighborhood of the point contains a copy of the entire figure. For example, imagine the figure

formed by inscribing a square within another square, rotated by 45°. Then inside the inner square, inscribe another square in the same manner, and so on ad infinitum. A figure is strictly self-similar if it is self-similar at every point. Equivalently, this means that the figure can be decomposed into some number of disjoint pieces, each of which is an exact copy of the entire figure. The figure below is self-similar at the center of the squares – every ball around the center, no matter how small, will contain a complete copy of the entire figure. However, this is the only point where the figure is self-similar – at most points, the figure looks, at some magnification, like either a straight line or a corner where four lines meet, so this is not complicated enough to be a fractal[4].

Fractional Dimension

Fractals have Fractional dimensions. The notion of dimension is very familiar, but surprisingly subtle. Intuitively, we know that a line or curve is one-dimensional, a plane or surface is two-dimensional and space is 3-dimensional. Mathematicians consider a figure to be one-dimensional if it can be cut into pieces which each look like a piece of a line, two-dimensional if it can be cut into pieces which look like a piece of a plane, and so forth. But this rough notion of dimension doesn't work for fractals.

Iterative formation

Fractals are created mainly with iterative stages; to make fractal, consider a geometric shape such as a line or triangle and apply the operations on the shape, whereby a complicated shape will come which must be applied on the new shape. At this time, there will be more complicated shape. Repeat this operation to the end. It seems that it can continue it to the end. But any iterative operations on the shapes do not end in emergence of fractals. For instance, segment a line and continue this to the end, whereby a fractal will not be created. In following, iterative stages will be examined in a fractal. Here the example is the Sierpinski triangle, introduced in 1916 by the Polish mathematician Waclaw Sierpinski. Begin with a solid equilateral triangle. Joining the midpoints of the three sides gives another equilateral triangle – remove it. You will be left with three equilateral triangles, with each pair meeting at a corner. Repeat the process with each of these equilateral triangles, and continue to repeat the process on each new smaller solid triangle left at each stage. Several early stages of the construction are shown below.

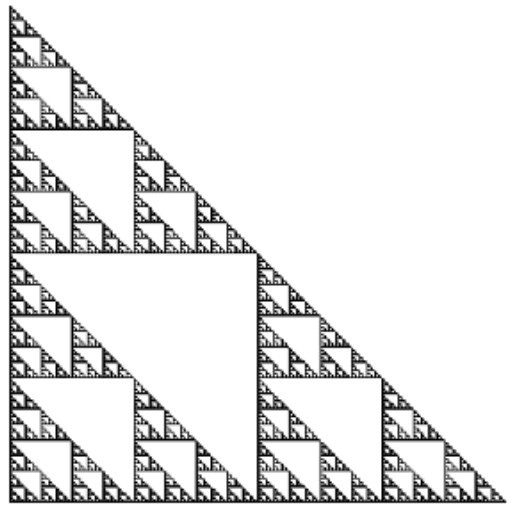

Figure 1. A sample of self-similarity (Dadashvand, 2014, p. 6)

We can also produce fractals with a predetermined similarity dimension. For example, to produce a fractal with dimension 3/2, we need log (n)/log(k) = 3/2; so we need to have some x where n=x3 and k=x2. An example of such an arrangement, and the first few stages of the resulting fractal, is shown below [4].

Overview of fractal in architecture

Vernacular architecture created by the people of the world has emerged as a fractal. The important buildings of the past architecture and vernacular architecture throughout the world follow critical mathematical harmony that one is fractal structure. Cities especially the most interesting cities are fractal. In large cities such as Paris, London or Venice, everything from strip roads and streets to the placement of trees are fractal. This problem has been examined by the scholars including Michelle Baty and Frank

Figure 2. Iterative formation (Sierpinski triangle) (Dadash vand, 2014, p. 6)

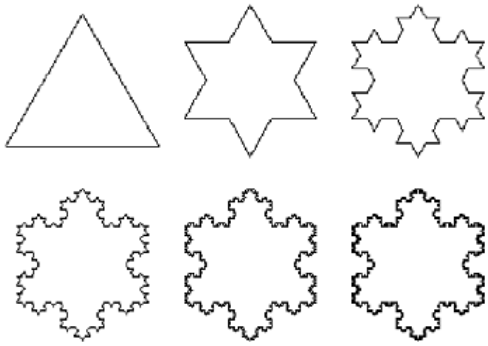

Figure 3. Design fractals with similarity dimension

Figure 4. the greatest Hindu temple in Indonesia(Ghomeishi, Tabiji, Alizadeh, 2014, p. 7)

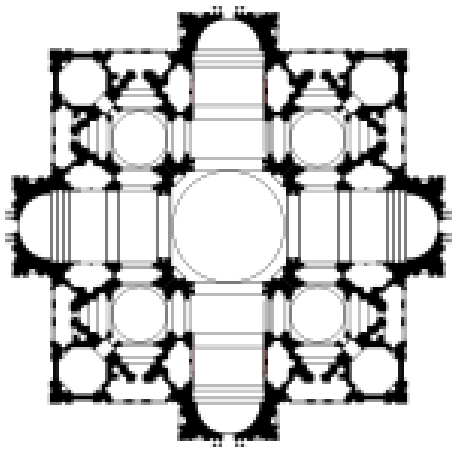

Figure 5. church of San Pietro-architect: Bramante (Rahim and Hosseini , 2009, p. 166)

Houser. Fractals have two tangible features showing complexity in any scale change and not smooth edges and intersections but jagged and intertwined [6]. For instance, the present research can mention the fractal buildings such as various medieval palaces, unbalanced and disproportionate churches at eighteenth century , Hindu temples and works of Frank Lloyd Wright or Louis Sullivan that have one direct and tangible feature of fractal geometry. It cannot be still considered as the works that had been made merely super-fractal. Architecture and memorable buildings of the eastern south Asian and India display the fractal structures. In these buildings, any leading tower is enclosed with a series of towers and each of those towers is enclosed with smaller towers, which this process continues to eight stages or more. They extended these buildings by increasing their height and placed a holy building at the depth of it. All the hindu architecture assumes as thousands of trees with thousands of branches that fruits have grown on each branch and each fruit has followed the pattern of branches and the branches have followed the pattern of tree[7].

George Hersey, Professor of History of Art-University Bill, knows the plan of church

Figure 6. fractal in mogharnas and Iranian architecture decorations(Rahimi & Hosseini, 2009, p. 165)

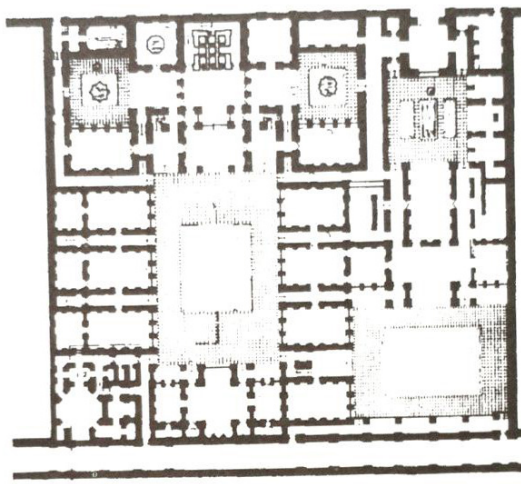

Figure 7. Using fractal geometry in Iranian house: The iteration of the same squares at various sizes (Source: author);

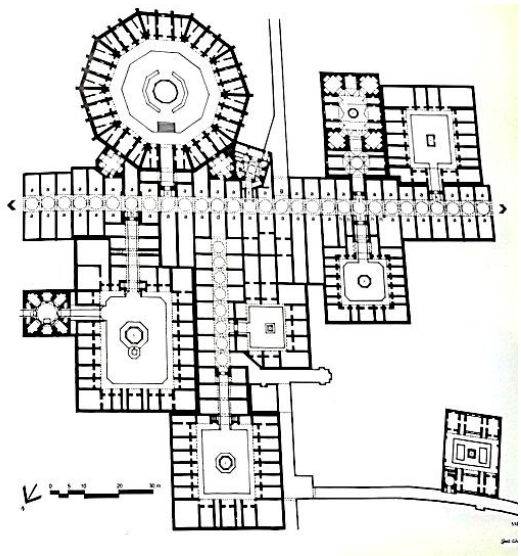

Figure 8. Using fractal geometry in the bazaar: iteration of the same squares at various sizes (source: the author)

of San Pietro by Donato Bramante with the fractal features such as iterative mode at different scales. symmetry of the interior spaces and crossed arms in four sides have dents which form the leading cube of church(figure 9). arms of smaller cross also include smaller dents[1].

The buildings of the past and vernacular architecture throughout the world have numerous commonalities in sense which we perceive that one of the reasons lies on type of scaling in them. Hierarchical feature at various scales has given this ability to consider the largest scale including cities to the smallest scale including details of design and maintain the linkage between the whole and part at various scales. At the middle and micro scale, it can refer to use of fractal geometry in architectural structures. Literature of fractal structures in the single buildings dates back to the geometric temples, the churches at the 18th century, Iranian fire temples, Islamic mosques and even before this century when the man formed his settlements by mimicry from nature around him.

Overview of fractal in Iran's architecture

Fractal in generalities: plan and symbols of

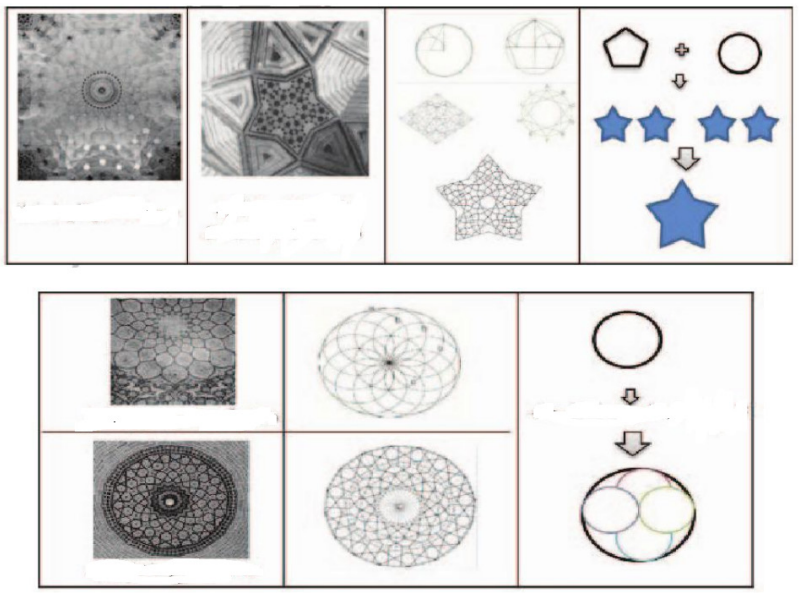

Figure 9. Taj al-Mulk dome, Mosque of Isfahan, the principle of self-similarity through the proportionality between the whole and part (Belian and Sattar Zadeh, 2011, p. 88)

Iranian architecture

Fractal geometry can be witnessed in Iran's past architecture. In architecture of Iran's mosques, use of particular arches in leading ivan and use of that arch at smaller scale in the entrance and use of those arches at another scale in both sides of ivans are clear. Numerous fractal sets are witnessed in Iran's markets[8].

The most superior sample of time fractals develop when the iteration operations are fulfilled in them in a way that the ultimate form approaches to the early form. In other words, Stationary includes the shapes that keep up with iteration, that is, the more the ultimate form gets similar to the early form, the stationary will be more excellent. Sierpinski triangle refers to the most excellent Iterative formation. With an outlook to these shapes, it can perceive that iteration in them exists in the most tangible form. Formation of a major part of Iranian architecture such as Iranian market and Iranian house can be elaborated based on fractal features such as iteration and square-shaped form, deduced that iteration of similar squares at different sizes is obviously witnessed in the geometric structure of these buildings that have spread based on symmetry and rotation along the diameter. Heritable point in geometric structure of this architecture lies on similarity of the ultimate form of plan with the components inside it so that iteration has occurred in them[9].

Fractal in details and decorations

In general, decorations (motifs) in Iranian architecture develop from a basic form, i.e. the basic form which can be exposed to transformation by expansion to different forms, under which a basic form can be the basis for formation of a complicated form and motif which deforms by transformation to a complicated form. All the Iranian motifs are classified to two groups of geometric and non-geometric motifs that have similarity with natural fractal **shapes in the features below:**

-Self-similarity

Henry believes that the pattern of Iran's geometric decorations uses an iteration principle in which the self-similar shapes are decomposed to smaller copies of them. Complicated geometry of Islamic architecture indicates the artists' effort to express feelings through complicated geometric plans which encompass iteration, tune, scale and composition [10].

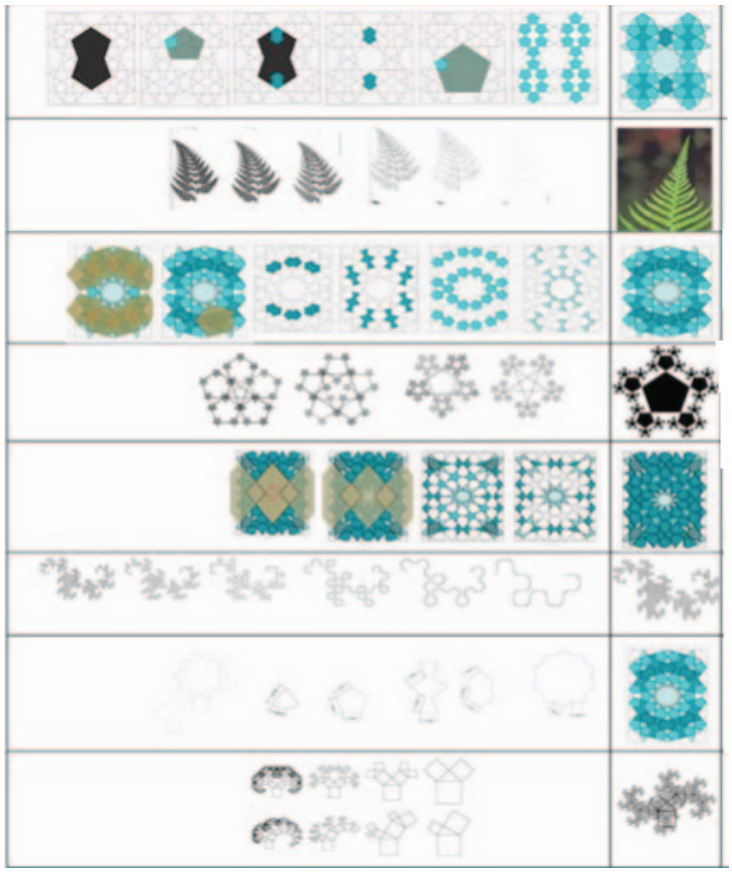

Figure 10. Display of self-similarity, iteration, symmetry and proportionality in Islamic architecture (Belian, Sattar zadeh, 2011, p. 92)

Geometric structure inside the domes indicates that they have been replete with the information on Euclidean three-dimensional space geometry; these plans by means of self-similarity indicate that these artists were informed of concept of fractal geometry [10].

Sub-scale

As seen in image above, fractals seem at the same scale; the fractals have been developed from the sub-sets which include larger sets. These sets have been developed from smaller sub-sets. These sub-sets are similar to larger sets which are called sub-scale [11].

Fractal dimensions

Mandelbrot proposed a simple but radical way to qualify fractal geometry through a fractal dimension based on a discussion of the length of the coast of England. The dimension is a statistical quantity that gives an indication of how completely a fractal appears to fill space, as one zooms down to finer scales. This definition is a simplification of the Hausdorff dimension that Mandelbrot used as a basis. We focus on this one and briefly mentions box-counting dimension because of its wide practical applications. However, it should be noted that there are many specific definitions of fractal dimensions, such as Hausdorff dimension, Rényi dimensions, box-counting dimension and correlation dimension, etc, and none of them should be treated as the universal one.

In figure below, Fractal dimensions are 1.456 like previous figure because any alternative piece has been developed from five elements which have been downsized with ratio of 1/3. Therefore, however the fractal plan in two previous images has a different form, but it has the same fractal dimension (1.456).

dimension

The reasons to used fractal geometry in Islam-

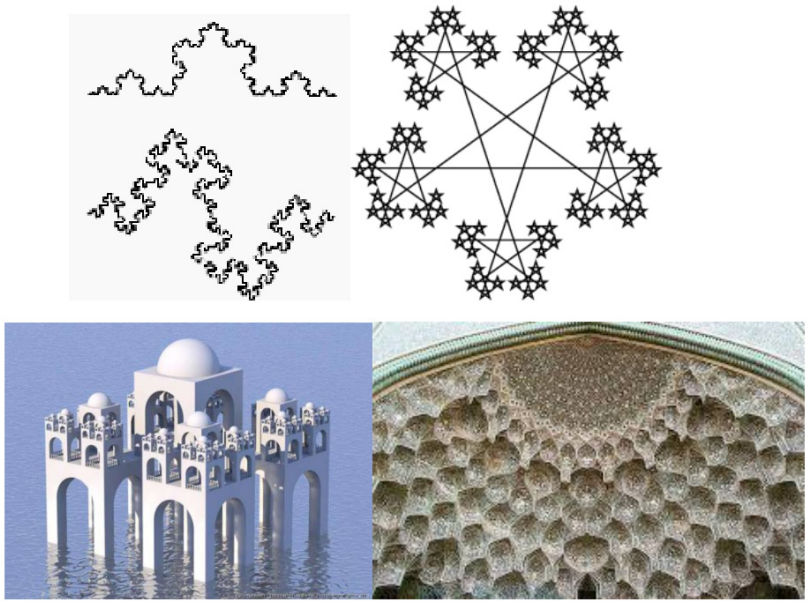

Figure 11. A sample of fractal geometry in terms of sub-scale (Sarami, Hadian pour & Nosrati, 2014, p. 8)

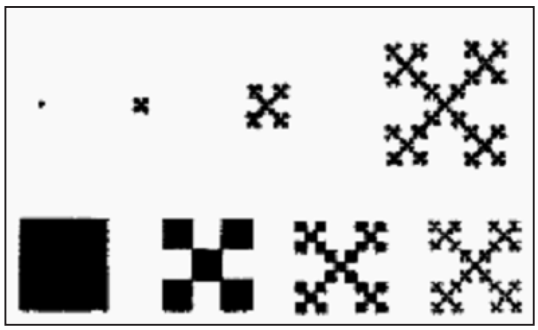

Figure 12. A sample of fractal geometry in terms of Fractal dimensions (Mirian, 2011, p. 115)

ic architecture and motifs

Concerning emergence of fractals in Iranian and Islamic architecture and art which results in fractal patterns, there are numerous conceptual and applied reasons of which it can refer to the following:

-use of this geometry as a creative instrument: geometry plays a major role in design of Iranian architecture. From the perspective of external function, use of geometry as the art for creation of shapes, patterns and proportions reminds the great architect of the world and calls the images. Therefore, the art of geometry has had a key element to make relationship between the building and the ideas that the designer has in his mind.

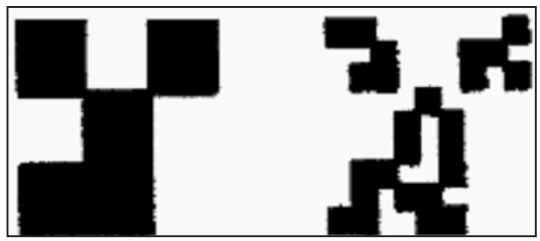

Figure 13. Fractal geometry in terms of Fractal

-use of fractal geometry for structural sustainability

From perspective of internal function, geometry as the science to select structural dimensions such as height, length and width of building and the structural components govern the structural behavior of the building. The geometry from rules of nature raises a structural sustainability.

-use of the fractal geometry to create details at various scales

Use of count box dimension addressed by Carl Boil in the book "fractal geometry" in architecture and design has been a principle of the nature design principles, so that its linkage with fractal geometry is the feature for scaling details in natural fractals. It can elaborate this principle in this way that size of details has been selected regarding the distance from the object and the extent of size which can be

seen by our vision system. Use of this principle can display fractal geometry in human scale. Indeed, this principle criticizes on buildings with flat walls that are not seen just as a tall wall in the modern age[14].

Visualization of the heaven and the human environment

The plant and decorative motifs in the buildings represent the heaven and heaven plants, however, they date back to pre-Islamic age. However the decorations do not play a major role in stability of building, they have been inseparable part of Iranian architecture.

Use of fractal geometric motifs as unity in plurality

To traditional architect, geometric patterns are as plurality forms in unity. Iterative patterns are the symbol for infinite and endless idea and the beauty which is observed in geometric patterns reflects a deeper geometric order, i.e. cosmic rules. Spiritual man tries to discover geometric patterns as a means to get close to God[13]. For this, without any knowledge, the nature and fractal patterns are inspired to reflect this iteration and plurality to achieve unity.

Conclusion

The investigations indicate that the nature is inspired in the works of architects, thus the architects have used fractal geometry inspired of geometry of nature for making geometrical shapes in building. Use of fractal geometry in architecture indicates the man's tendency to works of creation. It can observe that fractal geometry has been common since the formation of Iranian art and architecture and titles entitled fractal geometry is the new interpretation in the recent contemporary period. From point of view of Iranian architect, abstract art manifests unity in plurality. Iranian architects used the fractal geometry for several reasons including use of geometry for creation of diversity, collection, analysis of structural stability, use of this geometry to create details at various scales, visualization of heaven and human living environment and geometric motifs as unity in plurality. According to the investigations, it can observe that as fractal geometry can be used in macro-scales such as urbanization, it can be used in micro-scales such as architecture (plan and façade) and details of architecture(motifs and decorations). Ultimately it can deduce that fractal geometry of nature can be the best way for design of qualitative architecture, simple fractal form cannot create a value by mimicry from nature. Geometry of nature can pave the way for qualitative architecture and spaces well suited to new human needs.

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