

# Basic Electrical Engineering

By: M. Shahraki

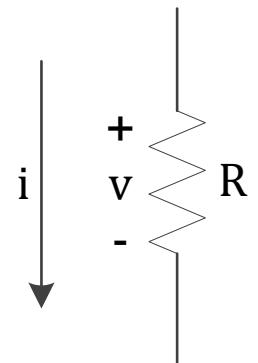


University of Sistan & Baluchestan  
Faculty of Electrical and Computer Engineering  
Department of Electrical & Electronics Engineering

# Transient state

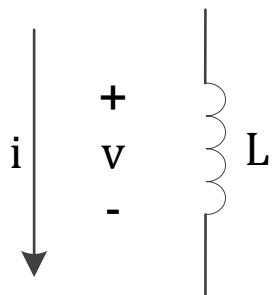
حالت گذرا

رفتار عناصر مدار



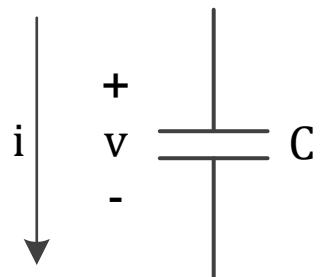
$$v = Ri$$

$$i = \frac{1}{R} v = Gv$$



$$v = L \frac{di}{dt}$$

$$i = \frac{1}{L} \int v dt + i_0$$



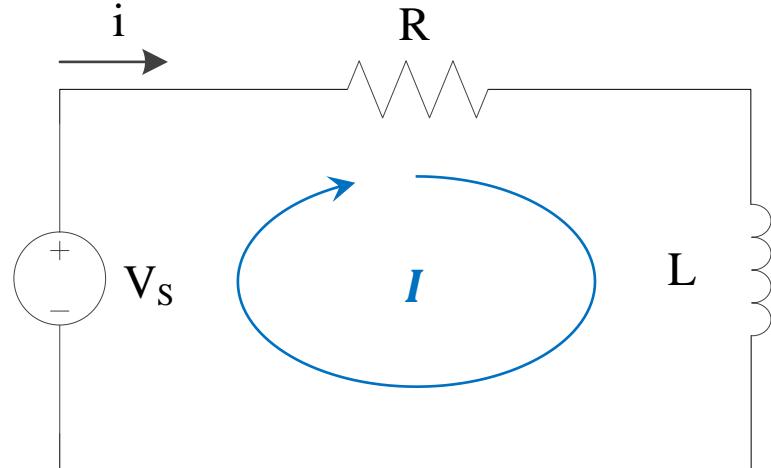
$$v = \frac{1}{C} \int i dt + v_0$$

$$i = C \frac{dv}{dt}$$



# Transient state

حالت گذرا



حالت دائمی ثابت

$$-V_S + V_R + V_L = 0$$

$$V_R = RI$$

$$V_L = L \frac{di}{dt}$$

حالت دائمی ثابت  $\leftarrow$  مدار مدت زمان زیادی بدون تغییر است  $\leftarrow$  تغییرات وجود ندارد

$$-V_S + RI + 0 = 0$$

$$I = \frac{V_S}{R}$$

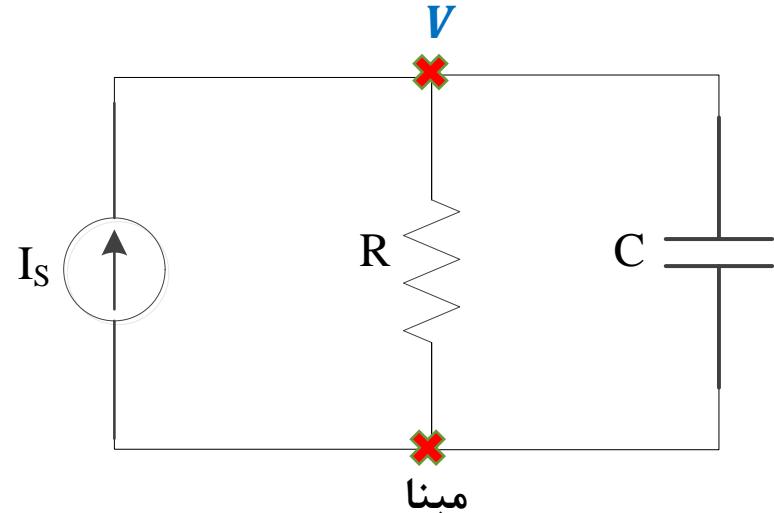
$$I_L = I_R = \frac{V_S}{R}$$

سلف در زمان بی نهایت  $\leftarrow$  اتصال کوتاه با جریان  $I_0$  است.



# Transient state

حالت گذرا



$$I_C = 0 \quad \frac{dV}{dt} = 0$$

$$-I_S + \frac{V}{R} + 0 = 0$$

$$V = RI_S$$

$$V_C = V_R = RI_S$$

حالت دائمی ثابت

$$-I_S + I_R + I_C = 0$$

$$I_R = \frac{V}{R} \quad I_C = C \frac{dV}{dt}$$

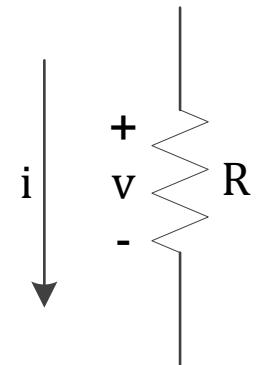
حالت دائمی ثابت  $\leftarrow$  مدار مدت زمان زیادی بدون تغییر است  $\leftarrow$  تغییرات وجود ندارد

خازن در زمان بی نهایت  $\leftarrow$  اتصال باز با ولتاژ  $V_0$  است.



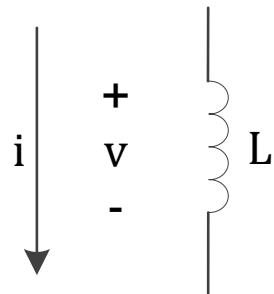
# Transient state

حالت گذرا



$$V_R = RI_R$$

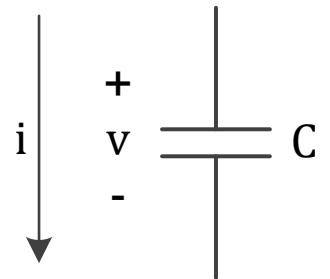
$$I_R = \frac{1}{R} V_R$$



$$V_L = 0$$

$$I_L = cte$$

حالت دائمی ثابت (رفتار در زمان بی نهایت)



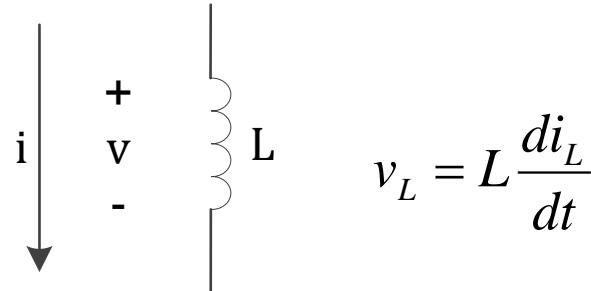
$$I_C = 0$$

$$V_C = cte$$

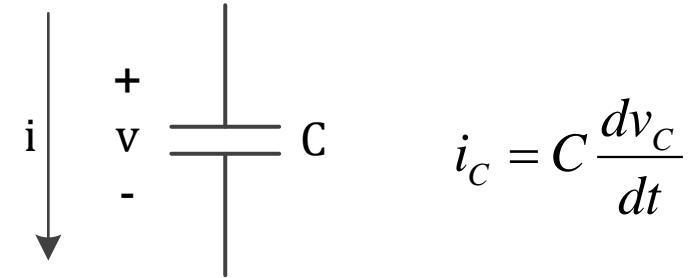


# Transient state

حالت گذرا

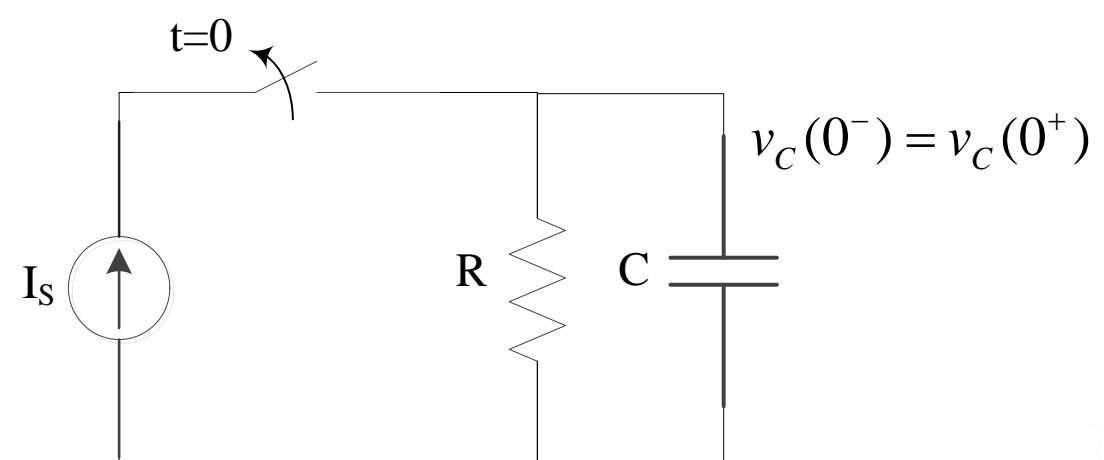
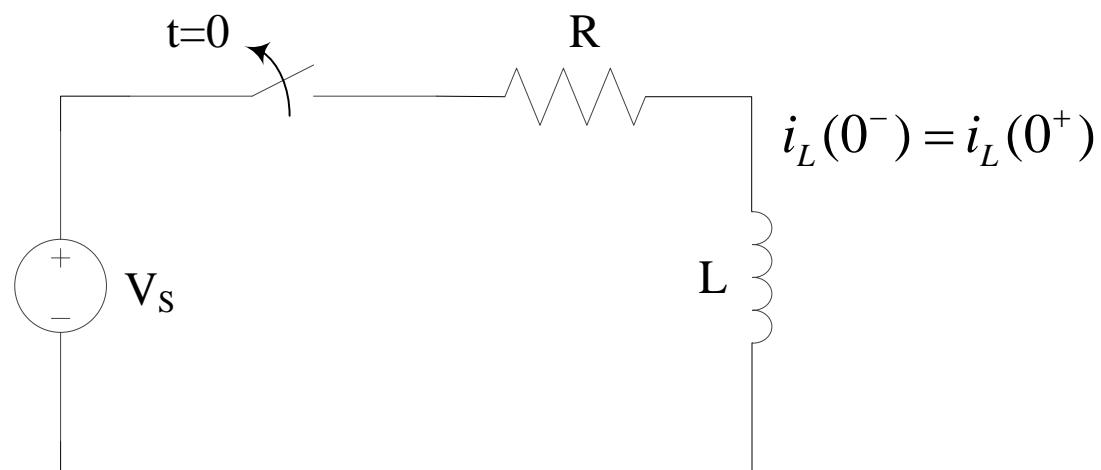


حالت گذرا



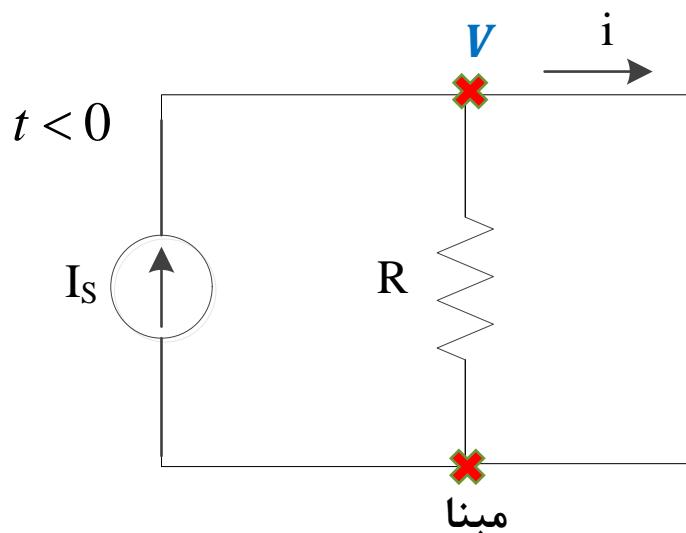
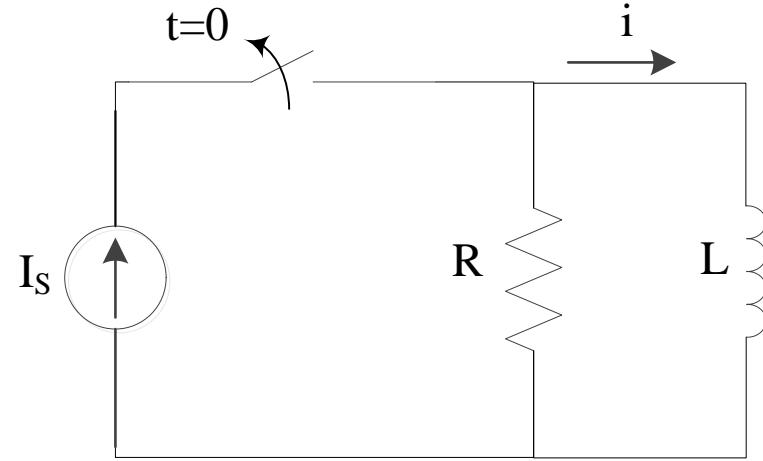
$v_L \neq \infty$  جریان سلف تعییر ناگهانی ندارد

$i_C \neq \infty$  ولتاژ خازن تعییر ناگهانی ندارد



# Transient state

حالت گذرا

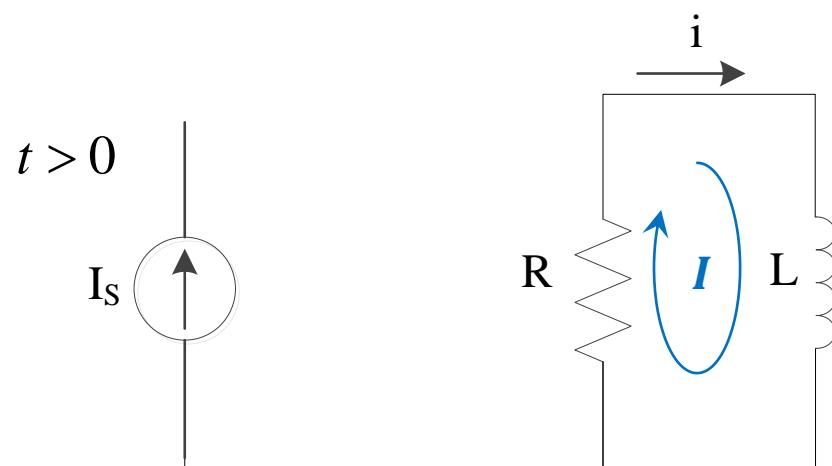


حالت گذرا

سلف اتصال کوتاه

$$-I_s + \frac{0}{R} + I = 0$$

$$I = I_s \quad i_L(0^-) = I_s$$



$$Ri + L \frac{di}{dt} = 0 \quad i_L(0^+) = i_L(0^-) = I_s$$

# Transient state

حالت گذرا

حالت گذرا

$$Ri + L \frac{di}{dt} = 0$$

$$\frac{di}{i} = -\frac{R}{L} dt$$

$$i_L(0^+) = I_S$$

$$\int_{i(0^+)}^{i(t)} \frac{di}{i} = - \int_0^t \frac{R}{L} dt$$

$$\ln[i(t)] - \ln[i(0^+)] = -\frac{R}{L}[t - 0]$$

$$\ln \frac{i(t)}{i(0^+)} = -\frac{R}{L} t$$

$$\frac{i(t)}{i(0^+)} = e^{-\frac{R}{L}t}$$

$$i(t) = i(0^+) e^{-\frac{R}{L}t}$$

$$i(t) = I_S e^{-\frac{R}{L}t}$$

$$i(t) = I_0 e^{-\frac{t}{\tau}}$$

$$I_0 = I_S$$

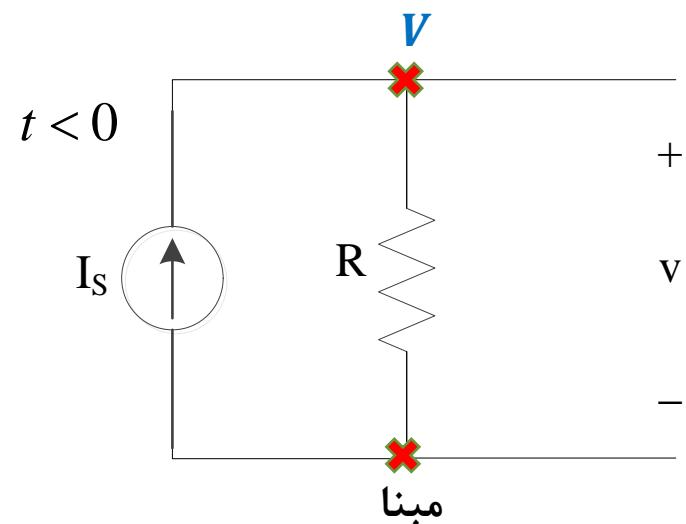
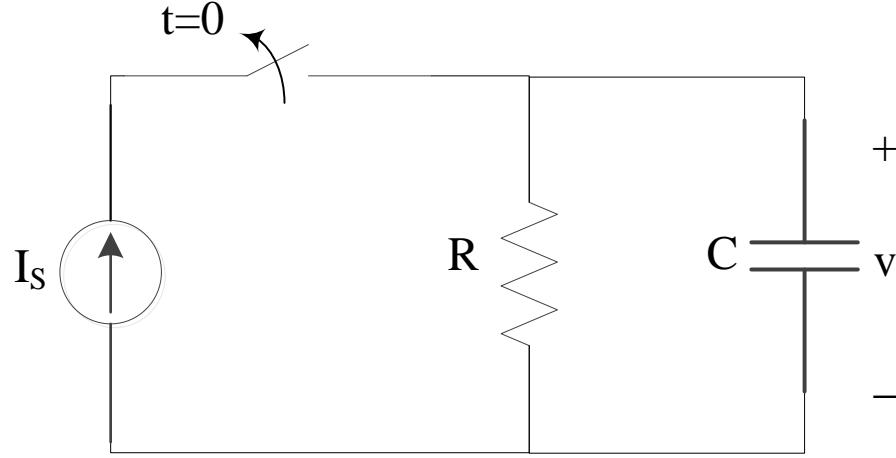
$$\tau = \frac{L}{R}$$

ثابت زمانی



# Transient state

حالت گذرا

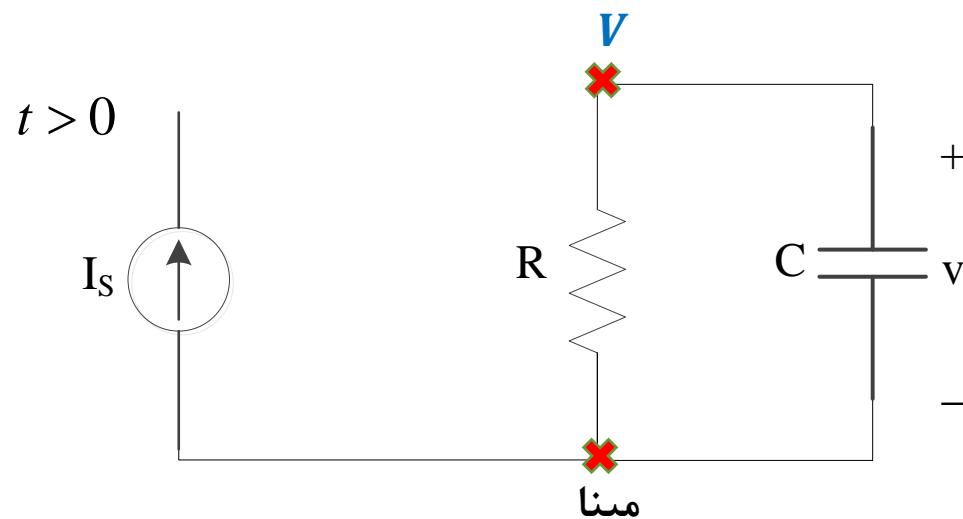


حالت گذرا

خازن اتصال باز

$$-I_S + \frac{V}{R} = 0$$

$$V = RI_S \quad v_C(0^-) = RI_S$$



$$i_R + i_C = 0$$

$$\frac{v}{R} + C \frac{dv}{dt} = 0$$

$$v_C(0^+) = v_C(0^-) = RI_S$$



# Transient state

حالت گذرا

$$\frac{v}{R} + C \frac{dv}{dt} = 0$$

$$\frac{dv}{v} = -\frac{1}{RC} dt$$

$$v_C(0^+) = v_C(0^-) = RI_s$$

حالت گذرا

$$\int_{v(0^+)}^{v(t)} \frac{dv}{v} = - \int_0^t \frac{1}{RC} dt$$

$$\ln[v(t)] - \ln[v(0^+)] = -\frac{1}{RC}[t - 0]$$

$$\ln \frac{v(t)}{v(0^+)} = -\frac{1}{RC} t$$

$$\frac{v(t)}{v(0^+)} = e^{-\frac{1}{RC}t}$$

$$v(t) = v(0^+) e^{-\frac{1}{RC}t}$$

$$v(t) = RI_s e^{-\frac{1}{RC}t}$$

$$v(t) = V_0 e^{-\frac{t}{\tau}}$$

$$V_0 = RI_s$$

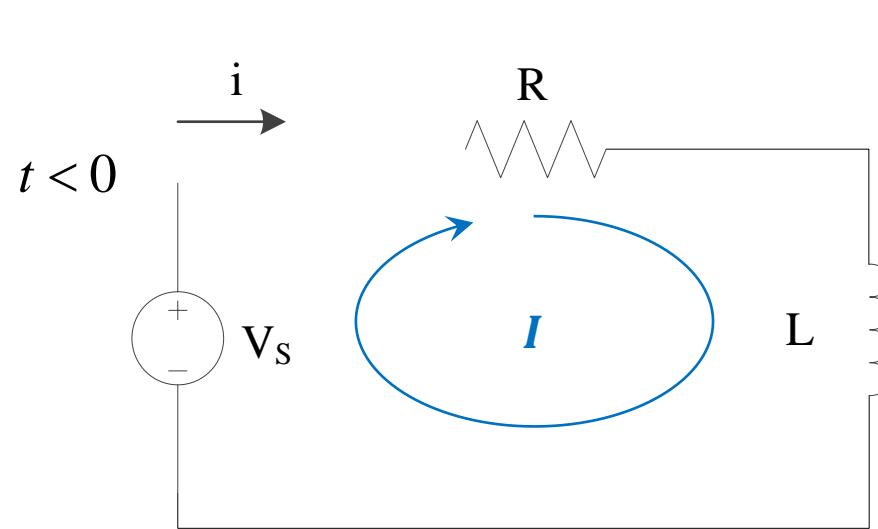
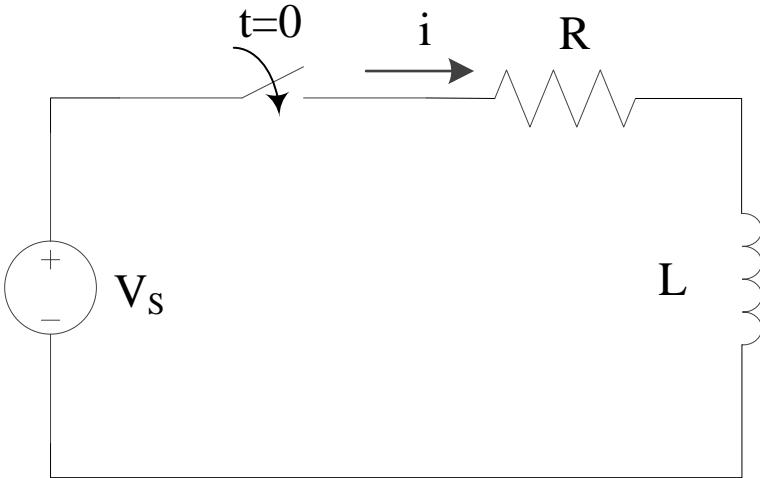
$$\tau = RC$$

ثابت زمانی



# Transient state

حالت گذرا

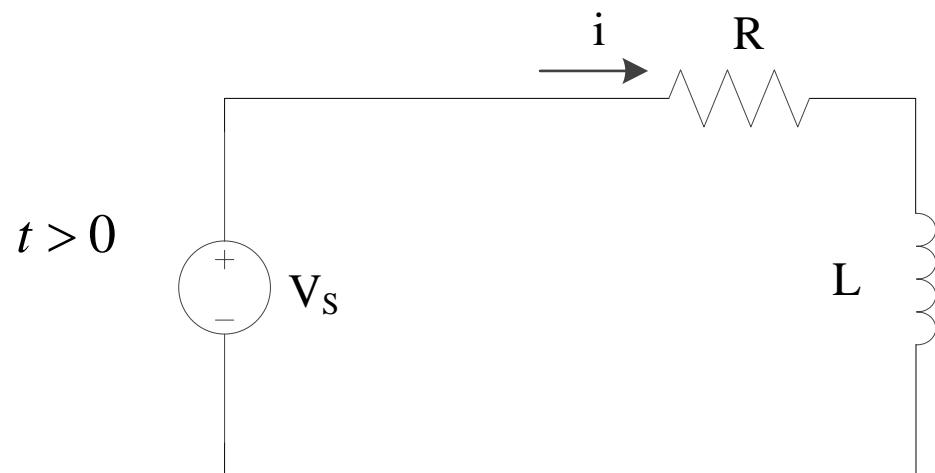


حالت گذرا

سلف اتصال کوتاه

$$I = 0$$

$$i_L(0^-) = 0$$



$$-V_s + V_R + V_L = 0$$

$$i_L(0^+) = i_L(0^-) = 0$$

$$Ri + L \frac{di}{dt} = V_s$$



# Transient state

حالت گذرا

$$Ri + L \frac{di}{dt} = V_s$$

جواب عمومی

$$Ri + L \frac{di}{dt} = 0$$

$$\frac{di}{i} = -\frac{R}{L} dt$$

حالت گذرا

$$\int_{i(0^+)}^{i(t)} \frac{di}{i} = - \int_0^t \frac{R}{L} dt$$

$$\ln[i(t)] - \ln[i(0^+)] = -\frac{R}{L}[t - 0]$$

$$\ln \frac{i(t)}{i(0^+)} = -\frac{R}{L} t$$

$$\frac{i(t)}{i(0^+)} = e^{-\frac{R}{L}t}$$

$$i(t) = i(0^+) e^{-\frac{R}{L}t}$$

$$i(t) = K_1 e^{-\frac{R}{L}t}$$

$$Ri + L \frac{di}{dt} = V_s$$

جواب خصوصی معادله

$$i = cte = K_2$$

$$i(t) = K_2 + K_1 e^{-\frac{R}{L}t}$$



# Transient state

حالت گذرا

$$i(t) = K_2 + K_1 e^{-\frac{R}{L}t}$$

$$i_L(0^+) = 0$$

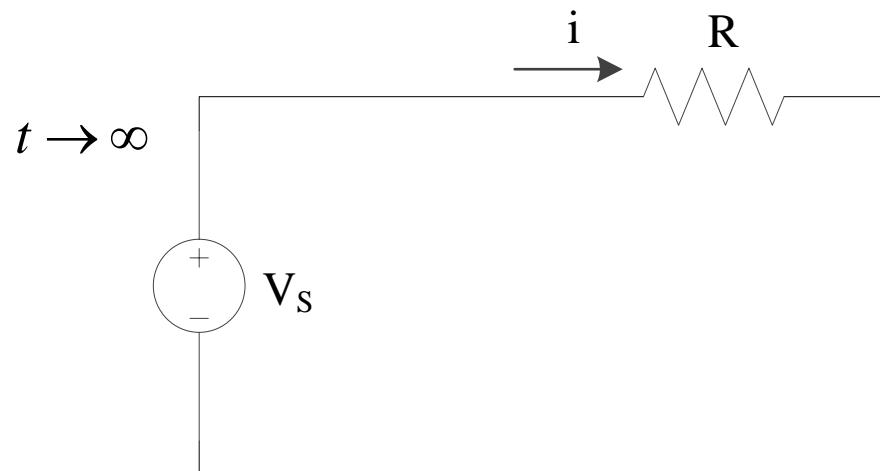
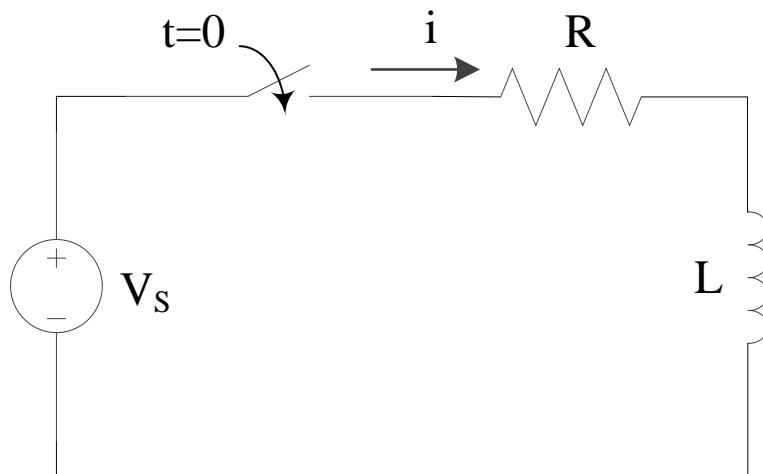
$$i_L(\infty) = \frac{V_s}{R}$$

حالت گذرا

$$K_2 = \frac{V_s}{R}$$

$$K_1 + K_2 = 0$$

$$i(t) = \frac{V_s}{R} (1 - e^{-\frac{R}{L}t})$$



سلف اتصال کوتاه

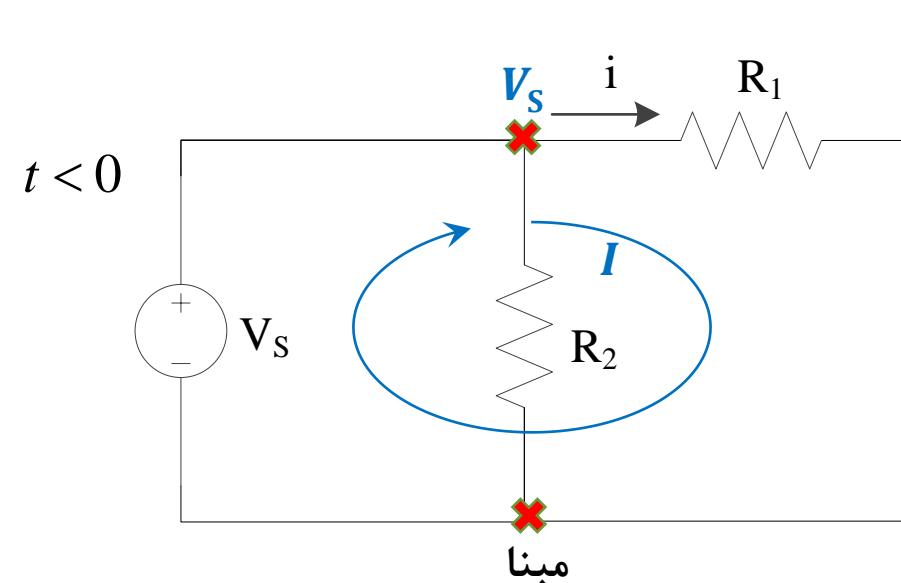
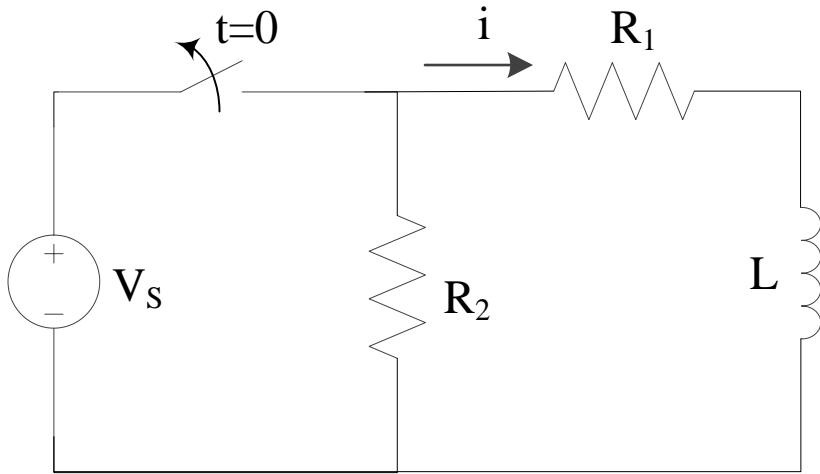
$$I = \frac{V_s}{R}$$

$$i_L(\infty) = \frac{V_s}{R}$$



# Transient state

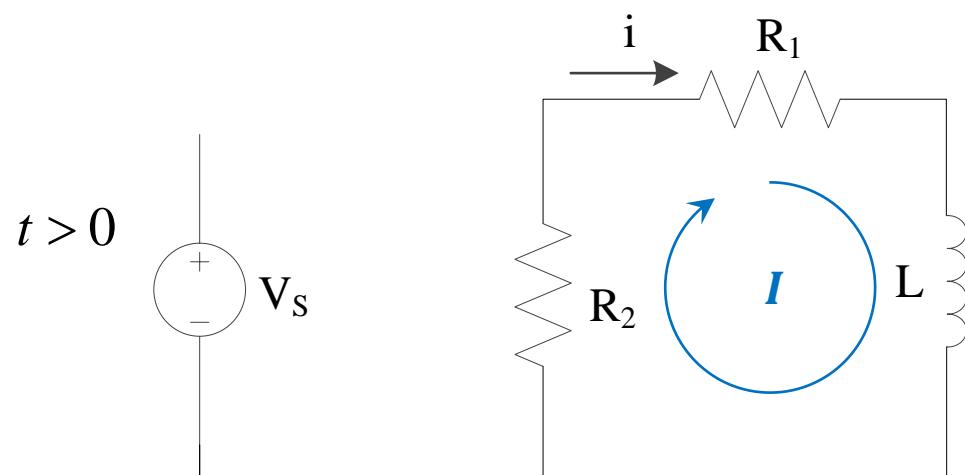
حالت گذرا



سلف اتصال کوتاه

$$I = \frac{V_s}{R_1}$$

$$i_L(0^-) = \frac{V_s}{R_1}$$



$$R_1 i + R_2 i + L \frac{di}{dt} = 0 \quad i_L(0^+) = i_L(0^-) = \frac{V_s}{R_1}$$



# Transient state

حالت گذرا

$$R_1 i + R_2 i + L \frac{di}{dt} = 0$$

$$\frac{di}{i} = -\frac{(R_1 + R_2)}{L} dt$$

$$i_L(0^+) = \frac{V_S}{R_1}$$

حالت گذرا

$$\int_{i(0^+)}^{i(t)} \frac{di}{i} = - \int_{0^+}^t \frac{(R_1 + R_2)}{L} dt \quad \ln[i(t)] - \ln[i(0^+)] = -\frac{(R_1 + R_2)}{L} [t - 0] \quad \ln \frac{i(t)}{i(0^+)} = -\frac{(R_1 + R_2)}{L} t$$

$$\frac{i(t)}{i(0^+)} = e^{-\frac{(R_1 + R_2)}{L} t}$$

$$i(t) = i(0^+) e^{-\frac{(R_1 + R_2)}{L} t}$$

$$i(t) = \frac{V_S}{R_1} e^{-\frac{(R_1 + R_2)}{L} t}$$

$$i(t) = I_0 e^{-\frac{t}{\tau}}$$

$$I_0 = \frac{V_S}{R_1}$$

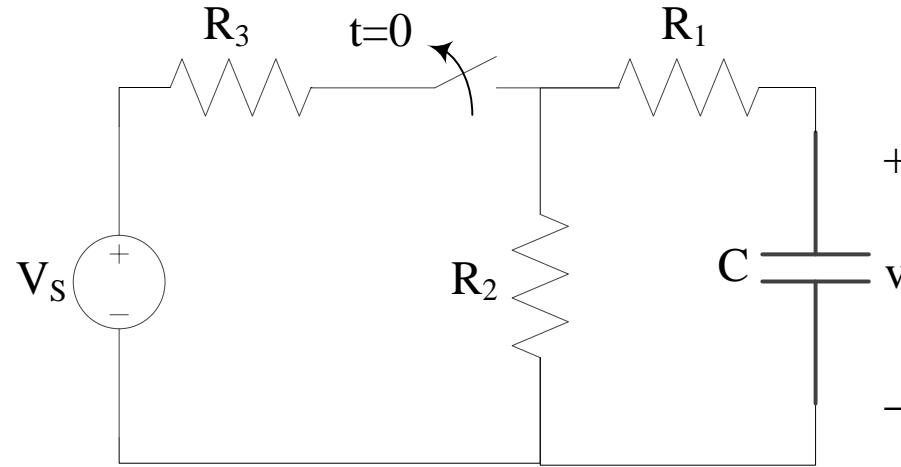
$$\tau = \frac{L}{(R_1 + R_2)}$$

ثابت زمانی

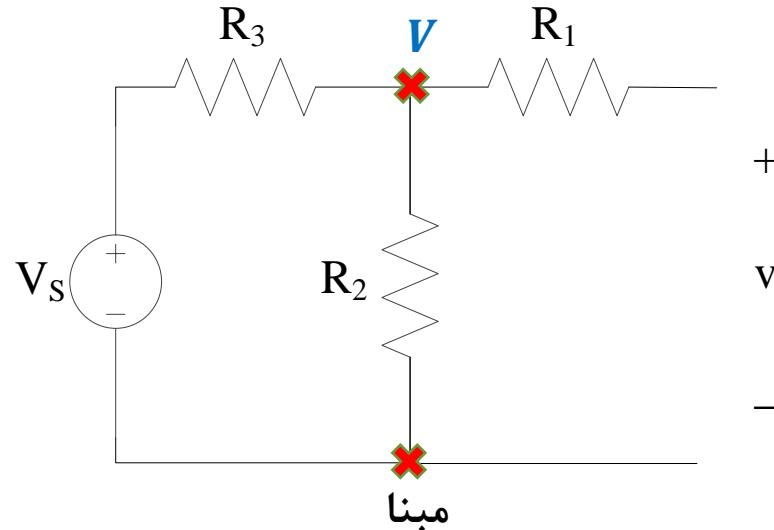


# Transient state

حالت گذرا



$t < 0$



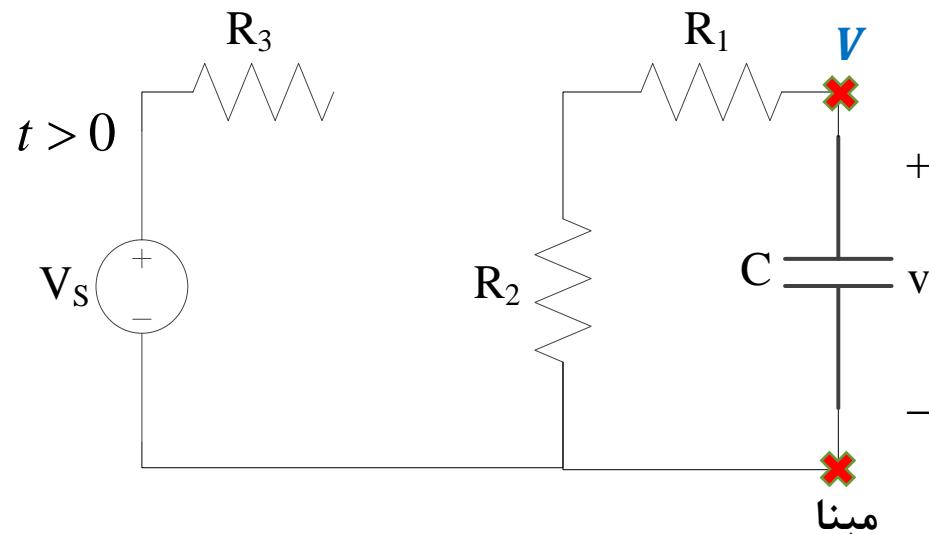
حالت گذرا

خازن اتصال باز

$$\frac{V - V_s}{R_3} + \frac{V}{R_2} = 0$$

$$V = \frac{R_2}{R_2 + R_3} V_s$$

$$v_C(0^-) = \frac{R_2}{R_2 + R_3} V_s$$



$$i_{R1} + i_C = 0$$

$$\frac{v}{R_1 + R_2} + C \frac{dv}{dt} = 0$$

$$v_C(0^+) = v_C(0^-) = \frac{R_2}{R_2 + R_3} V_s$$



# Transient state

حالت گذرا

$$\frac{v}{R_1 + R_2} + C \frac{dv}{dt} = 0 \quad \frac{dv}{v} = -\frac{1}{(R_1 + R_2)C} dt$$

$$v_C(0^+) = v_C(0^-) = \frac{R_2}{R_2 + R_3} V_s$$

حالت گذرا

$$\int_{v(0^+)}^{v(t)} \frac{dv}{v} = - \int_{0^+}^t \frac{1}{(R_1 + R_2)C} dt \quad \ln[v(t)] - \ln[v(0^+)] = -\frac{1}{(R_1 + R_2)C} t \quad \ln \frac{v(t)}{v(0^+)} = -\frac{1}{(R_1 + R_2)C} t$$

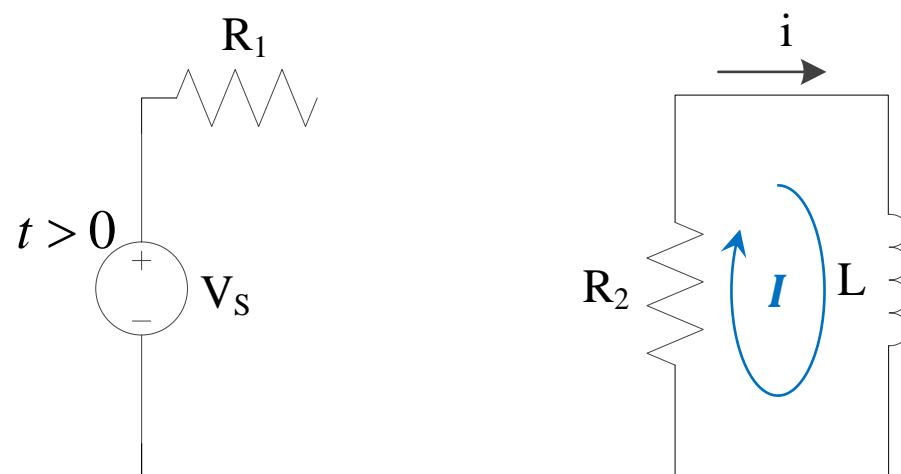
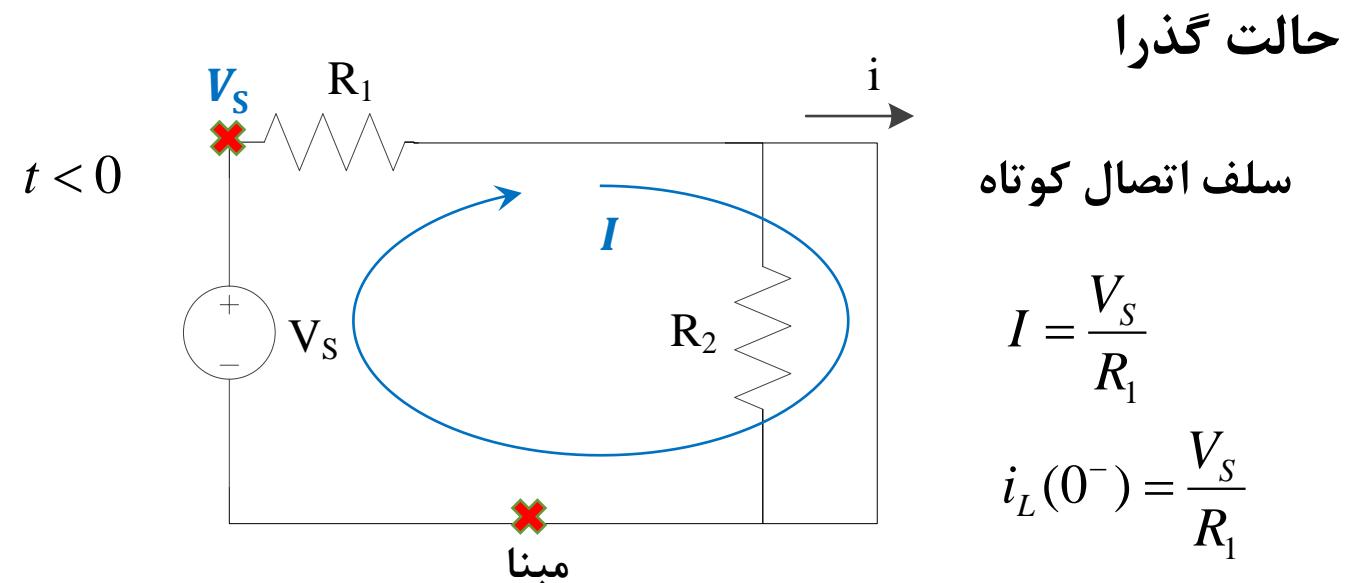
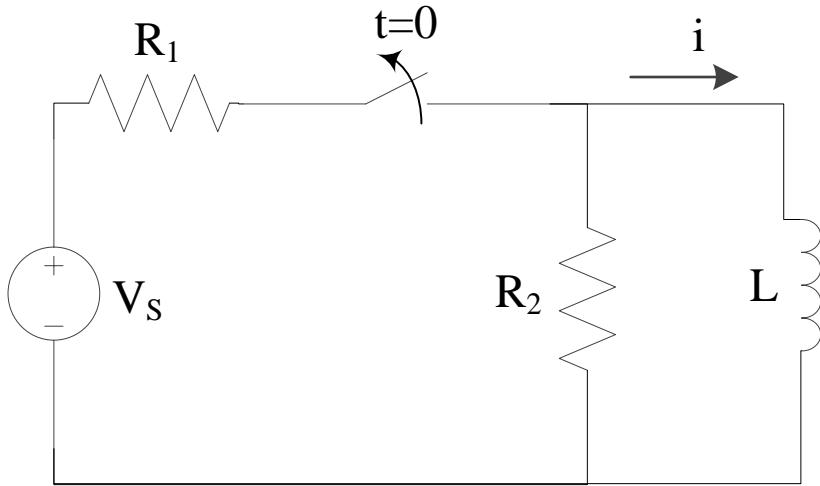
$$\frac{v(t)}{v(0^+)} = e^{-\frac{1}{(R_1 + R_2)C} t} \quad v(t) = v(0^+) e^{-\frac{1}{(R_1 + R_2)C} t} \quad v(t) = \frac{R_2}{R_2 + R_3} V_s e^{-\frac{1}{(R_1 + R_2)C} t}$$

$$v(t) = V_0 e^{-\frac{t}{\tau}} \quad V_0 = \frac{R_2}{R_2 + R_3} V_s \quad \tau = (R_1 + R_2)C \quad \text{ثابت زمانی}$$



# Transient state

حالت گذرا



$$R_2 i + L \frac{di}{dt} = 0 \quad i_L(0^+) = i_L(0^-) = \frac{V_s}{R_1}$$

# Transient state

حالت گذرا

$$R_2 i + L \frac{di}{dt} = 0$$

$$i_L(0^+) = \frac{V_S}{R_1}$$

حالت گذرا

$$i(t) = I_0 e^{-\frac{t}{\tau}}$$

$$I_0 = \frac{V_S}{R_1}$$

$$\tau = \frac{L}{R_2}$$

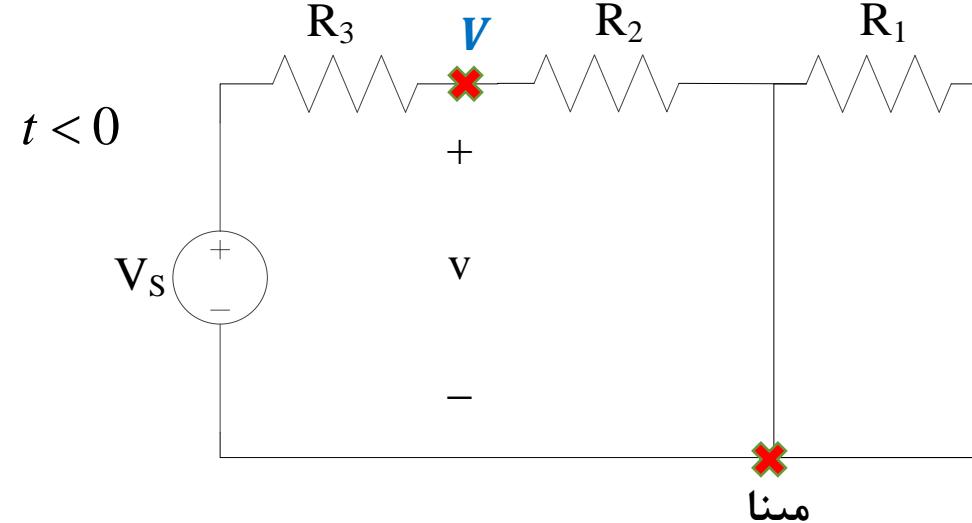
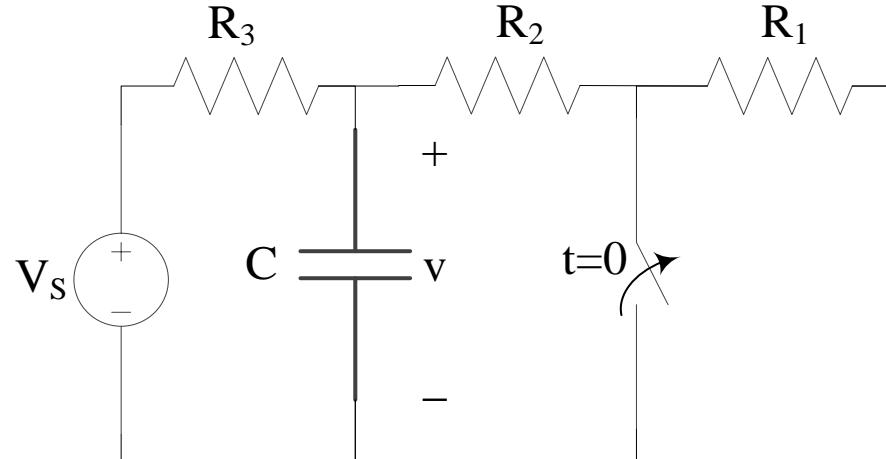
ثابت زمانی

$$i(t) = \frac{V_S}{R_1} e^{-\frac{R_2 t}{L}}$$



# Transient state

حالت گذرا



حالت گذرا

خازن اتصال باز

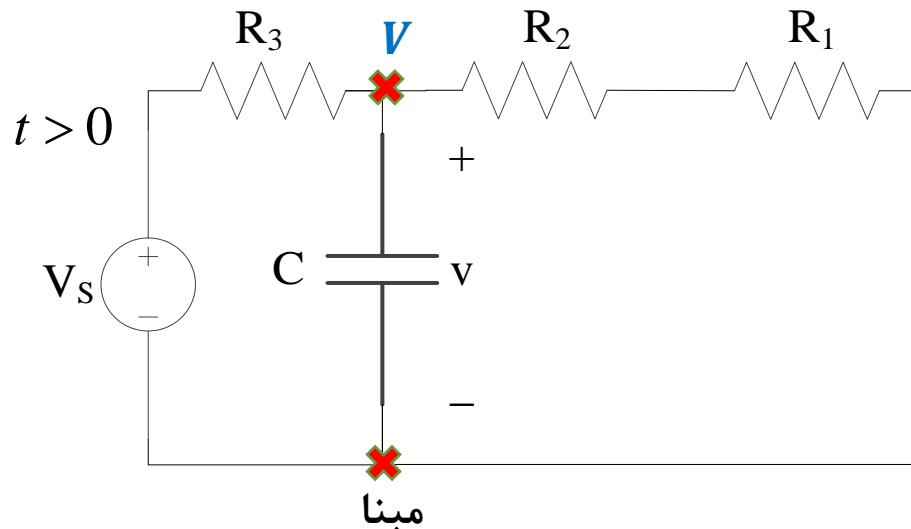
$$\frac{V - V_s}{R_3} + \frac{V}{R_2} = 0$$

$$V = \frac{R_2}{R_2 + R_3} V_s$$

$$v_C(0^-) = \frac{R_2}{R_2 + R_3} V_s$$

$$\frac{v - V_s}{R_3} + \frac{v}{R_1 + R_2} + C \frac{dv}{dt} = 0$$

$$v_C(0^+) = v_C(0^-) = \frac{R_2}{R_2 + R_3} V_s$$



# Transient state

حالت گذرا

$$\frac{v - V_s}{R_3} + \frac{v}{R_1 + R_2} + C \frac{dv}{dt} = 0$$

$$v_C(0^+) = \frac{R_2}{R_2 + R_3} V_s$$

حالت گذرا

$$\frac{v}{R_3} + \frac{v}{R_1 + R_2} + C \frac{dv}{dt} = \frac{V_s}{R_3}$$

$$v \left( \frac{1}{R_3} + \frac{1}{R_1 + R_2} \right) + C \frac{dv}{dt} = \frac{V_s}{R_3}$$

$$\frac{1}{R_T} = \left( \frac{1}{R_3} + \frac{1}{R_1 + R_2} \right)$$

$$R_T = R_3 \parallel (R_1 + R_2)$$

$$\frac{v}{R_T} + C \frac{dv}{dt} = \frac{V_s}{R_3}$$

$$v(t) = K_1 + K_2 e^{-\frac{t}{\tau}} \quad \tau = [(R_1 + R_2) \parallel R_3] C \quad \text{ثابت زمانی}$$

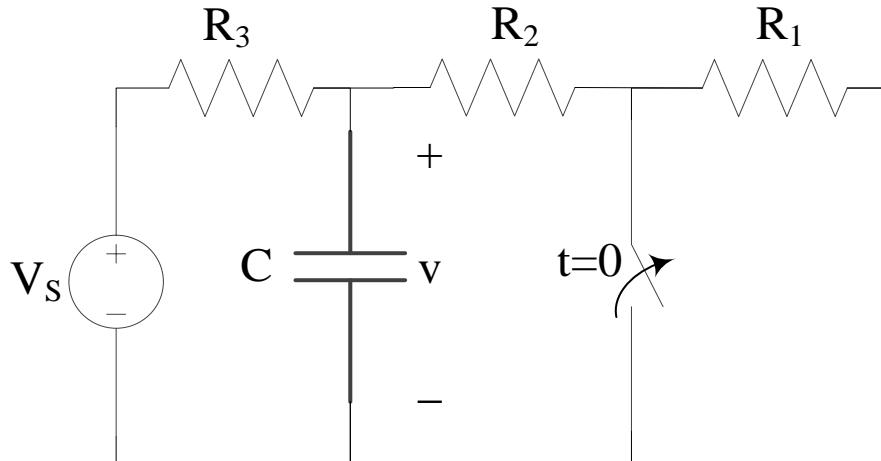
$$v_C(0^+) = \frac{R_2}{R_2 + R_3} V_s$$

$$K_1 + K_2 = \frac{R_2}{R_2 + R_3} V_s$$



# Transient state

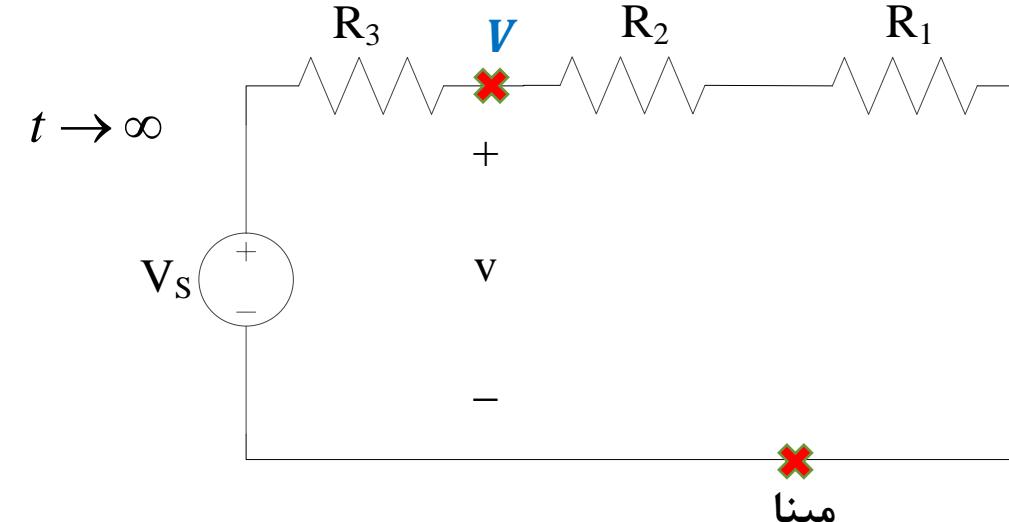
حالت گذرا



$$v(t) = K_1 + K_2 e^{-\frac{t}{\tau}}$$

$$K_1 = \frac{R_1 + R_2}{R_1 + R_2 + R_3} V_s$$

$$K_2 = \frac{R_2}{R_2 + R_3} V_s - \frac{R_1 + R_2}{R_1 + R_2 + R_3} V_s$$



حالت گذرا

خازن اتصال باز

$$\frac{V - V_s}{R_3} + \frac{V}{R_1 + R_2} = 0$$

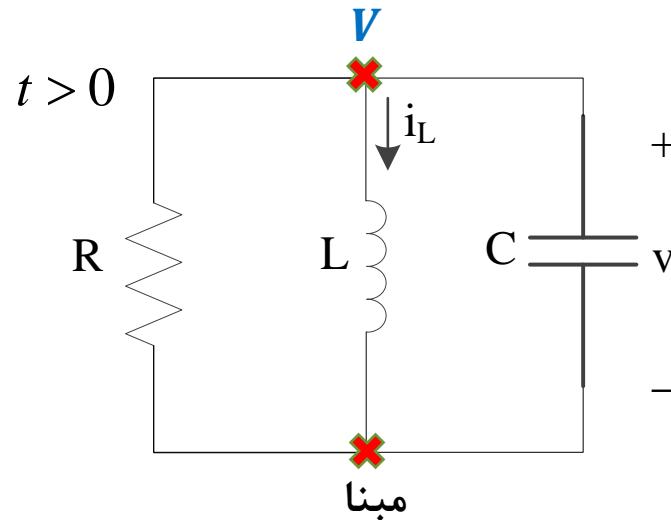
$$V = \frac{R_1 + R_2}{R_1 + R_2 + R_3} V_s$$

$$v_C(\infty) = \frac{R_1 + R_2}{R_1 + R_2 + R_3} V_s$$



# Transient state

حالت گذرا



مدارهای **RLC** موازی

$$i_L(0^+) = i_0$$

$$v_C(0^+) = v_0$$

$$i_R + i_L + i_C = 0$$

$$\frac{v}{R} + \frac{1}{L} \int_{0^+}^t v dt + i_L(0^+) + C \frac{dv}{dt} = 0$$

$$\frac{1}{R} \frac{dv}{dt} + \frac{v}{L} + C \frac{d^2v}{dt^2} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{v}{LC} = 0$$



# Transient state

حالت گذرا

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{v}{LC} = 0$$

$$v = Ke^{st}$$

مدارهای RLC موازی

$$Ks^2 e^{st} + \frac{Ks}{RC} e^{st} + \frac{Ke^{st}}{LC} = 0$$

$$Ke^{st} \left( s^2 + \frac{1}{RC} s + \frac{1}{LC} \right) = 0$$

$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

$$\alpha = \frac{1}{2RC} \quad \omega_0^2 = \frac{1}{LC}$$

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

معادله مشخصه

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$



# Transient state

حالت گذرا

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC}$$

$$\omega_0^2 = \frac{1}{LC}$$

**مدارهای RLC موازی**

$$\alpha > \omega_0$$

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha - \beta_n$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + \beta_n$$

دو ریشه حقیقی و مجزا

$$v = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$v = e^{-\alpha t} [K_1 e^{-\beta_n t} + K_2 e^{\beta_n t}]$$

پاسخ فوق میرا

$$\alpha < \omega_0$$

$$s_1 = -\alpha - j\sqrt{\omega_0^2 - \alpha^2} = -\alpha - j\omega_n$$

$$s_2 = -\alpha + j\sqrt{\omega_0^2 - \alpha^2} = -\alpha + j\omega_n$$

دو ریشه مختلط مزدوج

$$v = e^{-\alpha t} [K_1 e^{-j\omega_n t} + K_2 e^{+j\omega_n t}]$$

پاسخ زیرمیرا

$$\alpha = \omega_0$$

$$s_1 = -\alpha$$

$$s_2 = -\alpha$$

دو ریشه حقیقی یکسان

$$v = e^{-\alpha t} [K_1 t + K_2]$$

پاسخ میرای بحرانی



# Transient state

حالت گذرا

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC}$$

$$\omega_0^2 = \frac{1}{LC}$$

مدارهای RLC موازی

$$\alpha = 0$$

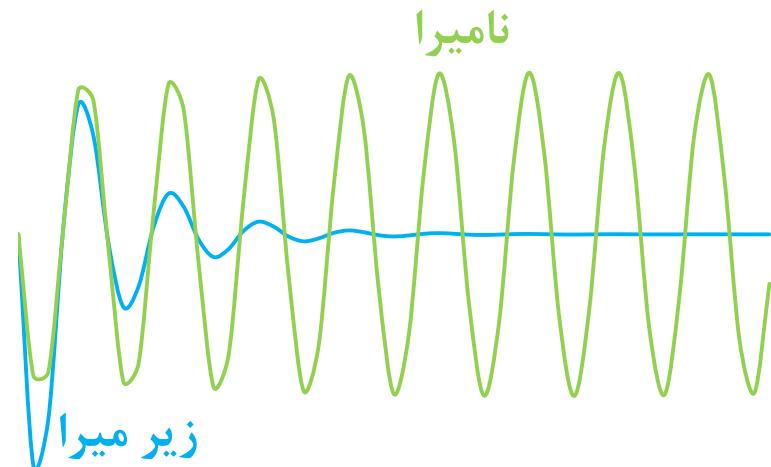
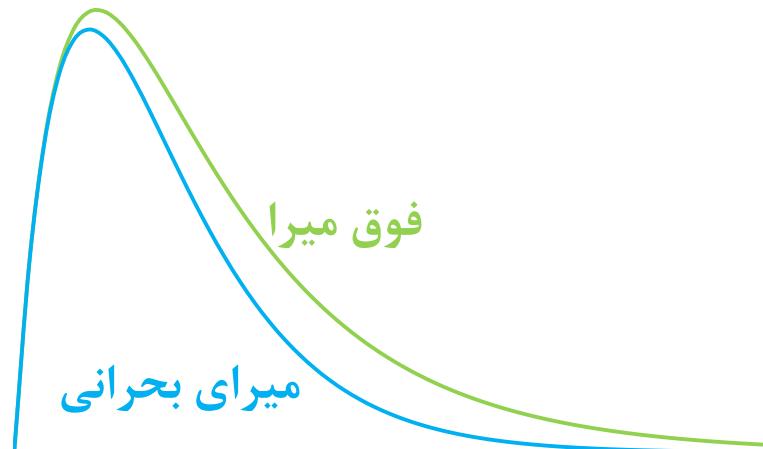
$$s_1 = -j\omega_0$$

$$v = [K_1 e^{-j\omega_0 t} + K_2 e^{j\omega_0 t}]$$

دو ریشه موهومی

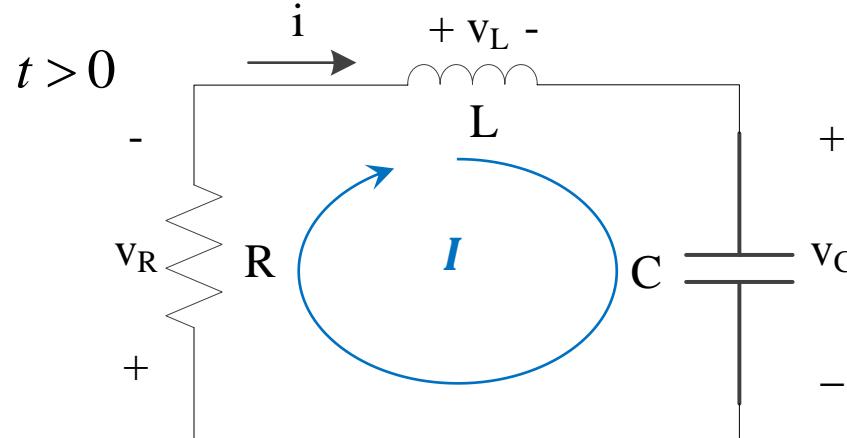
$$s_2 = +j\omega_0$$

پاسخ نامیرا



# Transient state

حالت گذرا



مدارهای RLC سری

$$v_R + v_L + v_C = 0$$

$$Ri + L \frac{di}{dt} + \frac{1}{C} \int_{0^+}^t idt + v_C(0^+) = 0$$

$$R \frac{di}{dt} + L \frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$$



# Transient state

حالت گذرا

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$$

$$i = Ke^{st}$$

مدارهای RLC سری

$$Ks^2 e^{st} + \frac{RKs}{L} e^{st} + \frac{Ke^{st}}{LC} = 0$$

$$Ke^{st} \left( s^2 + \frac{R}{L}s + \frac{1}{LC} \right) = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\alpha = \frac{R}{2L} \quad \omega_0^2 = \frac{1}{LC}$$

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

معادله مشخصه

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$



# Transient state

حالت گذرا

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L}$$

$$\omega_0^2 = \frac{1}{LC}$$

**مدارهای RLC سری**

$$\alpha > \omega_0$$

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha - \beta_n$$

$$i = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

دو ریشه حقیقی و مجزا

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + \beta_n$$

$$i = e^{-\alpha t} [K_1 e^{-\beta_n t} + K_2 e^{\beta_n t}]$$

پاسخ فوق میرا

$$\alpha < \omega_0$$

$$s_1 = -\alpha - j\sqrt{\omega_0^2 - \alpha^2} = -\alpha - j\omega_n$$

$$i = e^{-\alpha t} [K_1 e^{-j\omega_n t} + K_2 e^{j\omega_n t}]$$

دو ریشه مختلط مزدوج

$$s_2 = -\alpha + j\sqrt{\omega_0^2 - \alpha^2} = -\alpha + j\omega_n$$

پاسخ زیرمیرا

$$\alpha = \omega_0$$

$$s_1 = -\alpha$$

$$i = e^{-\alpha t} [K_1 t + K_2]$$

دو ریشه حقیقی یکسان

$$s_2 = -\alpha$$

پاسخ میرای بحرانی



# Transient state

حالت گذرا

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L}$$

$$\omega_0^2 = \frac{1}{LC}$$

مدارهای RLC سری

$$\alpha = 0$$

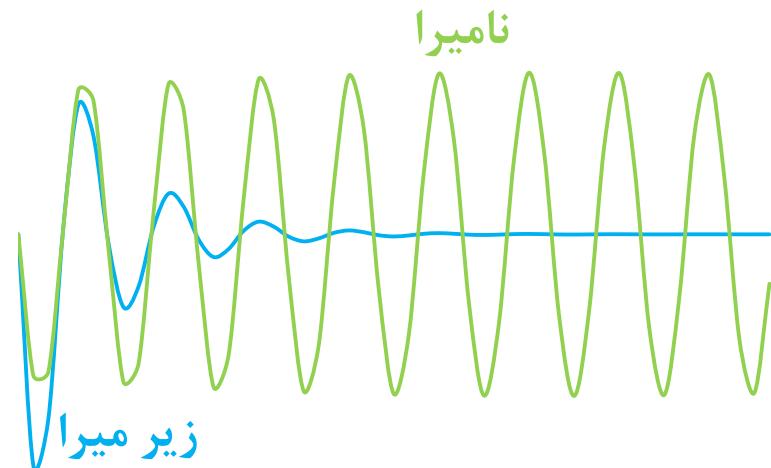
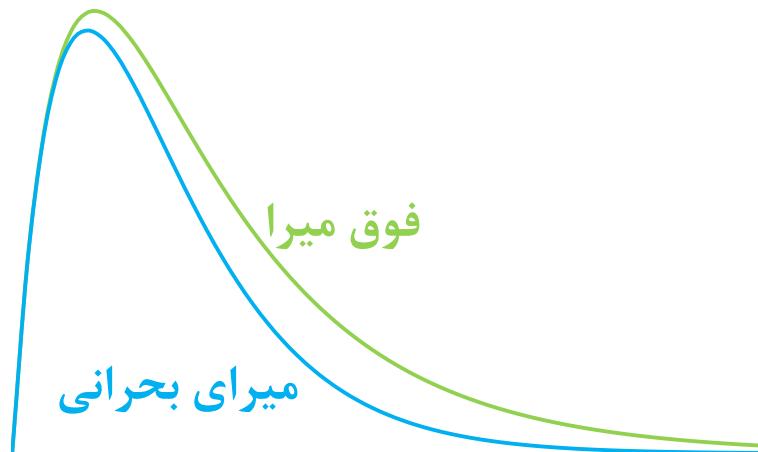
$$s_1 = -j\omega_0$$

$$i = [K_1 e^{-j\omega_0 t} + K_2 e^{j\omega_0 t}]$$

دو ریشه موهومی

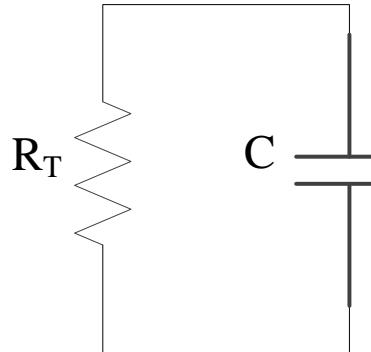
$$s_2 = +j\omega_0$$

پاسخ نامیرا



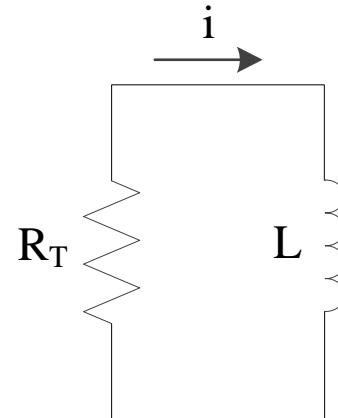
# Transient state

حالت گذرا



$$v(t) = K_1 + K_2 e^{-\frac{t}{\tau}}$$

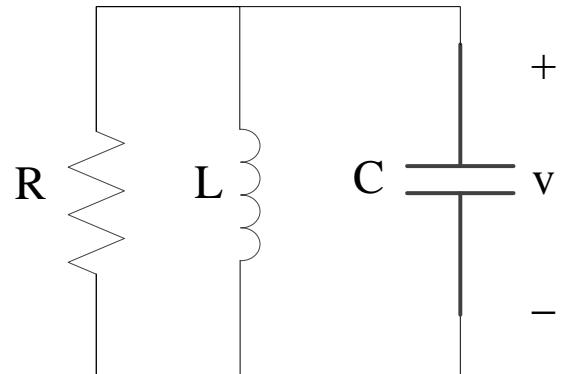
$$\tau = R_T C$$



$$i(t) = K_1 + K_2 e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R_T}$$

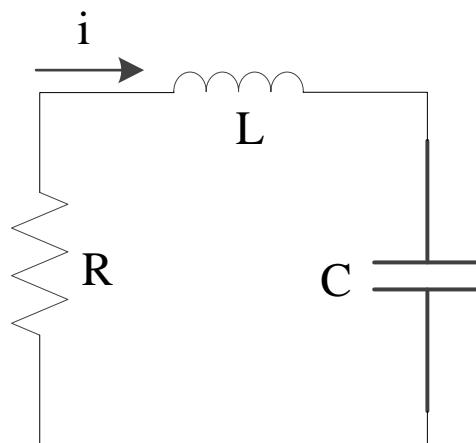
حالت گذرا



$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$\alpha = \frac{1}{2RC}$$

$$\omega_0^2 = \frac{1}{LC}$$



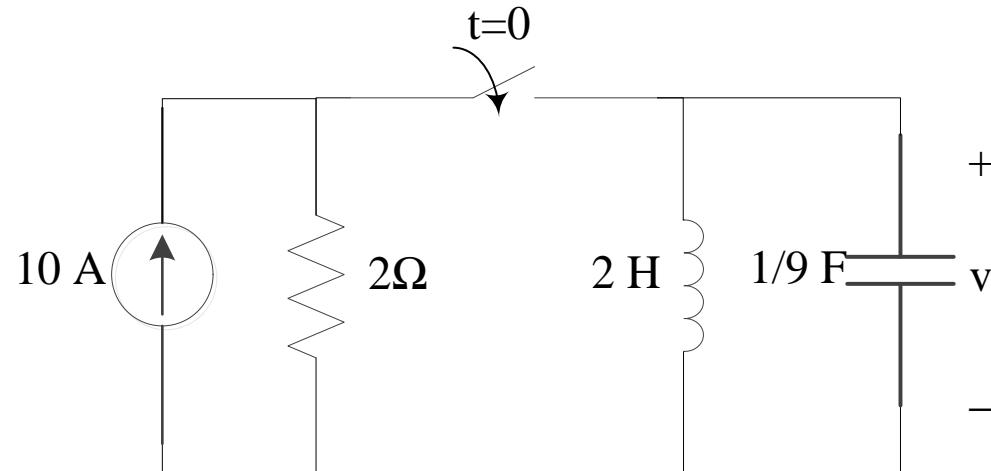
$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$\alpha = \frac{R}{2L}$$

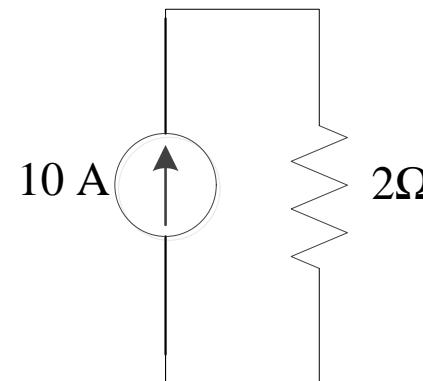
$$\omega_0^2 = \frac{1}{LC}$$

# Transient state

حالت گذرا



$t < 0$



مدارهای RLC موازی

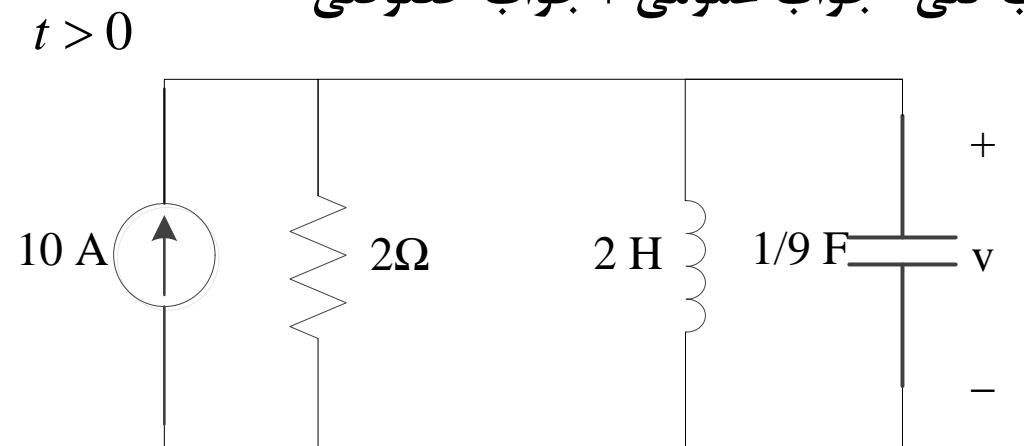
خازن اتصال باز،  
سلف اتصال کوتاه

$$i_L(0^+) = 0$$

$$v_C(0^+) = 0$$

جواب کلی = جواب عمومی + جواب خصوصی

$$s^2 + 2\alpha s + \omega_0^2 = 0$$



$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 2 \times \frac{1}{9}} = \frac{9}{4}$$

$$\alpha > \omega_0$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{2 \times \frac{1}{9}} = \frac{9}{2}$$

$$\omega_0 = \sqrt{\frac{9}{2}}$$



# Transient state

حالت گذرا

$$\alpha = \frac{9}{4}$$

$$\omega_0 = \sqrt{\frac{9}{2}}$$

$$\alpha > \omega_0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

مدارهای RLC موازی

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha - \beta_n$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + \beta_n$$

$$s_1 = -\frac{9}{4} - \sqrt{\frac{81}{16} - \frac{9}{2}}$$

$$s_2 = -\frac{9}{4} + \sqrt{\frac{81}{16} - \frac{9}{2}}$$

$$v = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$v = K_1 e^{-3t} + K_2 e^{-1.5t}$$

$$v = K_1 e^{-3t} + K_2 e^{-1.5t} + K_3$$

$$i_L(0^+) = i_0$$

$$v_C(0^+) = v_0$$

$$v_C(\infty) = ?$$

جواب عمومی

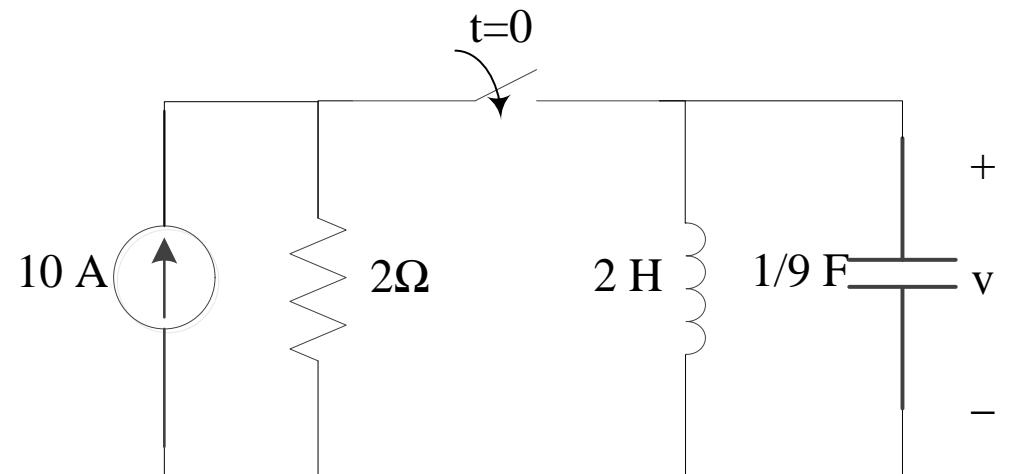
جواب کلی

شرایط مرزی

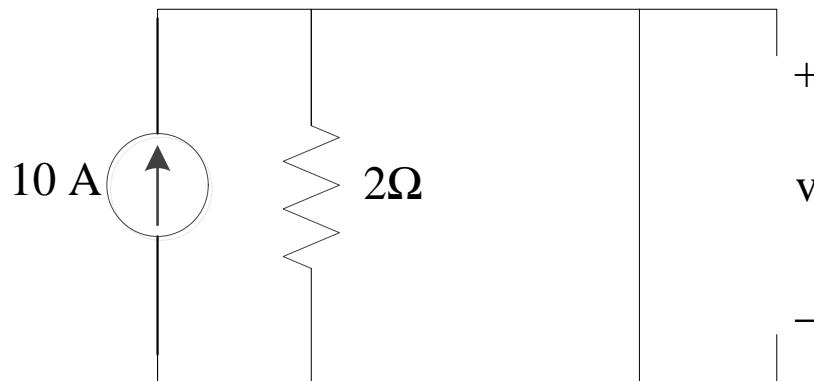


# Transient state

حالت گذرا



$t \rightarrow \infty$



مدارهای RLC موازی

خازن اتصال باز،  
سلف اتصال کوتاه

$$v(\infty) = 0$$

$$v = K_1 e^{-3t} + K_2 e^{-1.5t} + K_3$$

$$i_L(0^+) = 0$$

$$v(0^+) = 0$$

$$v(\infty) = 0$$

$$v(\infty) = 0 \longrightarrow K_3 = 0$$

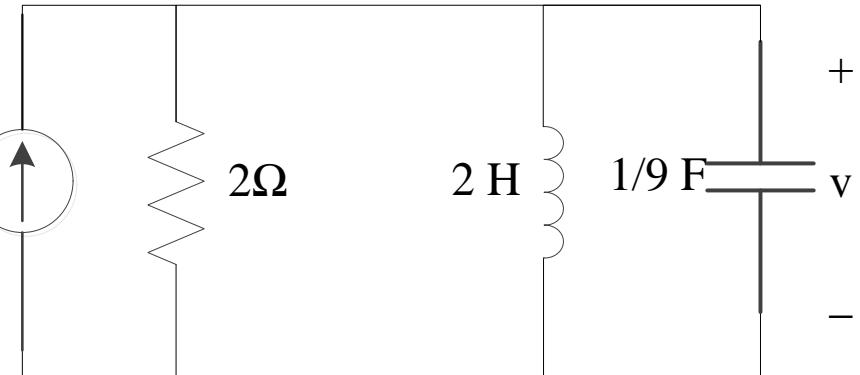
$$v = K_1 e^{-3t} + K_2 e^{-1.5t}$$



# Transient state

حالت گذرا

$t > 0$



مدارهای RLC موازی

$$v = K_1 e^{-3t} + K_2 e^{-1.5t}$$

$$i_L(0^+) = 0 \quad v(0^+) = 0$$

$$\frac{dv}{dt} = -3K_1 e^{-3t} - 1.5K_2 e^{-1.5t}$$

$$v(0^+) = 0$$

$$i_C(0^+) = ?$$

$$v(0^+) = 0$$

$$i_R(0^+) = \frac{v(0^+)}{R} = 0$$

$$i_L(0^+) = 0$$



$$K_1 + K_2 = 0$$

$$i_C(0^+) = 10 A$$

$$i_C(0^+) = C \frac{dv(0^+)}{dt}$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{10}{\frac{1}{9}} = 90$$

$$\frac{dv(0^+)}{dt} = -3K_1 - 1.5K_2$$

$$-3K_1 - 1.5K_2 = 90$$



# Transient state

حالت گذرا

$$v = K_1 e^{-3t} + K_2 e^{-1.5t}$$

مدارهای RLC موازی

$$K_1 + K_2 = 0$$

$$K_1 = -K_2$$

$$K_1 = -60$$

$$-3K_1 - 1.5K_2 = 90$$

$$3K_2 - 1.5K_2 = 90$$

$$1.5K_2 = 90$$

$$K_2 = 60$$

$$v = -60e^{-3t} + 60e^{-1.5t}$$

$$v = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$i_L = \frac{1}{L} \int_0^t v dt + i_L(0^+) = \frac{1}{L} \int_0^t K_1 e^{s_1 t} + K_2 e^{s_2 t} + i_L(0^+) dt$$

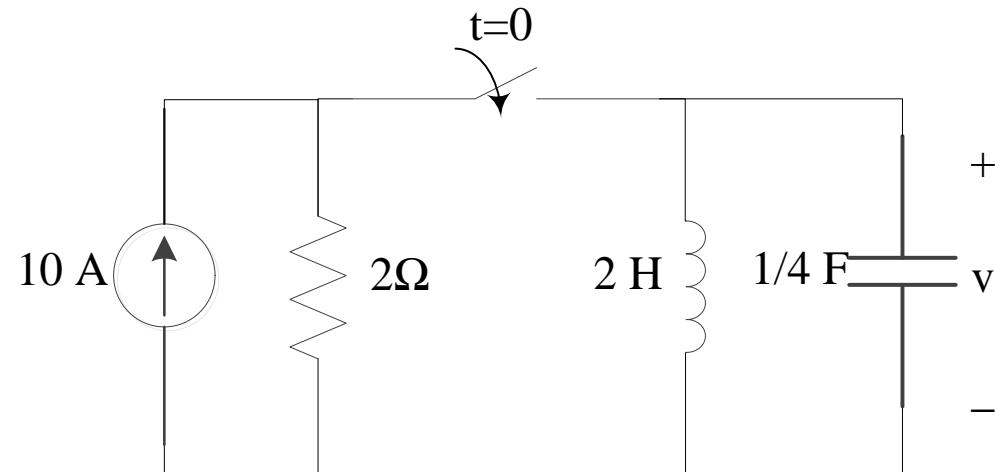
$$i_L = \frac{K_1}{L s_1} [e^{s_1 t} - 1] + \frac{K_2}{L s_2} [e^{s_2 t} - 1] + i_L(0^+)$$

$$i_L = 10e^{-3t} - 20e^{-1.5t} + 10$$

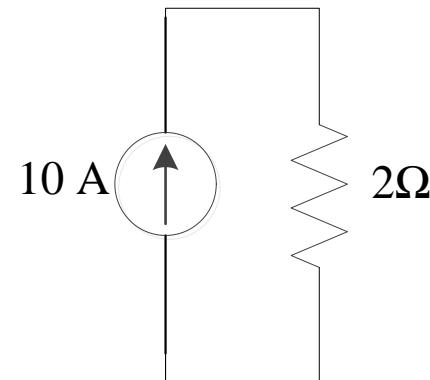


# Transient state

حالت گذرا



$t < 0$



مدارهای RLC موازی

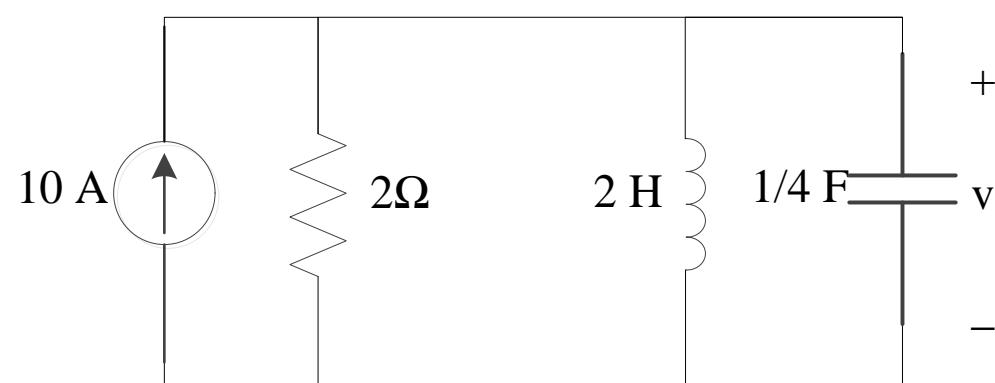
خازن اتصال باز،  
سلف اتصال کوتاه

$$i_L(0^+) = 0$$

$$v_C(0^+) = 0$$

$t > 0$  جواب کلی = جواب عمومی + جواب خصوصی

$$s^2 + 2\alpha s + \omega_0^2 = 0$$



$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 2 \times \frac{1}{4}} = 1$$

$$\alpha < \omega_0$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{2 \times \frac{1}{4}} = 2$$

$$\omega_0 = \sqrt{2}$$



# Transient state

حالت گذرا

$$\alpha = 1$$

$$\omega_0 = \sqrt{2}$$

$$\alpha < \omega_0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

مدارهای RLC موازی

$$s_1 = -\alpha - j\sqrt{\omega_0^2 - \alpha^2} = -\alpha - j\omega_n$$

$$s_1 = -1 - j\sqrt{2-1}$$

$$s_1 = -1 - j1$$

$$s_2 = -\alpha + j\sqrt{\omega_0^2 - \alpha^2} = -\alpha + j\omega_n$$

$$s_2 = -1 + j\sqrt{2-1}$$

$$s_2 = -1 + j1$$

$$v = e^{-\alpha t} [K_1 e^{-j\omega_n t} + K_2 e^{+j\omega_n t}]$$

$$v = Ke^{-t} \cos(t + \theta)$$

جواب عمومی

$$v = Ke^{-\alpha t} \cos(\omega_n t + \theta)$$

$$v = Ke^{-t} \cos(t + \theta) + K_3$$

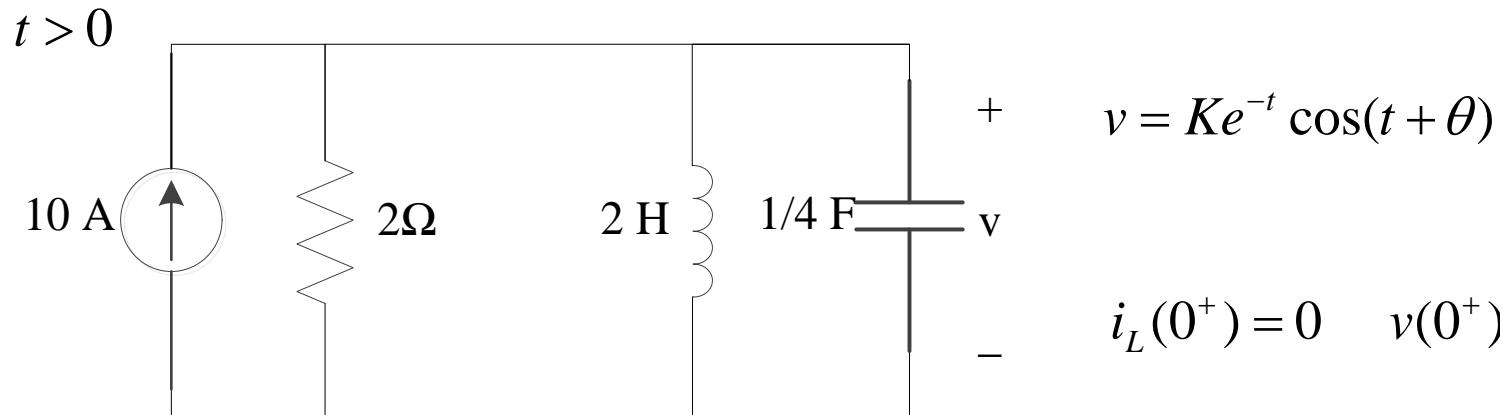
جواب کلی

$$v_C(\infty) = 0 \quad \longrightarrow \quad K_3 = 0$$



# Transient state

حالت گذرا



مدارهای RLC موازی

$$v = Ke^{-t} \cos(t + \theta)$$

$$\frac{dv}{dt} = -Ke^{-t} \cos(t + \theta) - Ke^{-t} \sin(t + \theta)$$

$$v(0^+) = 0$$

$$i_C(0^+) = ?$$

$$v(0^+) = 0$$

$$i_R(0^+) = \frac{v(0^+)}{R} = 0$$

$$i_L(0^+) = 0$$



$$K \cos(\theta) = 0$$

$$i_C(0^+) = 10 \text{ A}$$

$$i_C(0^+) = C \frac{dv(0^+)}{dt}$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{10}{1/4} = 40$$

$$\frac{dv(0^+)}{dt} = -K \cos(\theta) - K \sin(\theta) \rightarrow -K \cos(\theta) - K \sin(\theta) = 40$$



# Transient state

حالت گذرا

مدارهای RLC موازی

$$v = K e^{-t} \cos(t + \theta)$$

$$K \cos(\theta) = 0 \longrightarrow \cos(\theta) = 0 \longrightarrow \theta = \frac{\pi}{2}$$

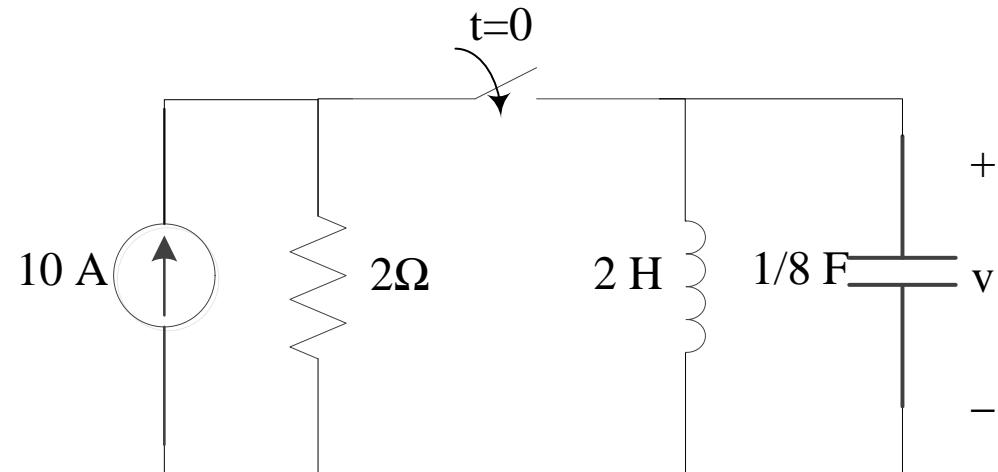
$$-K \cos(\theta) - K \sin(\theta) = 40 \longrightarrow K \sin(\theta) = -40 \longrightarrow K = -40$$

$$v = -40e^{-t} \cos\left(t + \frac{\pi}{2}\right)$$

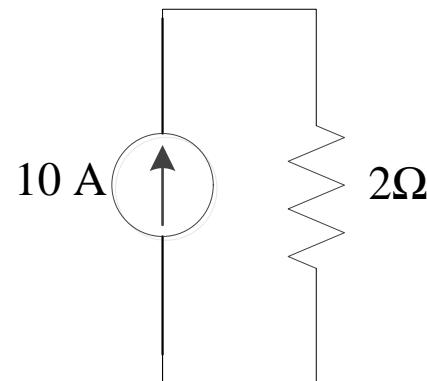


# Transient state

حالت گذرا



$t < 0$



مدارهای RLC موازی

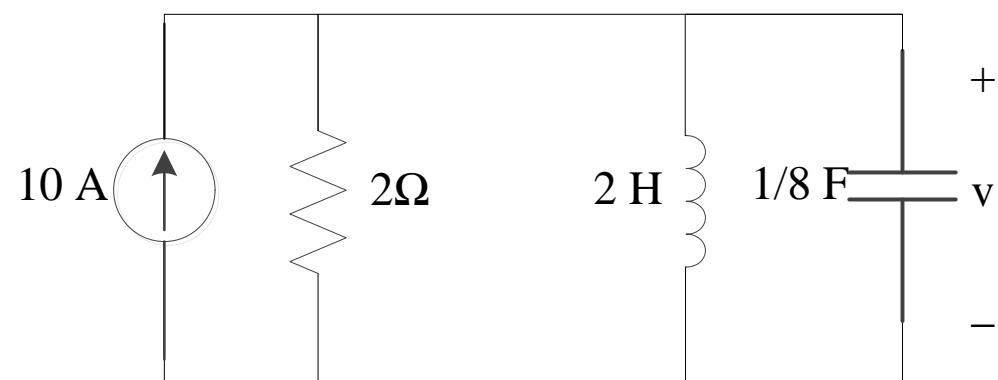
خازن اتصال باز،  
سلف اتصال کوتاه

$$i_L(0^+) = 0$$

$$v_C(0^+) = 0$$

$t > 0$  جواب کلی = جواب عمومی + جواب خصوصی

$$s^2 + 2\alpha s + \omega_0^2 = 0$$



$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 2 \times \frac{1}{8}} = 2$$

$$\alpha = \omega_0$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{2 \times \frac{1}{8}} = 4$$

$$\omega_0 = 2$$



# Transient state

حالت گذرا

$$\alpha = 2$$

$$\omega_0 = 2$$

$$\alpha = \omega_0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

مدارهای RLC موازی

$$s_1 = -\alpha$$

$$s_1 = -2$$

$$s_2 = -\alpha$$

$$s_2 = -2$$

$$v = e^{-\alpha t} [K_1 t + K_2]$$

$$v = e^{-2t} [K_1 t + K_2]$$

جواب عمومی

$$v = e^{-2t} [K_1 t + K_2] + K_3$$

جواب کلی

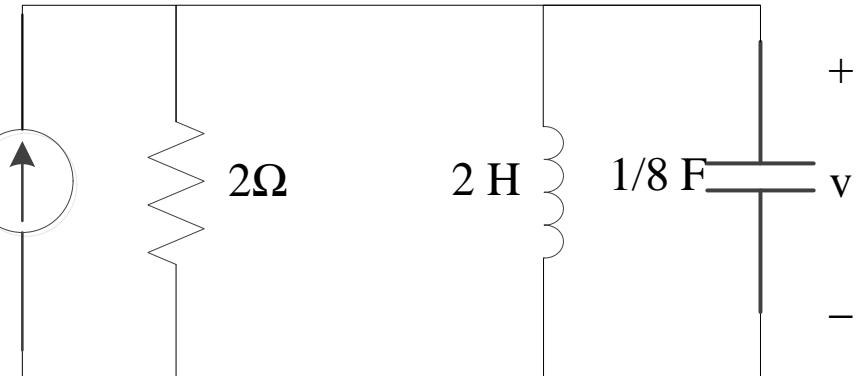
$$v_C(\infty) = 0 \quad \longrightarrow \quad K_3 = 0$$



# Transient state

حالت گذرا

$t > 0$



$$v = e^{-2t}[K_1 t + K_2]$$

$$i_L(0^+) = 0 \quad v(0^+) = 0$$

مدارهای RLC موازی

$$\frac{dv}{dt} = K_1 e^{-2t} - 2e^{-2t}[K_1 t + K_2]$$

$$v(0^+) = 0$$

$$i_C(0^+) = ?$$

$$v(0^+) = 0$$

$$i_R(0^+) = \frac{v(0^+)}{R} = 0$$

$$i_L(0^+) = 0$$



$$K_2 = 0$$

$$\frac{dv(0^+)}{dt} = K_1 - 2[0 + K_2]$$



$$K_1 - 2K_2 = 80$$



# Transient state

حالت گذرا

مدارهای RLC موازی

$$v = e^{-2t} [K_1 t + K_2]$$

$$K_2 = 0$$

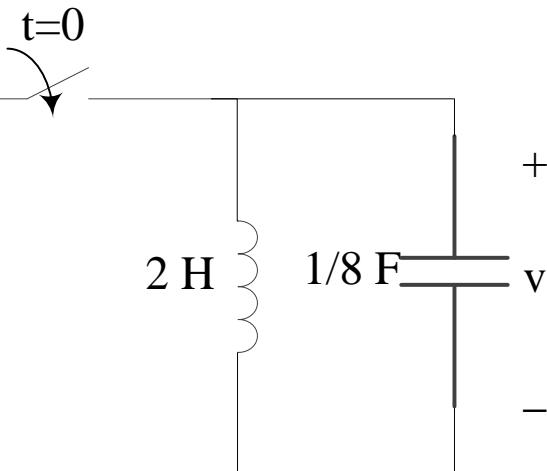
$$K_1 - 2K_2 = 80 \quad \longrightarrow \quad K_1 = 80$$

$$v = 80te^{-2t}$$

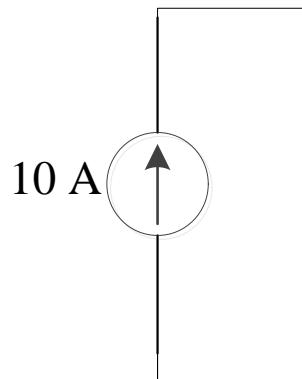


# Transient state

حالت گذرا



$t < 0$



مدارهای RLC موازی

خازن اتصال باز،  
سلف اتصال کوتاه

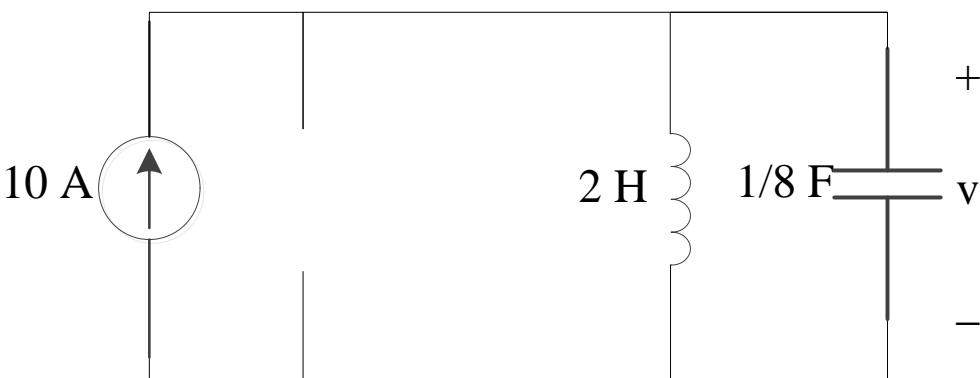
$$i_L(0^+) = 0$$

$$v_C(0^+) = 0$$

جواب کلی = جواب عمومی + جواب خصوصی

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$t > 0$



$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times \infty \times \frac{1}{8}} = 0$$

$$\alpha = 0$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{2 \times \frac{1}{8}} = 4$$

$$\omega_0 = 2$$



# Transient state

حالت گذرا

$$\alpha = 0$$

$$\omega_0 = 2$$

$$\alpha = 0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

مدارهای RLC موازی

$$s_1 = -j\omega_0$$

$$s_1 = -j2$$

$$s_2 = j\omega_0$$

$$s_2 = j2$$

$$v = [K_1 e^{-j\omega_0 t} + K_2 e^{j\omega_0 t}]$$

$$v = K \cos(2t + \theta)$$

$$v = K \cos(\omega_0 t + \theta)$$

$$v = K \cos(2t + \theta) + K_3$$

جواب عمومی

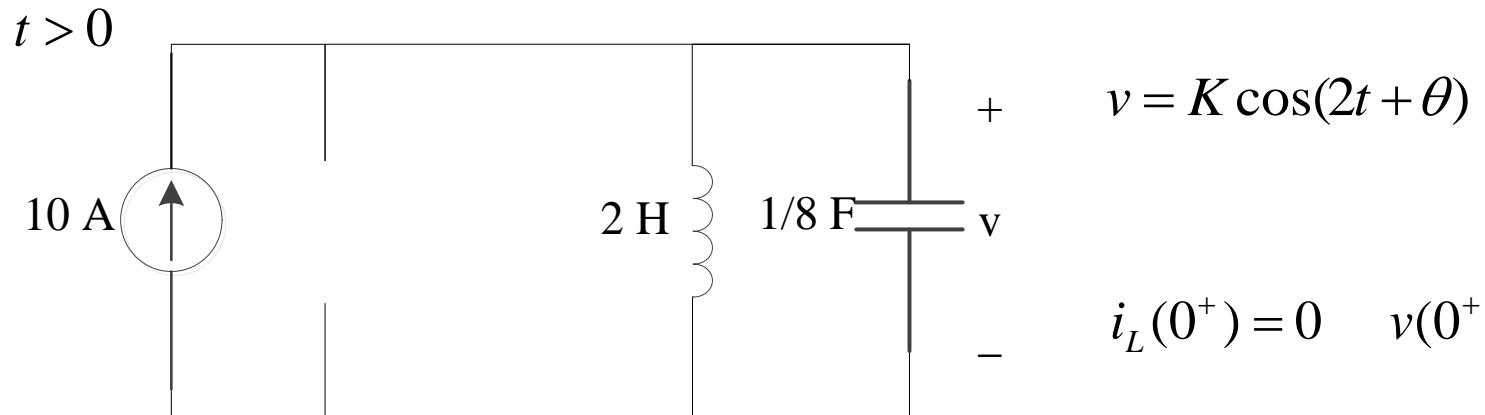
جواب کلی

$$v_C(\text{Stimulation}) = 0 \longrightarrow K_3 = 0$$



# Transient state

حالت گذرا



مدارهای RLC موازی

$$\frac{dv}{dt} = -2K \sin(2t + \theta)$$

$$v(0^+) = 0 \quad | \quad i_C(0^+) = ? \quad v(0^+) = 0 \quad \longrightarrow \quad i_R(0^+) = \frac{v(0^+)}{R} = 0 \quad i_L(0^+) = 0$$

$$\downarrow \quad | \quad i_C(0^+) = 10 A \quad i_C(0^+) = C \frac{dv(0^+)}{dt} \quad \longrightarrow \quad \frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{10}{\frac{1}{8}} = 80$$

$$K \cos(\theta) = 0 \quad | \quad \frac{dv(0^+)}{dt} = -2K \sin(0 + \theta) \quad \longrightarrow \quad -2K \sin(\theta) = 80$$



# Transient state

حالت گذرا

مدارهای RLC موازی

$$v = K \cos(2t + \theta)$$

$$K \cos(\theta) = 0 \longrightarrow \cos(\theta) = 0 \longrightarrow \theta = \frac{\pi}{2}$$

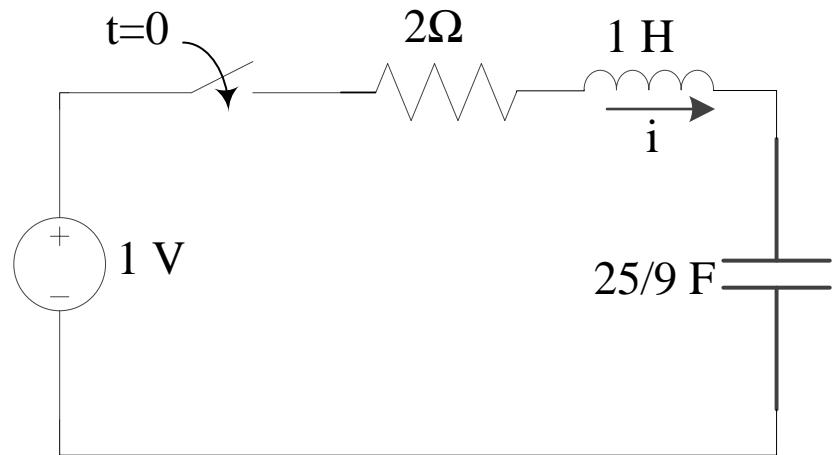
$$-2K \sin(\theta) = 80 \longrightarrow K \sin(\theta) = -40 \longrightarrow K = -40$$

$$v = -40 \cos(t + \frac{\pi}{2})$$

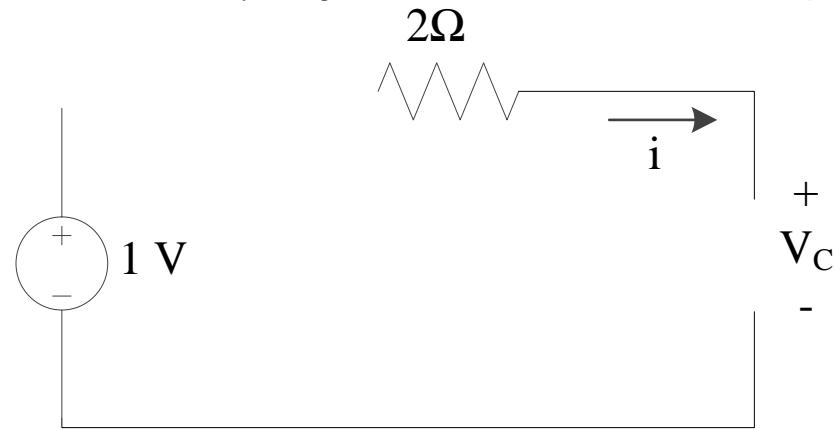


# Transient state

حالت گذرا



$t < 0$



مدارهای RLC سری

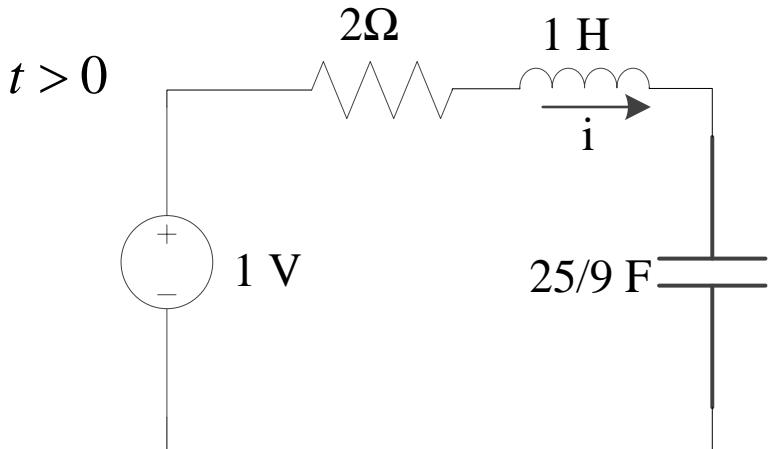
خازن اتصال باز،  
سلف اتصال کوتاه

$$i_L(0^+) = 0$$

$$v_C(0^+) = 0$$

جواب کلی = جواب عمومی + جواب خصوصی

$$s^2 + 2\alpha s + \omega_0^2 = 0$$



$$\alpha = \frac{R}{2L} = \frac{2}{2 \times 1} = 1$$

$$\alpha > \omega_0$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{1 \times 25/9} = 9/25$$

$$\omega_0 = 3/5$$



# Transient state

حالت گذرا

$$\alpha = 1$$

$$\omega_0 = \frac{3}{5}$$

$$\alpha > \omega_0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

مدارهای RLC سری

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha - \beta_n$$

$$s_1 = -1 - \sqrt{1 - \frac{9}{25}}$$

$$s_1 = -1.8$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + \beta_n$$

$$s_2 = -1 + \sqrt{1 - \frac{9}{25}}$$

$$s_2 = -0.2$$

$$i = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$i = K_1 e^{-1.8t} + K_2 e^{-0.2t}$$

جواب عمومی

$$i = K_1 e^{-1.8t} + K_2 e^{-0.2t} + K_3$$

جواب کلی

$$i_L(0^+) = i_0 \quad v_C(0^+) = v_0$$

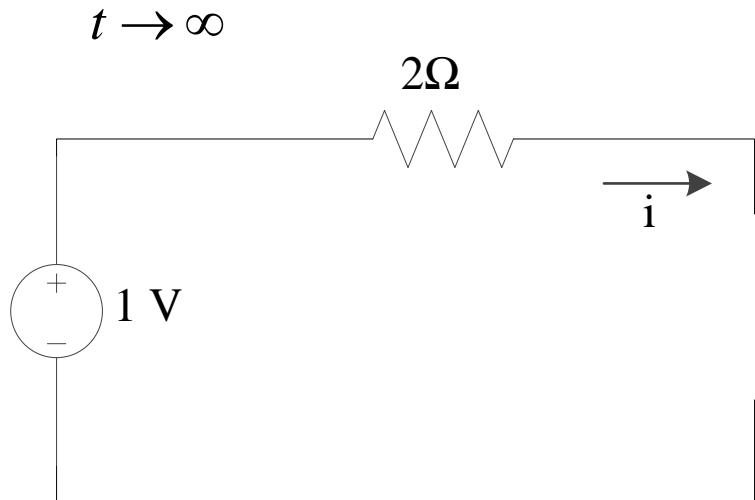
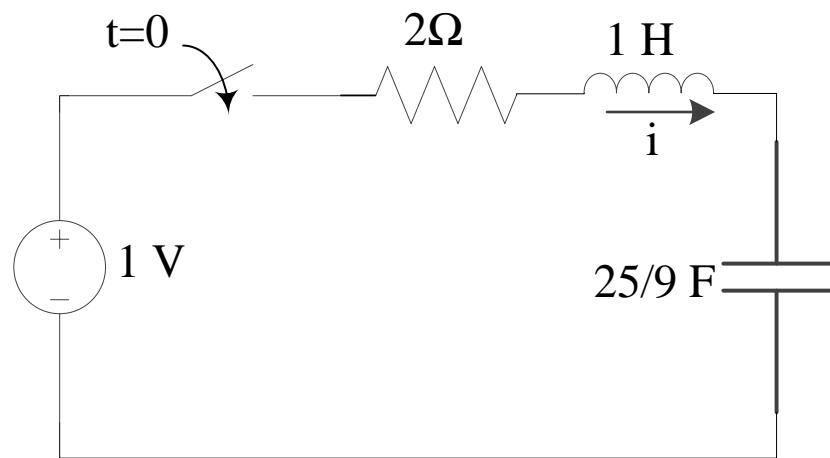
$$i_L(\infty) = ?$$

شرط مرزی



# Transient state

حالت گذرا



مدارهای RLC سری

خازن اتصال باز،  
سلف اتصال کوتاه

$$i(\infty) = 0$$

$$i = K_1 e^{-1.8t} + K_2 e^{-0.2t} + K_3$$

$$i_L(0^+) = 0$$

$$v(0^+) = 0$$

$$i(\infty) = 0$$

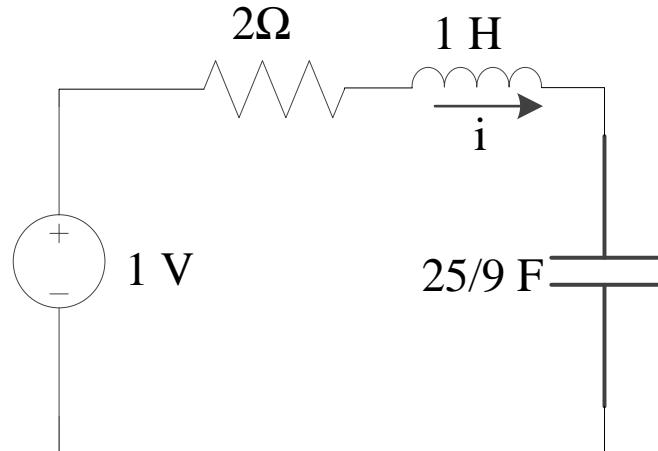
$$i(\infty) = 0 \longrightarrow K_3 = 0$$

$$i = K_1 e^{-1.8t} + K_2 e^{-0.2t}$$

# Transient state

حالت گذرا

$t > 0$



مدارهای RLC سری

$$i = K_1 e^{-1.8t} + K_2 e^{-0.2t}$$

$$i_L(0^+) = 0 \quad v(0^+) = 0$$

$$\frac{di}{dt} = -1.8K_1 e^{-1.8t} - 0.2K_2 e^{-0.2t}$$

$$i(0^+) = 0$$

$$v_L(0^+) = ?$$

$$i(0^+) = 0$$

$$v_R(0^+) = Ri(0^+) = 0$$

$$v_C(0^+) = 0$$



$$K_1 + K_2 = 0$$

$$\frac{di(0^+)}{dt} = -1.8K_1 - 0.2K_2$$

$$v_L(0^+) = L \frac{di(0^+)}{dt}$$

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{1}{1} = 1$$

$$-1.8K_1 - 0.2K_2 = 1$$



# Transient state

حالت گذرا

$$i = K_1 e^{-1.8t} + K_2 e^{-0.2t}$$

مدارهای RLC سری

$$K_1 + K_2 = 0$$

$$K_1 = -K_2$$

$$K_1 = -0.625$$

$$-1.8K_1 - 0.2K_2 = 1$$

$$1.8K_2 - 0.2K_2 = 1$$

$$1.6K_2 = 1$$

$$K_2 = 0.625$$

$$i = -0.625e^{-1.8t} + 0.625e^{-0.2t}$$

$$v_C = \frac{1}{C} \int_0^t i dt + v_C(0^+)$$

$$i = K_1 e^{-s_1 t} + K_2 e^{-s_2 t}$$

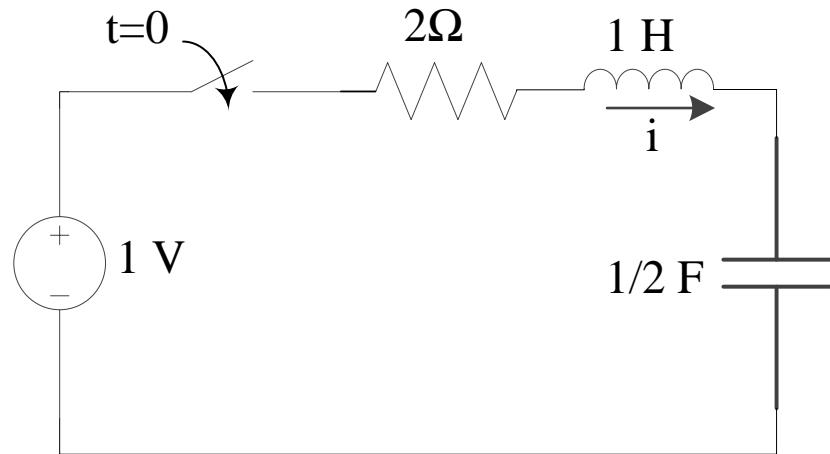
$$v_c = \frac{-K_1}{s_1 C} [e^{-s_1 t} - e^0] + \frac{-K_2}{s_2 C} [e^{-s_2 t} - e^0] + 0$$

$$v_c = 1 - 1.125e^{-0.2t} + 0.125e^{-1.8t}$$

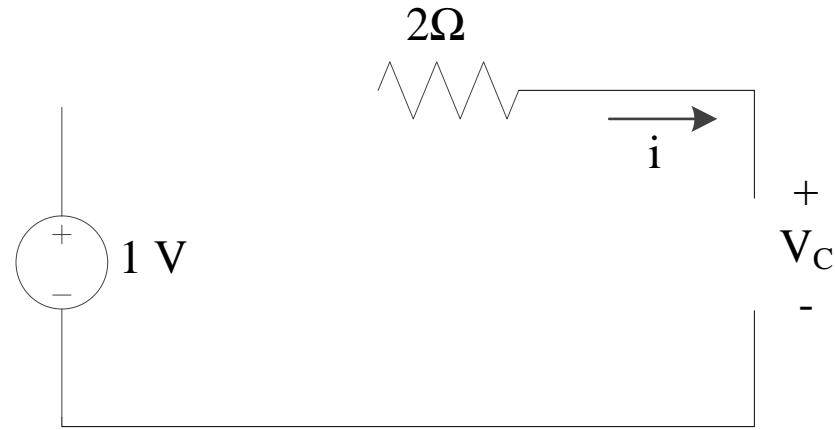


# Transient state

حالت گذرا



$t < 0$



مدارهای RLC سری

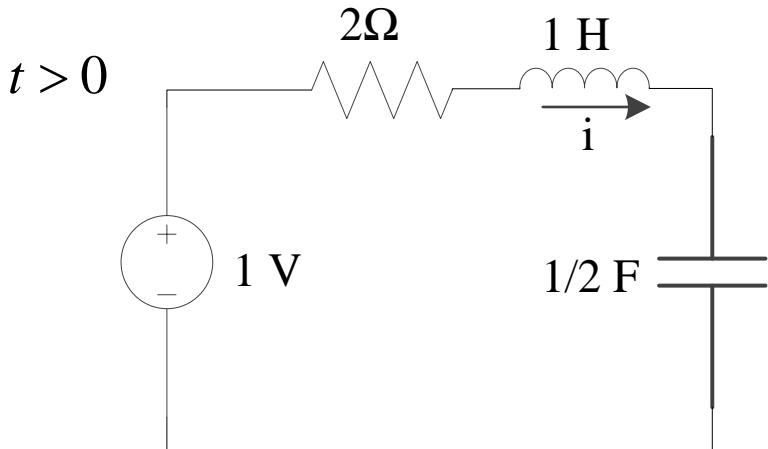
خازن اتصال باز،  
سلف اتصال کوتاه

$$i_L(0^+) = 0$$

$$v_C(0^+) = 0$$

جواب کلی = جواب عمومی + جواب خصوصی

$$s^2 + 2\alpha s + \omega_0^2 = 0$$



$$\alpha = \frac{R}{2L} = \frac{2}{2 \times 1} = 1$$

$$\alpha < \omega_0$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{1 \times \frac{1}{2}} = 2$$

$$\omega_0 = \sqrt{2}$$



# Transient state

حالت گذرا

$$\alpha = 1$$

$$\omega_0 = \sqrt{2}$$

$$\alpha < \omega_0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

مدارهای RLC سری

$$s_1 = -\alpha - j\sqrt{\omega_0^2 - \alpha^2} = -\alpha - j\omega_n$$

$$s_1 = -1 - j\sqrt{2-1}$$

$$s_1 = -1 - j1$$

$$s_2 = -\alpha + j\sqrt{\omega_0^2 - \alpha^2} = -\alpha + j\omega_n$$

$$s_2 = -1 + j\sqrt{2-1}$$

$$s_2 = -1 + j1$$

$$i = e^{-\alpha t} [K_1 e^{-j\omega_n t} + K_2 e^{+j\omega_n t}]$$

$$i = K e^{-t} \cos(t + \theta)$$

جواب عمومی

$$i = K e^{-\alpha t} \cos(\omega_n t + \theta)$$

$$i = K e^{-t} \cos(t + \theta) + K_3$$

جواب کلی

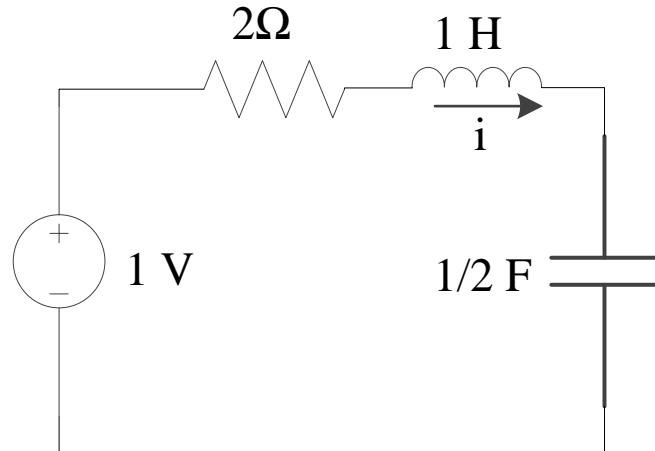
$$v_C(\infty) = 0 \quad \longrightarrow \quad K_3 = 0$$



# Transient state

حالت گذرا

$t > 0$



مدارهای RLC سری

$$i = Ke^{-t} \cos(t + \theta)$$

$$i_L(0^+) = 0 \quad v(0^+) = 0$$

$$\frac{di}{dt} = -Ke^{-t} \cos(t + \theta) - Ke^{-t} \sin(t + \theta)$$

$$i(0^+) = 0$$

$$v_L(0^+) = ?$$

$$i(0^+) = 0$$

$$v_R(0^+) = Ri(0^+) = 0$$

$$v_C(0^+) = 0$$



$$K \cos(\theta) = 0$$

$$v_L(0^+) = 1V$$

$$v_L(0^+) = L \frac{di(0^+)}{dt}$$

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{1}{1} = 1$$

$$\frac{di(0^+)}{dt} = -K \cos(\theta) - K \sin(\theta) \rightarrow -K \cos(\theta) - K \sin(\theta) = 1$$



# Transient state

حالت گذرا

## مدارهای RLC سری

$$i = K e^{-t} \cos(t + \theta)$$

$$K \cos(\theta) = 0 \longrightarrow \cos(\theta) = 0 \longrightarrow \theta = \frac{\pi}{2}$$

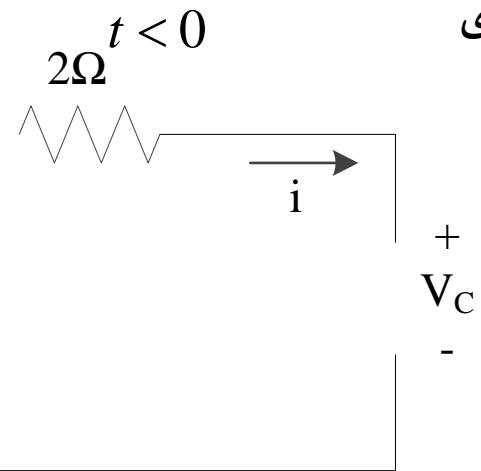
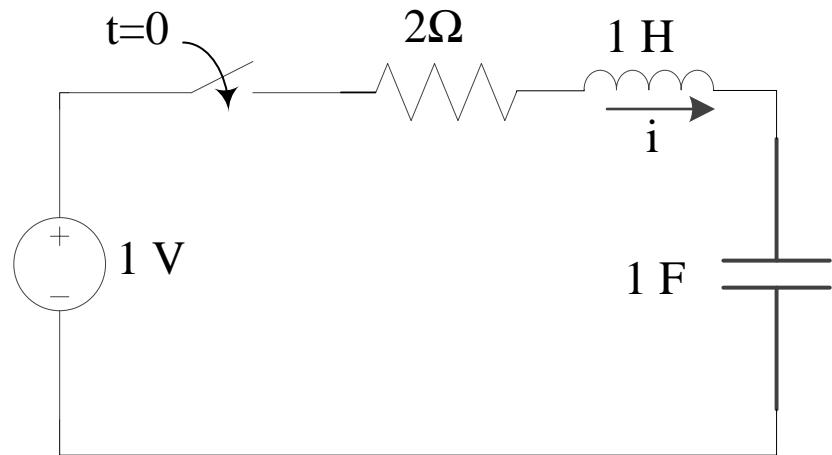
$$-K \cos(\theta) - K \sin(\theta) = 1 \longrightarrow -K \sin(\theta) = 1 \longrightarrow K = -1$$

$$i = -e^{-t} \cos\left(t + \frac{\pi}{2}\right)$$



# Transient state

حالت گذرا



مدارهای RLC سری

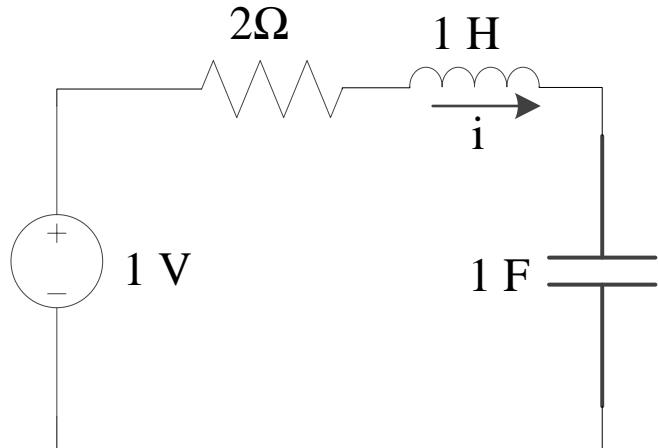
خازن اتصال باز،  
سلف اتصال کوتاه

$$i_L(0^+) = 0$$

$$v_C(0^+) = 0$$

جواب کلی = جواب عمومی + جواب خصوصی

$$s^2 + 2\alpha s + \omega_0^2 = 0$$



$$\alpha = \frac{R}{2L} = \frac{2}{2 \times 1} = 1$$

$$\alpha = \omega_0$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{1 \times 1} = 1$$

$$\omega_0 = 1$$



# Transient state

حالت گذرا

$$\alpha = 1$$

$$\omega_0 = 1$$

$$\alpha = \omega_0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

مدارهای RLC سری

$$s_1 = -\alpha$$

$$s_1 = -1$$

$$s_2 = -\alpha$$

$$s_2 = -1$$

$$i = e^{-\alpha t} [K_1 t + K_2]$$

$$i = e^{-t} [K_1 t + K_2]$$

جواب عمومی

$$i = e^{-t} [K_1 t + K_2] + K_3$$

جواب کلی

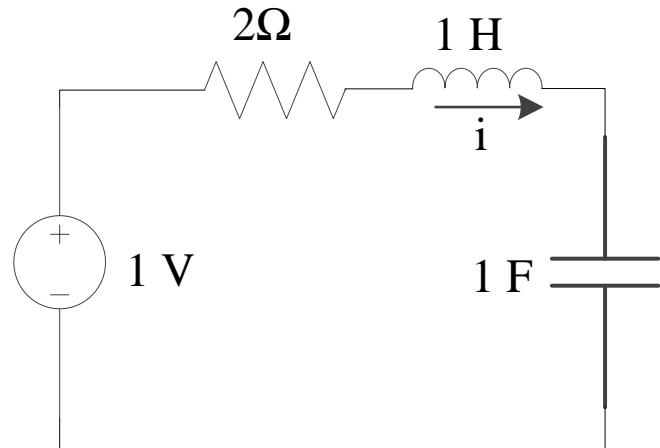
$$i_L(\infty) = 0 \quad \longrightarrow \quad K_3 = 0$$



# Transient state

حالت گذرا

$t > 0$



مدارهای RLC سری

$$i = e^{-t}[K_1 t + K_2]$$

$$i_L(0^+) = 0 \quad v(0^+) = 0$$

$$\frac{dv}{dt} = K_1 e^{-t} - e^{-t}[K_1 t + K_2]$$

$$i(0^+) = 0$$

$$v_L(0^+) = ?$$

$$i(0^+) = 0$$

$$v_R(0^+) = Ri(0^+) = 0$$

$$v_C(0^+) = 0$$



$$K_2 = 0$$

$$v_L(0^+) = 1V$$

$$v_L(0^+) = L \frac{di(0^+)}{dt}$$

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{1}{1} = 1$$

$$\frac{di(0^+)}{dt} = K_1 - [0 + K_2]$$



$$K_1 - K_2 = 1$$



# Transient state

حالت گذرا

مدارهای RLC سری

$$i = e^{-t} [K_1 t + K_2]$$

$$K_2 = 0$$

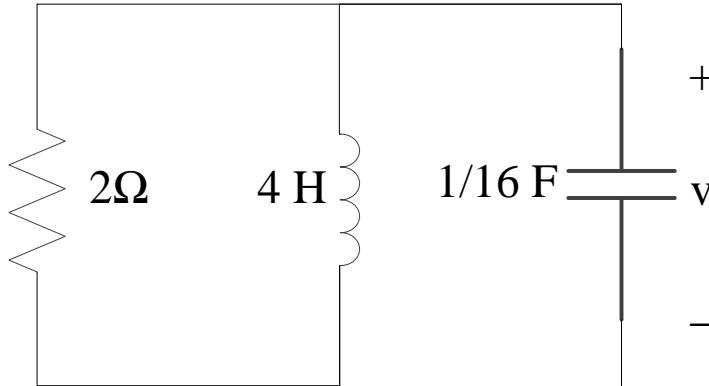
$$K_1 - K_2 = 1 \quad \longrightarrow \quad K_1 = 1$$

$$i = t e^{-t}$$



# Transient state

حالت گذرا



$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 2 \times \frac{1}{16}} = 4$$

$$\alpha > \omega_0$$

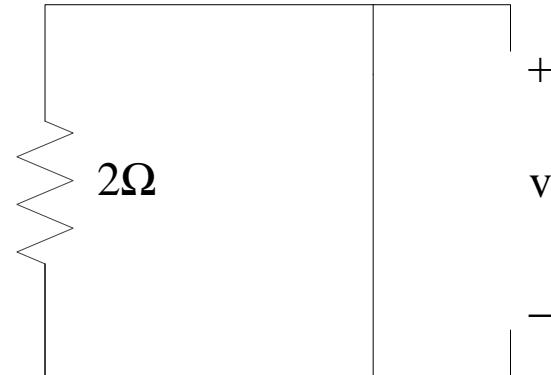
$$\omega_0^2 = \frac{1}{LC} = \frac{1}{4 \times \frac{1}{16}} = 4$$

$$\omega_0 = 2$$

$$s_1 = -7.46 \quad s_2 = -0.54$$

$$v = K_1 e^{-7.46t} + K_2 e^{-0.54t} + K_3$$

$$t \rightarrow \infty$$



مدارهای RLC موازی

خازن اتصال باز،  
سلف اتصال کوتاه

$$v(\infty) = 0$$

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha - \beta_n$$

$$s_1 = -4 - \sqrt{16 - 4}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + \beta_n$$

$$s_2 = -4 + \sqrt{16 - 4}$$



# Transient state

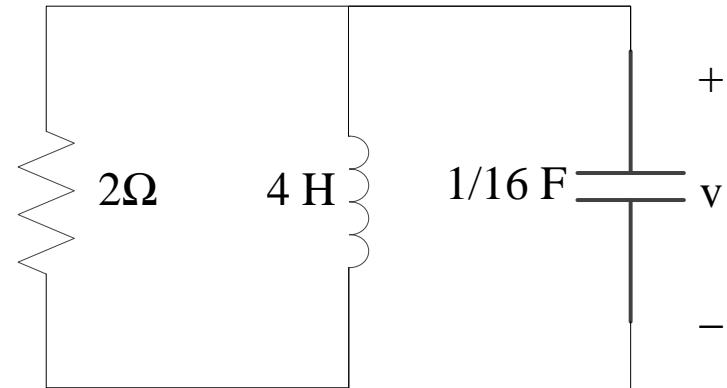
حالت گذرا

$$v = K_1 e^{-7.46t} + K_2 e^{-0.54t} + K_3$$

$$i_L(0^+) = -1A$$

$$v_C(0^+) = 2V$$

$$v_C(\infty) = 0$$



**مدارهای RLC موازی**

$$i_R(0^+) + i_L(0^+) + i_C(0^+) = 0$$

$$v(0^+) = 2V \quad \longrightarrow \quad i_R(0^+) = \frac{v(0^+)}{R} = \frac{2}{2} = 1A \quad i_L(0^+) = -1A \quad i_C(0^+) = -1 - (-1) = 0A$$

$$i_C(0^+) = C \frac{dv_C(0^+)}{dt} \quad \longrightarrow \quad \frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = 0 \quad \frac{dv}{dt} = -7.46K_1 e^{-7.46t} - 0.54K_2 e^{-0.54t}$$

$$\frac{dv}{dt}(0^+) = -7.46K_1 - 0.54K_2 = 0$$



# Transient state

حالت گذرا

مدارهای **RLC** موازی

$$v = K_1 e^{-7.46t} + K_2 e^{-0.54t} + K_3$$

$$v_C(\infty) = 0$$

$$K_3 = 0$$

$$v_C(0^+) = 2V$$

$$K_1 + K_2 = 2$$

$$K_2 = 2.15$$

$$\frac{dv}{dt}(0^+) = 0$$

$$-7.46K_1 - 0.54K_2 = 0$$

$$K_1 = -0.15$$

$$v = -0.15e^{-7.46t} + 2.15e^{-0.54t}$$

