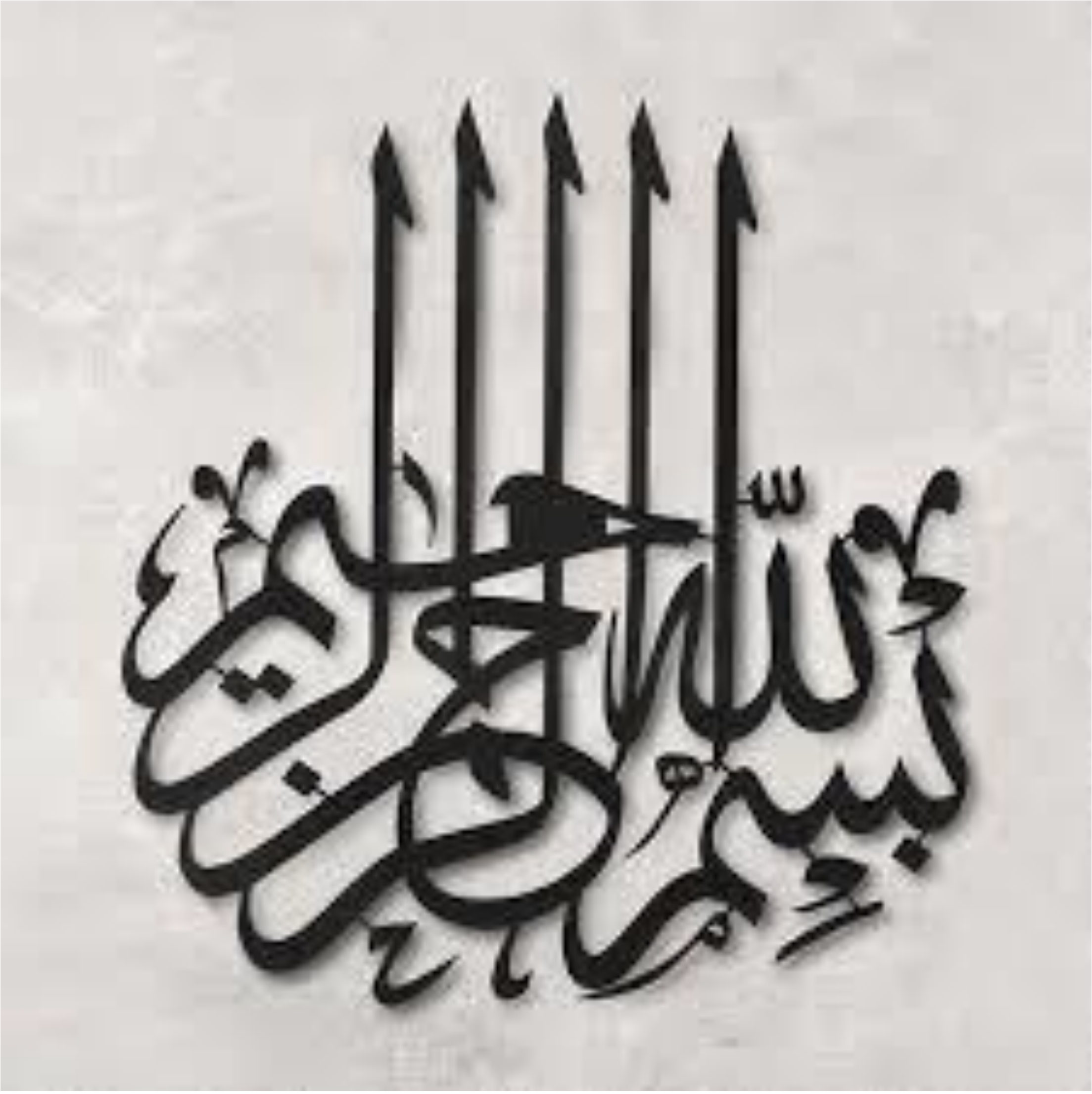
بسم الله الرحمن الرحيم

CHAPTER

3

p–n Junction

Forward and reverse bias

- The previous discussions are for a *p–n* junction at thermal equilibrium without external bias.
- The equilibrium energy band diagram, shown again in *Fig. 8a*, illustrates that the total electrostatic potential across the junction is V_{bi}.
- The corresponding potential energy difference from the p-side to the n-side is qV_{bi}.
- If we apply a positive voltage V_F to the p-side with respect to the n-side, the p–n junction becomes forward-biased, as shown in *Fig.8b*.

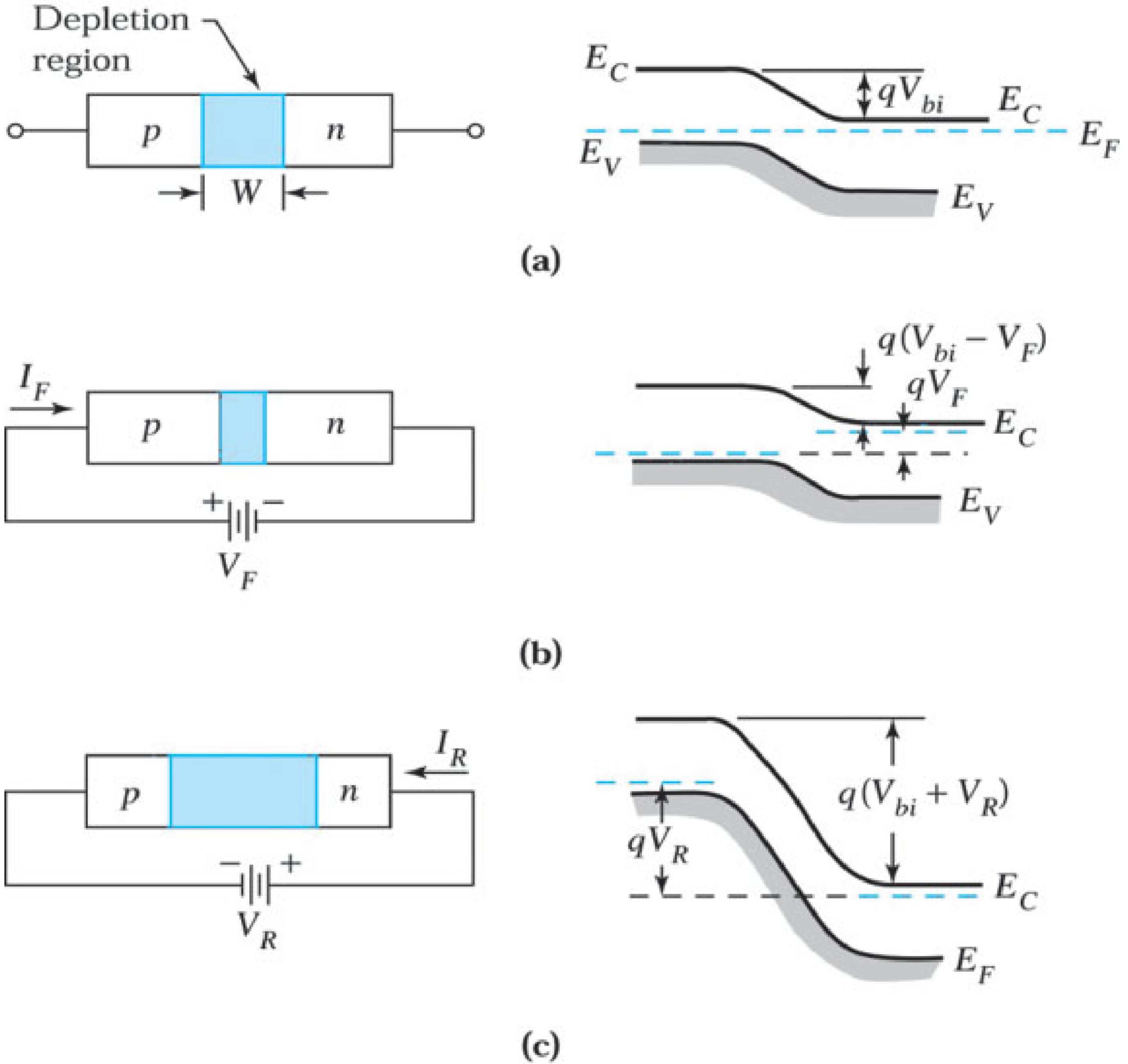

Fig. 8 **Schematic representations of depletion layer width and energy band diagrams of a *p–n* junction under various biasing conditions. (*a*) Thermal-equilibrium condition. (*b*) Forward-bias condition. (*c*) Reverse-bias condition.**

- The total electrostatic potential across the junction decreases by V_F; that is, it is replaced with $V_{bi} - V_F$.

$$\boxed{W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_{bi}}\,.} \qquad (20)$$

- Thus, forward bias reduces the depletion layer width.

- By contrast, as shown in *Fig. 8c*, if we apply positive voltage V_R to the n-side with respect to the p-side, the p–n junction now becomes reverse-biased and the total electrostatic potential across the junction increases by V_R: that is, it is replaced by $V_{bi} + V_R$.

Animation: https://www.pveducation.org/pvcdrom/pn-junctions/bias-of-pn-junctions

- Here, we find that reverse bias increases the depletion layer width.

- Substituting these voltage values in Eq. 21 yields the depletion layer widths as a function of the applied voltage for a one-sided abrupt junction:

$$\boxed{W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_{bi}}\,.} \tag{20}$$

$$\boxed{W \cong x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}}\,.} \tag{21}$$

$$\boxed{W = \sqrt{\frac{2\varepsilon_s (V_{bi} - V)}{qN_B}}\,,} \tag{27}$$

- where N_B is the lightly doped bulk concentration and V is positive for forward bias and negative for reverse bias.

- Note that the depletion layer width W varies as the square root(ریشه دوم) of the total electrostatic potential difference across the junction.

- **3.2.2 Linearly Graded Junction**

- We first consider the case of thermal equilibrium.

- The impurity distribution for a linearly graded junction is shown in Fig. 9a. The Poisson's equation for the case is

$$\frac{d^2\psi}{dx^2} = -\frac{d\mathscr{E}}{dx} = -\frac{\rho_s}{\varepsilon_s} = -\frac{q}{\varepsilon_s}ax \qquad -\frac{W}{2} \le x \le \frac{W}{2}, \tag{28}$$

- where *a* is the impurity concentration gradient (in cm^{-4}) and *W* is the depletion-layer width
- We have assumed that mobile carriers are negligible in the depletion region.
- By integrating Eq. 28 once with the boundary conditions that the electric field is zero at $\pm W/2$, we obtain the electric-field distribution shown in Fig. 9b:

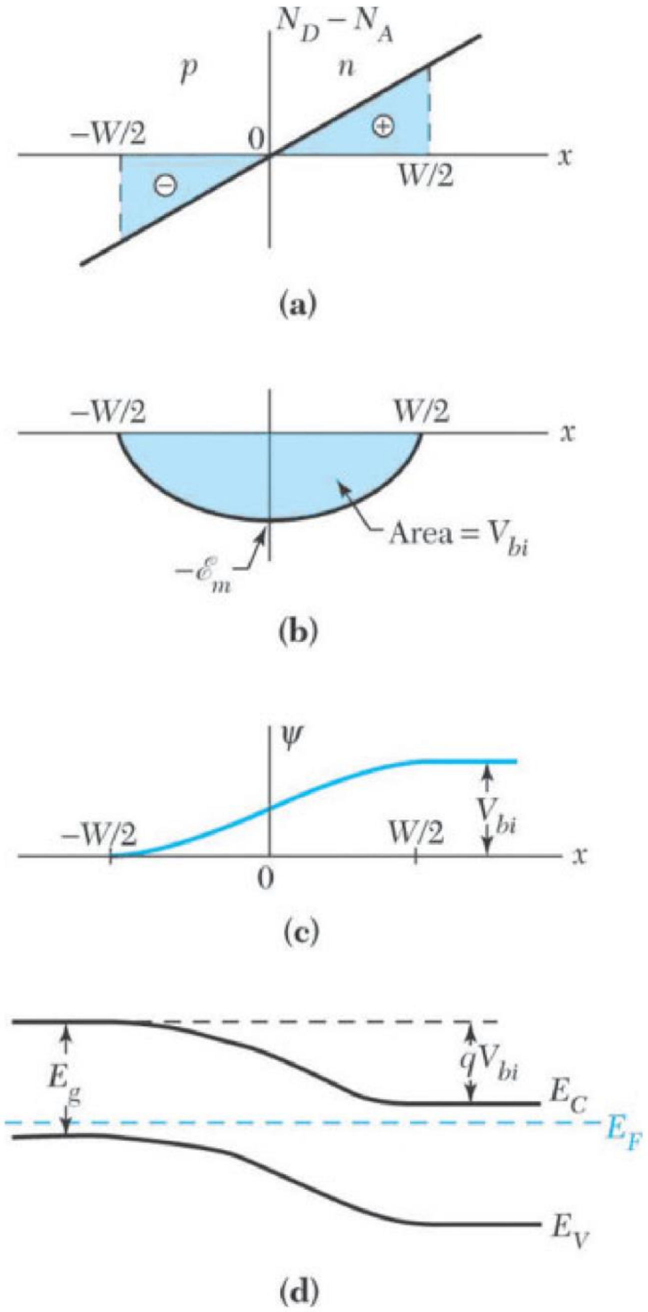

Fig. 9 Linearly graded junction in thermal equilibrium. (*a*) Impurity distribution. (*b*) Electric field distribution. (*c*) Potential distribution. (*d*) Energy band diagram.

$$\mathscr{E}(x) = -\frac{qa}{\varepsilon_s}\left[\frac{(W/2)^2 - x^2}{2}\right]. \tag{29}$$

The maximum field at x = 0 is

$$\mathscr{E}_m = \frac{qaW^2}{8\varepsilon_s}. \tag{29a}$$

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$$-\frac{d\mathcal{E}}{dx} = -\frac{q}{\varepsilon_s} a x \longrightarrow \int d\mathcal{E} = \int \frac{q}{\varepsilon_s} a x\, dx \quad (1)$$

الف) برای $-\frac{W}{2} \leq x < 0$

$$\xrightarrow{(1)} \int_0^{\mathcal{E}(x)} d\mathcal{E} = \frac{qa}{\varepsilon_s} \int_{-\frac{W}{2}}^{x} x\, dx$$

$$\longrightarrow \mathcal{E}(x) = \frac{qa}{2\varepsilon_s} x^2 \Big|_{-W/2}^{x} = \frac{qa}{2\varepsilon_s}\left(x^2 - \left(\frac{W}{2}\right)^2\right)$$

$$\longrightarrow \mathcal{E}(x) = -\frac{qa}{2\varepsilon_s}\left(\left(\frac{W}{2}\right)^2 - x^2\right) \rightarrow$$ همان رابطه ۲۹ کتاب

ب) برای $0 < x \leq \frac{W}{2}$

$$(1) \rightarrow \int_{\mathcal{E}(x)}^{0} d\mathcal{E} = \frac{qa}{\varepsilon_s} \int_{x}^{\frac{W}{2}} x\, dx \rightarrow 0 - \mathcal{E}(x) = \frac{qa}{2\varepsilon_s} x^2 \Big|_{x}^{\frac{W}{2}}$$

$$\longrightarrow \mathcal{E}(x) = -\frac{qa}{2\varepsilon_s}\left(\left(\frac{W}{2}\right)^2 - x^2\right) \rightarrow$$ همان رابطه ۲۹ کتاب

- Integrating Eq. 28 once again yields both the potential distribution and the corresponding energy band diagram shown in Figs. 9c and 9d, respectively.

- The built-in potential and the depletion layer width are given by

$$V_{bi} = \frac{qaW^3}{12\varepsilon_s} \tag{30}$$

and

$$W = \left(\frac{12\varepsilon_s V_{bi}}{qa}\right)^{1/3}. \tag{31}$$

اثبات رابطه ۳۰

$$\Psi = -\int \mathcal{E}\, dx = -\frac{qa}{2\varepsilon_0}\int \left(\frac{W^2}{4} - x^2\right) dx$$

$$= +\frac{qa}{2\varepsilon_0}\left(\frac{1}{4}W^2 x - \frac{x^3}{3}\right)\Bigg|_{-\frac{W}{2}}^{x}$$

$$= +\frac{qa}{2\varepsilon_0}\left[\left(\frac{1}{4}W^2 x - \frac{x^3}{3}\right) - \left(\frac{1}{4}W^2\left(-\frac{W}{2}\right) - \frac{1}{3}\left(-\frac{W}{2}\right)^3\right)\right]$$

$$\Psi = \left(\frac{1}{4}W^2 x - \frac{x^3}{3} + \frac{W^3}{8} - \frac{W^3}{24}\right)\left(+\frac{qa}{2\varepsilon_0}\right)$$

$$x = -\frac{W}{2} \Rightarrow \Psi = \left(\frac{1}{4}W^2\left(-\frac{W}{2}\right) - \frac{\left(-\frac{W}{2}\right)^3}{3} + \frac{W^3}{8} - \frac{W^3}{24}\right) \times \left(+\frac{qa}{2\varepsilon_0}\right)$$

$$\left(+\frac{qa}{2\varepsilon_0}\right)\left(\frac{-W^3}{8} + \frac{W^3}{24} + \frac{W^3}{24}\right) = 0$$

$$x = \frac{W}{2} \Rightarrow \Psi = \left(\frac{1}{4}\cancel{\left(+\frac{W}{2}\right)^2} W^2\left(\frac{W}{2}\right) - \frac{\left(\frac{W}{2}\right)^3}{3} + \frac{W^3}{8} - \frac{W^3}{24}\right)\left(+\frac{qa}{2\varepsilon_0}\right)$$

$$\Psi = +\frac{qa}{2\varepsilon_0}\left[\frac{W^3}{8} - \frac{W^3}{24} + \frac{W^3}{8} - \frac{W^3}{24}\right)$$

$$= +\frac{qa}{2\varepsilon_0}\left[\frac{W^3}{4} - \frac{W^3}{12}\right] = \frac{+qa}{2\varepsilon_0}\left(\frac{3W^3 - W^3}{12}\right)$$

$$= +\frac{qa}{2\varepsilon_0}\,\frac{W^3}{6} = +\frac{qaW^3}{12\varepsilon_0}$$

$$= V_{bi}$$

❖Since the values of the impurity concentrations at the edges of the depletion region (– *W/2* and *W/2*) are the same and both are equal to *aW/2*, the built-in potential for a linearly graded junction may be expressed in a form similar to Eq. 12

$$\boxed{V_{bi} = \psi_n - \psi_p = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right).} \tag{12}$$

$$V_{bi} = \frac{kT}{q}\ln\left[\frac{(aW/2)(aW/2)}{n_i^2}\right] = \frac{2kT}{q}\ln\left(\frac{aW}{2n_i}\right). \tag{32}$$

- When W is eliminated from Eqs. 31 and 32 yields the built-in potential as a function of a.

$$W = \left(\frac{12\varepsilon_s V_{bi}}{qa} \right)^{1/3}. \qquad (31)$$

$$V_{bi} = \frac{kT}{q} \ln\left[\frac{(aW/2)(aW/2)}{n_i^2} \right] = \frac{2kT}{q} \ln\left(\frac{aW}{2n_i} \right). \qquad (32)$$

- The results for silicon and gallium arsenide linearly graded junctions are shown in Fig.10.

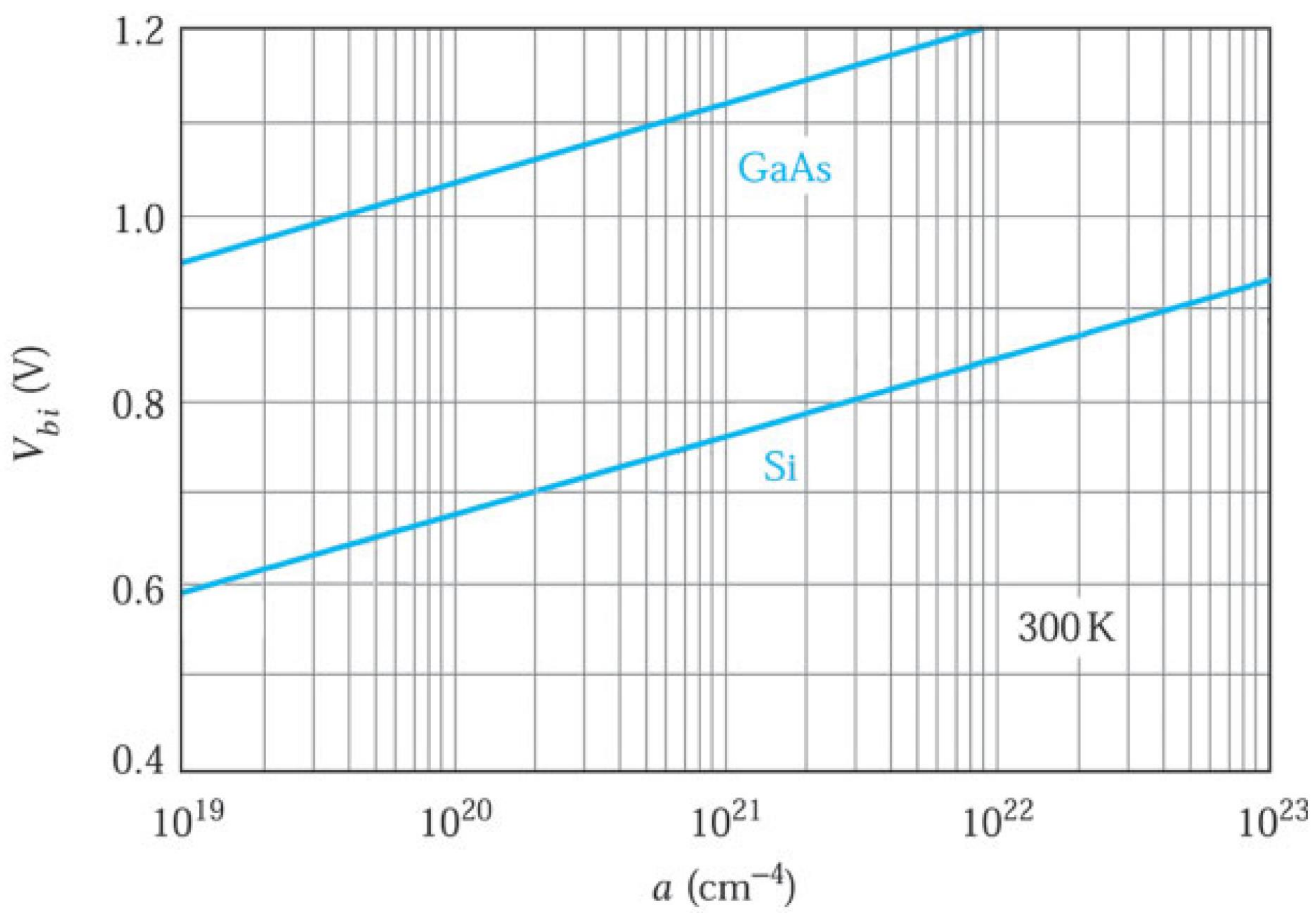

Fig. 10 **Built-in potential for a linearly graded junction in Si and GaAs as a function of impurity gradient.**

Forward and reverse bias

- When either forward or reverse bias is applied to the linearly graded junction, the variations of the depletion layer width and the energy band diagram will be similar to those shown in Fig. 8 for abrupt junctions.
- However, the depletion layer width will vary as $(V_{bi} - V)^{1/3}$, where V is positive for forward bias and negative for reverse bias.