Photonic Crystal Optics

What Is It All About?

- From:
  - Book: Photonics and Lasers An Introduction - Chapter 8
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Introduction (1)

- The conventional method
- **Utilize** Total Internal Reflection (TIR)

- **TIR** fundamental Principle Operation
- Nearly all Fiber Optic Devices
- Planar Waveguide Devices
Recently, for controlling the flow of light

**Utilize some modifications on microstructure of the cladding region**

![Diagram showing light propagation in modified cladding regions.]

- Light cannot propagate in there

\[
\text{The refractive index (n) varies periodically in space with a repetition distance on the order of the wavelength of light.}
\]
1D, 2D, 3D Photonic crystals
1D Photonic Crystals \{ Step-Index Grating (1) \}

- Bragg grating the simplest 1D PhC

\[ n_2 \] The refractive index of the slabs
\[ n_1 \] The refractive index of the medium between slabs
\[ \Delta n \equiv n_2 - n_1 \] The index difference
\[ \Lambda \] The center-to-center spacing of the slabs
\[ \Lambda/2 \] The thickness of each slab
\[ L = N\Lambda \] The total length of the photonic crystal
\[ E_i(x,t) \] The incident light wave’s electric field
1D Photonic Crystals \{ Step-Index Grating (2) \}

\[ E_t(x, t) = A \cos(\omega t - kx) = A \Re e^{i(\omega t - kx)} \] (1)

- The "Real Part" of equation
- The "angular frequency" of the light
- The wave vector magnitude
- Free-space wavelength

Assume \( \Delta n \ll 1 \rightarrow n_1 \approx n_2 \approx n \)

E field of light reflected from the \( j \)th interface at \( x = 0 \)

\[ E_{ri}(0, t) = \Re E_{ri} e^{i\omega t} \] (2)

- A complex amplitude
- A and a real number

At \( x = 0 \) \( E_{ri} = A \) and a real number

The total electric field of the reflected light by adding the contribution from each interface
1D Photonic Crystals \{ Step-Index Grating (3) \}

- The **Fresnel equations** for the reflected and transmitted $E$ fields

- For $p$ (parallel) polarization (TM)

$$\left( \frac{E_r}{E_i} \right)_\parallel = \frac{n_1 \cos \theta_2 - n_2 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1}$$

$$\left( \frac{E_t}{E_i} \right)_\parallel = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1} \quad \text{(chap 2.11)}$$

- For $s$ polarization (TE) \( (\text{senkrecht German for perpendicular}) \)

$$\left( \frac{E_r}{E_i} \right)_\perp = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

$$\left( \frac{E_t}{E_i} \right)_\perp = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \quad \text{(chap 2.12)}$$

- Reflections

  - Incident from index $n1$
  - Incident from index $n2$

The **Fresnel equations** at normal incidence $\theta = 0$

$$r = \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2} \approx -\frac{\Delta n}{2n} \quad \text{(3)}$$

$E_i$ and $E_r$ are evaluated just before and after reflection.
1D Photonic Crystals \{ Step-Index Grating (4) \}

- Reflections
  - Incident from index $n_1$
  - Incident from index $n_2$

  obey the same relation, except
  that $n_1$ and $n_2$ are interchanged
  so $r = \frac{\Delta n}{2n}$

- Assuming $|r| \ll 1$ \(\rightarrow\) The total reflected field at $x = 0$ due to reflections of the first type

$$\widetilde{E}_{r,12} = -\frac{\Delta n}{2n} A [1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad (4)$$

$$\delta - k(2\Lambda) - \frac{2\pi n}{\lambda_0} (2\Lambda) - \frac{4\pi nA}{\lambda_0} \quad (5)$$  \text{phase delay} due to propagation over the round-trip distance $2\Lambda$ between slabs

$$\widetilde{E}_{r,21} = \frac{\Delta n}{2n} A e^{-i\delta/2} [1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad (6)$$  \text{total reflected field of the second type}
1D Photonic Crystals \{ Step-Index Grating (5) \}

\[ \widetilde{E}_{r,21} = \frac{\Delta n}{2n} A e^{-i\delta/2} \left[ 1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta} \right] \quad (6) \]  

- Additional round-trip propagation distance \( 2\left(\Delta/2\right) \) for each type 2 reflection

✓ Adding the terms inside the square brackets
  - Visualize them as vectors
  - Each vector has the same magnitude
  - \( 0, -\delta, -2\delta, \ldots \) are the angles from the real axis

➢ First-Order Bragg diffraction \( (m = 1) \)

\[ \lambda_{B} = 2n\lambda \quad (7) \]  

Bragg wavelength in free-space
1D Photonic Crystals \{ Step-Index Grating (6) \}

- $\delta = 2\pi$ \quad \text{Satisfies Bragg condition} \quad \exp(-i\delta/2) = -1 \quad \tilde{E}_{r,12} = \tilde{E}_{r,21}$

- **Total reflected** complex amplitude from all interfaces

$$\tilde{E}_r = \tilde{E}_{r,12} + \tilde{E}_{r,21} = -\frac{\Delta n}{n} A [1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad \text{(8)}$$

- **N unity terms**

$$\tilde{E}_r = -\frac{\Delta n}{n} A N$$

- Power reflectivity

$$\chi = 0 \quad \tilde{E}_{rf} = \bar{A}$$

$$R_{\text{max}} = \left| \frac{\tilde{E}_r}{\tilde{E}_i} \right|^2 = \left| -\frac{\Delta n}{n} \right|^2 = N^2 \left( \frac{\Delta n}{n} \right)^2 \quad \text{(9)}$$

$$L = N\lambda \quad \frac{\lambda_B}{\lambda} = 2n\Lambda$$

$$R_{\text{max}} = \left( \frac{L}{\lambda} \frac{\Delta n}{n} \right)^2 = \left( \frac{2\Delta nL}{\lambda_B} \right)^2 \quad \text{(10)}$$

$$R_{\text{max}} = (\kappa L)^2 \quad \text{(11)}$$

peak reflectivity, **weak grating**

$$\kappa \equiv \frac{2\Delta n}{\lambda_B}$$

**Attenuation factor** of light as it propagates through the **Bragg grating**
1D Photonic Crystals \{ Step-Index Grating (7) \}

- The incident wavelength slightly detuned from $\lambda_B$:
  - Phases progressively further apart

\[ \vec{E}_r = \vec{E}_{r,12} + \vec{E}_{r,21} \]
\[ \frac{\Delta n}{n} \left[ 1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta} \right] \]

Phases difference $\phi = -\delta$

- If the wavelength changes by $\Delta \lambda$:
  - $N \Delta \phi = 2\pi$  \[ \Rightarrow \Delta \phi = -\Delta \delta \]
  - The grating reflectivity goes from a maximum to zero over a range of wavelengths $\Delta \lambda$

$\Delta \lambda$ spectral half-width of the Bragg resonance

Slightly off resonance small difference in phase between vectors

Vectors add to zero destructive interference
1D Photonic Crystals \{ Step-Index Grating (8) \}

- Evaluation of $\Delta \lambda$
  \[ \delta = k(2\lambda) = \frac{2\pi n}{\lambda_0} (2\lambda) = \frac{4\pi n\lambda}{\lambda_0} \] (5)
  \[ \Delta \phi = -\Delta \delta = -\frac{d\delta}{d\lambda} \Delta \lambda = \frac{4\pi n\lambda}{\lambda_0^2} \Delta \lambda \] (12)

- Near the Bragg resonance $\lambda_0 \approx \lambda_B$
  $\lambda_B = 2n\Lambda$
  $\Delta \phi = 2\pi/N$

- Quality factor
  \[ Q = \frac{\lambda}{\Delta \lambda} \rightarrow Q \approx N \]

- If the incident wavelength is detuned from $\lambda_B$ by more than $\Delta \lambda$
  continue to curl around in the complex plane
  a secondary maximum $N\Delta \phi \approx 3\pi$
  another zero $N\Delta \phi = 4\pi$
  spectral half-width weak grating

\[ \frac{2\pi}{N} = \frac{2\pi \lambda_B}{\lambda_0^2} \Delta \lambda \]

\[ \frac{\Delta \lambda}{\lambda_0} = \frac{1}{N} \]
1D Photonic Crystals { Step-Index Grating (9) }

This pattern continues with increasing detuning

Oscillatory dependence of reflectivity on wavelength

\[ \frac{\Delta \lambda}{\lambda_B} = \frac{1}{25} \]

Grating reflectivity \( R / R_{\max} \)

\[ \lambda_0 / \lambda_B \]
1D Photonic Crystals \{ Sinusoidal Index Grating (1) \}

- The refractive index \( n \) in a Bragg grating often \textbf{varies sinusoidally} with position

\[ \Lambda = \frac{\lambda_p}{2 \sin \theta} \]

\textbf{Separation between planes of maximum intensity}

\textit{fiber Bragg grating}
1D Photonic Crystals \{ Sinusoidal Index Grating \(2\) \}

- If **pump intensity** and **exposure time** is short enough
  - Resulting index variation may be expressed by the sinusoidal form

\[
n(x) = \bar{n} + \Delta n \cos\left(\frac{2\pi x}{\Lambda}\right)
\]

- **Bragg reflection** wavelength: \(\lambda_B = \frac{2\pi a}{\Lambda}\)
- Resonance half-width: \(\Delta\lambda = \frac{\lambda_0}{N}\)
- Peak reflectivity: \(R_{\text{max}} \approx (\kappa L)^2\)

**One small difference**
- Sinusoidal grating: \(\kappa \equiv \frac{\pi\Delta n}{\lambda_B}\)
- Step-index grating: \(\kappa \equiv \frac{2\Delta n}{\lambda_B}\)
1D Photonic Crystals \{ Sinusoidal Index Grating (2) \}

Arbitrary $\kappa L$ is of interest \( \rightarrow \) coupled mode theory

In the limit $\kappa L \ll 1$ \( \rightarrow \) $R_{\text{max}} \approx (\kappa L)^2$

In the limit $\kappa L \gg 1$ \( \rightarrow \) $R_{\text{max}} \rightarrow 1$.

\[
N_{\text{eff}} = \frac{L_{\text{eff}}}{\Lambda} = \left(\frac{\lambda_B}{\Delta n}\right)\frac{2n}{\lambda_B} = \frac{2n}{\Delta n}
\]

\[
\frac{\Delta \lambda}{\lambda_0} = \frac{1}{N_{\text{eff}}} = \frac{\Delta n}{2n}
\]

\[
\frac{\Delta \lambda}{\lambda_0} = \begin{cases} 
\frac{\lambda_0}{(2nL)} & \text{for } \kappa L \ll 1 \\
\frac{\Delta n}{(2n)} & \text{for } \kappa L \gg 1
\end{cases}
\]

\[
R_{\text{max}} = \tanh^2(\kappa L) \quad \text{(peak reflectivity, arbitrary } L)\]

\[E(x) = E_0 e^{-\kappa x}\]

E field at resonance decreases

$L_{\text{eff}} = \frac{\pi}{\kappa}$

incident light

reflected light

$N_{\text{eff}}$ layers interact with light
1D Photonic Crystals \{ Sinusoidal Index Grating (2) \}

- Most important application of Bragg gratings: mirrors in a laser cavity

- High reflectivity that is obtained for $\lambda L \gg 1$ → Selectivity for the laser output

- Fiber Bragg grating as a sensor → physical parameters:
  - Strain
  - Temperature change
  - Pressure change
1D Photonic Crystals \{ Photonic Band Gap (1) \}

- One of the important characteristics of a Bragg grating
  - Exponential attenuation of light for wavelengths close to $\lambda_B$
  - For a sufficiently long grating $\rightarrow$ 100% reflection

The range of wavelengths for which light is attenuated is referred to as the *stop band*.

A frequency gap in the *photon spectrum* is known as a *photonic band gap*.

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![Bragg reflection diagram](image)
1D Photonic Crystals \{ Photonic Band Gap (1) \}

- There are two values of the light frequency \( k = \frac{\pi}{\Lambda} \).
- The wave is not propagating, but is actually a standing wave.
- Physical origin of two frequency \( \omega \) intensity profiles.

**Effective index** relates frequency \( \omega \) wave vector \( \frac{c}{n_{\text{eff}}} \).

\[
\omega = \left( \frac{c}{n_{\text{eff}}} \right) k = \left( \frac{c}{n_{\text{eff}}} \right) \frac{\pi}{\Lambda}
\]

\( (23) \)
1D Photonic Crystals \{ Photonic Band Gap (2) \}

- Calculating $\Delta \omega$

\[
\omega = \frac{2\pi c}{\lambda_0}
\]

\[
\Delta \omega = -\frac{2\pi c}{\lambda_0^2} \Delta \lambda = -\frac{2\pi c}{\lambda_0} \cdot \frac{\Delta \lambda}{\lambda_0}
\]

\[
\frac{\Delta \omega}{\omega} = \frac{\Delta \lambda}{\lambda_0} = \frac{\Delta n}{2n}
\]

\[
\frac{\Delta \lambda}{\lambda_0} = \begin{cases} 
\frac{\lambda_0}{2nL} & \text{for } \kappa L \ll 1 \\
\frac{\Delta n}{2n} & \text{for } \kappa L \gg 1
\end{cases}
\]

\[
\frac{\text{frequency gap}}{\text{center frequency}} = \frac{2\Delta \omega}{\omega} = \frac{\Delta n}{n}
\]
1D Photonic Crystals \{ Localized Modes (1) \}

- If we have some frequencies within the photonic band gap
  
  Light wave's \( E \) field are exponentially attenuated

- If we create discontinuity or defect in the structure

  Light energy residing in optical mode confined to vicinity of the defect

- Simplest type of defect of a photonic crystal
  
  edge
  
  surface

- 1D Bragg grating defects
  
  begin point of grating

  removing one or more of the high-index layers
1D Photonic Crystals \{ Localized Modes (2) \}

Layers are removed \rightarrow\ The result is a localized mode

an **optical cavity** formed by **two semiinfinite** Bragg gratings
2D Photonic Crystals

2-D Triangular Lattice - Holes in a Substrate with higher n

2-D Square lattice - Rods in Air

Two symmetry directions x and x' are shown.
Planar Geometry (1)

- For 2D photonic crystals
  - Propagation mostly in the x–y plane
  - Propagate mostly along the z axis

1) Propagation in the x–y plane and Square Lattice

- Propagating along +X axis → refractive index spatially vary
  - Like 1D Bragg grating

- Bragg condition ✓ → light will be strongly reflected, and gap opens in dispersion relation

- 2D Difference with 1D Bragg grating → 2D is periodic in 2 direction x, x'}
Planar Geometry (2)

2 directions \[\rightarrow\] 2 Band gaps \[\rightarrow\] Different position and width

\(\omega\) \[\rightarrow\] \(\omega\)

\(\text{band gap for TM}\)

\(g_x\) \(2g_x\) \(k_x\)

\(g_m\) \(2g_m\) \(k_x'\)

\(\text{complete photonic band gap}\)

\((a)\) TM \(E \parallel \) rods

\(\text{No overlap – no CPBG}\)

\(\omega\) \[\rightarrow\] \(\omega\)

\(\text{no band gap for TE}\)

\(g_x\) \(2g_x\) \(k_x\)

\(g_m\) \(2g_m\) \(k_x'\)

\(\text{(b) TE}\)

\(E \perp \) rods
Planar Geometry (3)

**TE polarization**  
*E field along y and perpendicular to rods (with B along z)*

**TM polarization**  
*E field along z (with B along –y and perpendicular to the rods)*

- **Physical origin** of difference between **TM** and **TE** polarization

   - **Intensity peaks in the standing waves**
Planar Geometry (3)

confine light in two dimensions

band gaps for different directions and polarizations all overlap in some frequency range

1. A square lattice of dielectric rods

There is PBG for both TE and TM polarization

⇒ Does not overlap ⇒ No CPBG

2. A triangular lattice of air holes

Overlap ✓
2D Photonic Crystals - Fiber Geometry

2. Light propagating mostly in the \textit{z direction}

- component perpendicular to the \textit{rods} or \textit{air holes} are \textit{Negligible}

Usage: \textit{optical fiber}

- Guiding types in optical fibers
  - Total internal reflection (TIR)
  - Photonic band gap (PBG)
Guiding by Effective Index (1)

Early *photonic crystal fiber* called *holey fiber*

Single missing air hole, allows confinement of light by TIR

- Advantages
  - only one glass type
  - no doping
  - Cladding region being “doped” with air holes
Guiding by Effective Index (2)

- Light guidance of Photonic crystal fibers
- Single mode
- Multi mode

- Single-mode guidance of light in conventional step-index fiber

- Single-mode guidance for holey fiber

- Effective cladding index is not a constant \( V < 2.405 \)

\[
V = \frac{2\pi a}{\lambda_0} \sqrt{n_i^2 - n_2^2}
\]

- Shorter wavelengths, light is better confined and index is higher
- Longer wavelengths, diffraction prevents the light from being well confined

- Glass region between the air holes should be sufficiently wide \( V_{PCF} \leq \pi \) → \( \checkmark \) diffraction

holey fiber is also called Endlessly single-mode
Guiding by Effective Index (3)

- Boundaries between single-mode and multimode

This fibers can be scaled up

- high-power fiber lasers
- Frequency conversion

\[
\frac{d}{\Lambda} \quad \text{increases}
\]
\[
\frac{\lambda}{\Lambda} \quad \text{decreases}
\]
Guiding by Photonic Band Gap (1)

- Condition of Photonic band-gap in Crystal type (triangular lattice of air holes)
  \[ \varepsilon_r > 7.2 \]
  \[ \eta > 2.7 \]
  substrate material

- (n) Silica glass <1.5 \( \rightarrow \) Seems no Photonic band-gap guiding for holey fiber

- \( \varepsilon_r > 7.2 \) is only for light propagating perpendicular to the air holes (TM)

- For propagation parallel to holes (TE) if one use proper geometry of air holes

Complete PBG

Hollow-core fiber
Guiding by Photonic Band Gap (2)

- PBG
- Photonic crystal structure
- Propagating direction of light → β axial wave vector

- Modes with larger β → directed more nearly down the fiber axis

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Guiding by Photonic Band Gap (3)

- **Absorption** and **Rayleigh scattering** in the fiber core → much lower in hollow-core fiber
  - Less amplification for further distance

- Much higher **optical powers**

- Less **dispersion** → Very short optical pulses propagate without significant spreading in time

All because light is propagating mostly in **air**, rather than in the **glass**
In 1987, the concept of the 3-D photonic crystal was introduced independently by E. Yablonovitch and S. John.

- **E. Yablonovitch**: Improving the efficiency of semiconductor **laser** devices.
- **S. John**: "Localizing" or confining light to a small region of space.

A material with a **complete photonic band gap** can be achieved.

- **FCC cubic**: Diamond-type
  - *Need n > 1.87*
- **Opal**: Inverse opal
  - *Need n > 2.8 Narrow band gap*
3D Photonic Crystals (2)

- Creation Methods
  - "Top-down" approach

  - Yablonovite
  - Lincoln log

Figure 8-21 Yablonovite is formed by drilling holes into a dielectric at three precise angles, and results in a diamond-like structure. The holes can be created by exposure to X-rays through a mask.
3D Photonic Crystals (3)

- Creation Methods
  - "Top-down" approach

Figure 8-22 (a) The "Lincoln log" or "woodpile" structure has a diamond-type lattice symmetry, and exhibits a 3-D photonic band gap.
(b) Calculated density of states when \( n \) bars = 3.6 and fill 28% of the space.

density of states (number of propagating modes per unit frequency interval)
3D Photonic Crystals (4)

- Creation Methods

  - "bottom-up" or "self-assembly" approaches

    - **self-assembling** of **SiO₂ Spheres** with diameter of 100 to 1000 nm in the fcc structure

    - **Injecting** a high-index material between the spheres

    - **selective etching** of the original SiO₂ material

In "**Top-down**" approach for a given refractive index wider **band gap** can be achieved

In "**bottom-up**" approach better for **mass-production** levels