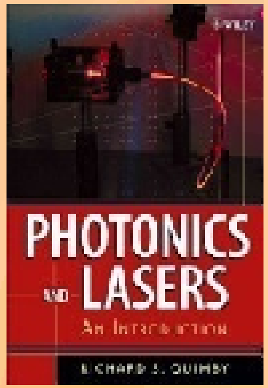
Photonic Crystal Optics

What Is It All About?

- From:
  - Book: **Photonics and Lasers An Introduction** - **Chapter 8**
  - Author: **Richard S. Quimby**



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DEC 2011

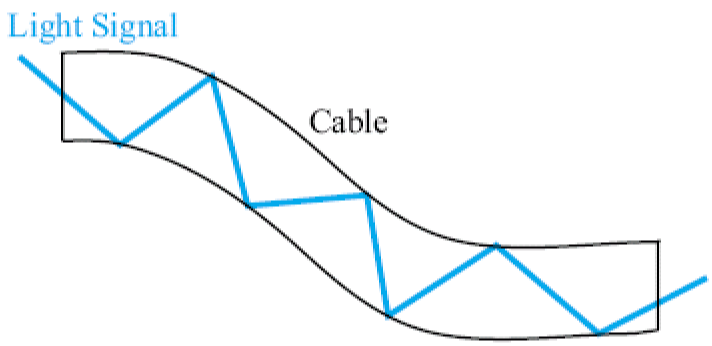
Photonic Crystal Optics

Contents

main topics in our discussion

Introduction (1)

- The conventional method **Utilize** → Total Internal Reflection (TIR)



- **TIR** fundamental Principle Operation
  - Nearly all Fiber Optic Devices
  - Planar Waveguide Devices

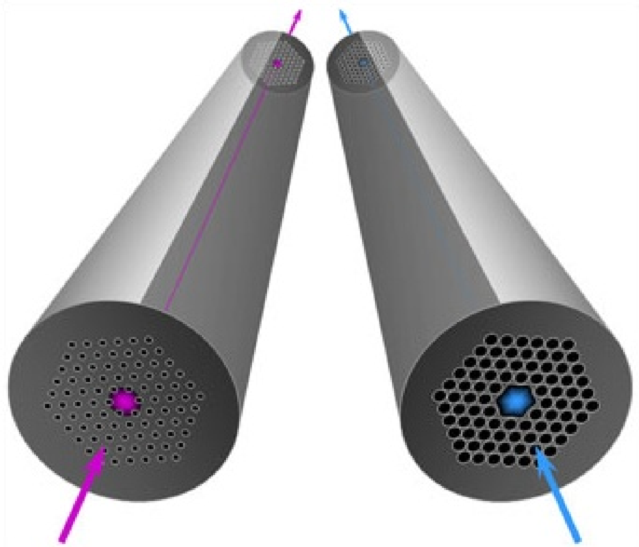
Introduction (2)

- Recently, for controlling the flow of light

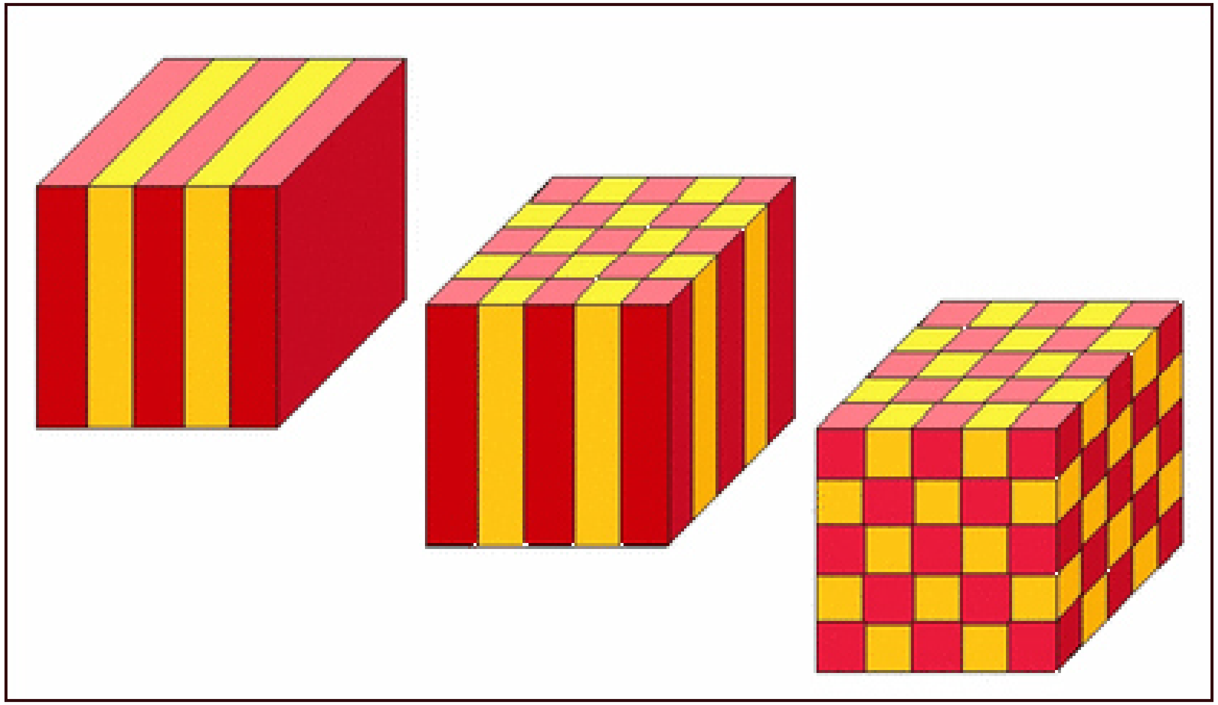
Utilize some modifications on microstructure of the cladding region

→ Light cannot propagate in there

{ The **refractive index (n)** varies periodically in space
with a repetition distance on the order of the wavelength of light }



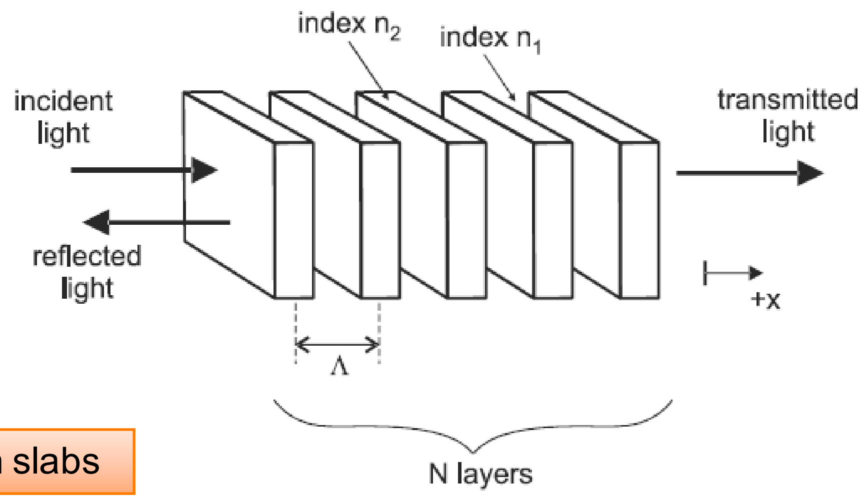
Introduction (3)



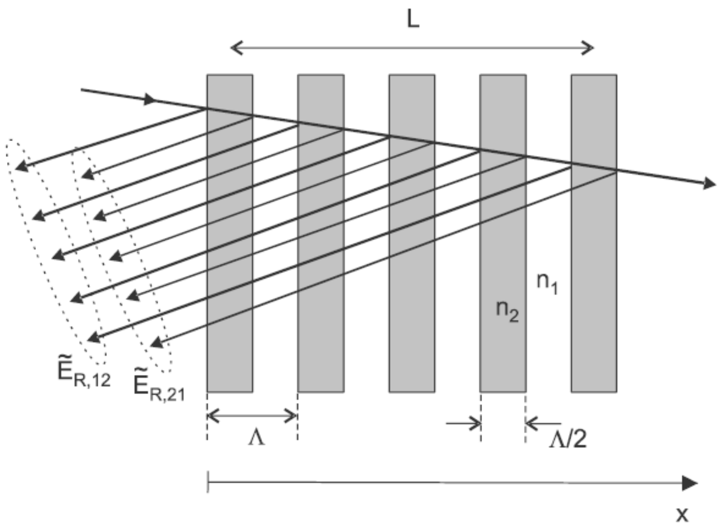
1D, 2D, 3D Photonic crystals

1D Photonic Crystals { Step-Index Grating (1) }

❖ Bragg grating the simplest 1D PhC



n_2	The refractive index of the slabs
n_1	The refractive index of the medium between slabs
$\Delta n \equiv n_2 - n_1$	The index difference
Λ	The center-to-center spacing of the slabs
$\Lambda/2$	The thickness of each slab
$L = N\Lambda$	The total length of the photonic crystal
$E_i(x,t)$	The **incident light** wave's electric field



1D Photonic Crystals { Step-Index Grating (2) }

$$E_i(x,t) = A\cos(\omega t - kx) = A\Re e^{i(\omega t - kx)} \quad (1)$$

$\Re$	The "**Real Part**" of equation
ω	The "**angular frequency**" of the light
$k = 2\pi n/\lambda 0$	The **wave vector** magnitude
$\lambda 0$	Free-space wavelength

❖ Assume $\Delta n \ll 1 \longrightarrow n_1 \approx n_2 \approx n$

E field of light reflected from the jth interface at $x = 0$

$$E_{rj}(0,t) \equiv \Re \tilde{E}_{rj} e^{i\omega t} \quad (2)$$

$\tilde{E}_{rj}$ — A complex amplitude → relative phase, magnitude

At $x = 0$ — $\tilde{E}_{rj} = A$ and a real number

The total electric field of the reflected light ⇒ by adding the contribution from each interface

1D Photonic Crystals { Step-Index Grating (3) }

- The *Fresnel equations for the reflected and transmitted E fields*

- *For **p** (parallel) polarization (TM)*

$$\left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$

$$\left(\frac{E_t}{E_i}\right)_{\parallel} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$

(chap 2.11)

- For **s** polarization (TE)
(*senkrecht* German for perpendicular)

$$\left(\frac{E_r}{E_i}\right)_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$\left(\frac{E_t}{E_i}\right)_{\perp} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

(chap 2.12)

- Reflections
  - Incident from index *n1*
  - Incident from index *n2*

The *Fresnel equations* at normal incidence $\theta = 0$

$$r = \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2} \cong -\frac{\Delta n}{2n} \quad (3)$$

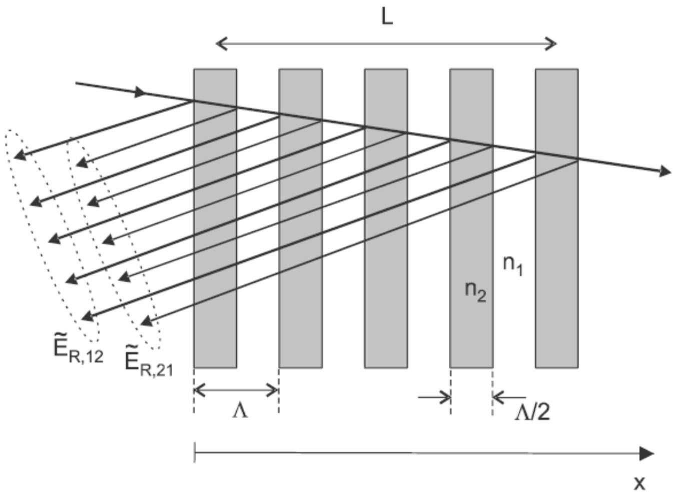
Ei and Er are evaluated just before and after reflection

1D Photonic Crystals { Step-Index Grating (4) }

- Reflections
  - Incident from index *n1*
  - Incident from index *n2*

obey the same relation, except that ***n1*** *and* ***n2*** *are interchanged*

so $r \approx \Delta n / 2n$



- Assuming $|r| \ll 1 \longrightarrow$ The total reflected field at $x = 0$ *due to reflections of the* ***first type***

$$\tilde{E}_{r,12} = -\frac{\Delta n}{2n} A[1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad (4)$$

$$\delta = k(2\Lambda) = \frac{2\pi\, n}{\lambda_0}(2\Lambda) = \frac{4\pi\, n\Lambda}{\lambda_0} \quad (5)$$

phase delay due to propagation over the round-trip distance 2Λ between slabs

$$\tilde{E}_{r,21} = \frac{\Delta n}{2n} A e^{-i\delta/2} [1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad (6)$$

total reflected field of the **second type**

1D Photonic Crystals { Step-Index Grating (5) }

$$\tilde{E}_{r,21} = \frac{\Delta n}{2n} A e^{-i\delta/2} \left[1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}\right] \quad (6)$$

total reflected field of the **second type**

❖ Additional round-trip propagation distance $2\left(\Lambda/2\right)$ for each **type 2** reflection

✓ Adding the terms inside the **square brackets**

❖ Visualize them as vectors

❖ Each vector has the same magnitude

❖ $0, -\delta, -2\delta, \ldots$ are the **angles from** the real axis

constructive interference

Bragg resonance, all vectors in phase

when $\delta = m2\pi$

m *is an integer* giving the order of diffraction

➢ First-Order Bragg diffraction (***m = 1***)

$$\lambda_B = 2n\Lambda \quad (7)$$

Bragg wavelength in free-space

1D Photonic Crystals { Step-Index Grating (6) }

- $\delta = 2\pi \xrightarrow{\text{Satisfies}}$ Bragg condition $\longrightarrow \exp(-i\delta/2) = -1 \longrightarrow \tilde{E}_{r,12} = \tilde{E}_{r,21}$

✓ **Total reflected** complex amplitude from all interfaces

$$\tilde{E}_r = \tilde{E}_{r,12} + \tilde{E}_{r,21} = -\frac{\Delta n}{n} A[1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad (8)$$

✓ *N* ***unity*** *terms*

$\delta = 2\pi$

$$\tilde{E}_r = -\frac{\Delta n}{n} AN$$

✓ Power reflectivity $\xrightarrow[\tilde{E}_{rj} = A]{x=0}$

$$R_{\max} = \left|\frac{\tilde{E}_r}{\tilde{E}_i}\right|^2 = \left|-\frac{\Delta n}{n} N\right|^2 = N^2\left(\frac{\Delta n}{n}\right)^2 \quad (9)$$

$$\xrightarrow[\lambda_B = 2n\Lambda]{L = N\Lambda} \quad R_{\max} = \left(\frac{L}{\Lambda}\frac{\Delta n}{n}\right)^2 = \left(\frac{2\Delta n L}{\lambda_B}\right)^2 \quad (10) \quad \longrightarrow \quad R_{\max} = (\kappa L)2 \quad (11) \quad \xrightarrow[\frac{\Delta n}{n} \ll 1]{\kappa L \ll 1} \quad \boxed{R_{\max} \ll 1}$$

peak reflectivity, **weak grating**

$\xrightarrow{\kappa \equiv 2\Delta n/\lambda B}$ **Attenuation factor** of light as it propagates through the **Bragg grating**

1D Photonic Crystals { Step-Index Grating (7) }

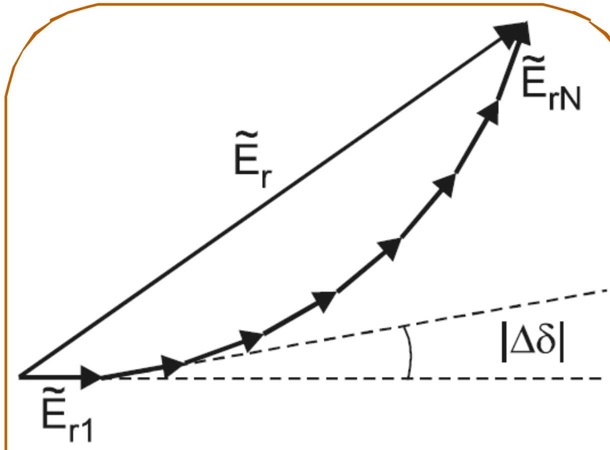
❖ The incident wavelength slightly detuned from λ_B

➡ **phases** progressively further apart

$$\tilde{E}_r = \tilde{E}_{r,12} + \tilde{E}_{r,21}$$

$$= -\frac{\Delta n}{n} A[1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad (8)$$

Phases difference $\phi = -\delta$

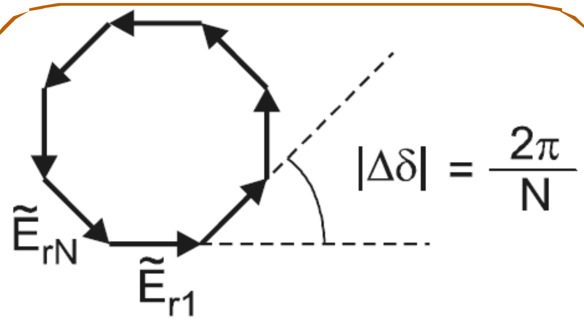


Slightly off resonance
small difference in phase
between vectors

❖ If the wavelength changes by $\Delta\lambda \longrightarrow \Delta\phi = -\Delta\delta$

$N\,\Delta\phi = 2\pi \longrightarrow$ The grating reflectivity goes from a **maximum** to **zero** over a range of wavelengths $\Delta\lambda$

$\Delta\lambda$ spectral half-width of the Bragg resonance



vectors add to zero
destructive interference

1D Photonic Crystals { Step-Index Grating (8) }

- Evaluation of $\Delta\lambda$

$$\delta = k(2\Lambda) = \frac{2\pi n}{\lambda_0}(2\Lambda) = \frac{4\pi n\Lambda}{\lambda_0} \qquad (5)$$

$$\Delta\phi = -\Delta\delta = -\frac{d\delta}{d\lambda}\Delta\lambda = \frac{4\pi n\Lambda}{\lambda_0^2}\Delta\lambda \qquad (12)$$

- Near the Bragg resonance $\lambda_0 \approx \lambda_B$

$$\lambda_B = 2n\Lambda$$

$$\Delta\phi = 2\pi/N$$

$$\frac{2\pi}{N} = \frac{2\pi\lambda_B}{\lambda_0^2}\Delta\lambda$$

$$\frac{\Delta\lambda}{\lambda_0} = \frac{1}{N}$$

spectral half-width

weak grating

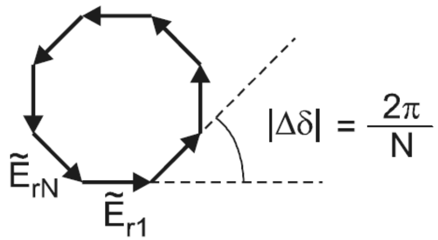
- *Quality factor* $\rightarrow$ $Q = \dfrac{\lambda}{\Delta\lambda}$ $\rightarrow$ $Q \approx N$

- If the incident wavelength is detuned from λ_B *by more than* $\Delta\lambda$

➡ continue to curl around in the complex plane

a secondary maximum $N\Delta\phi \simeq 3\pi$

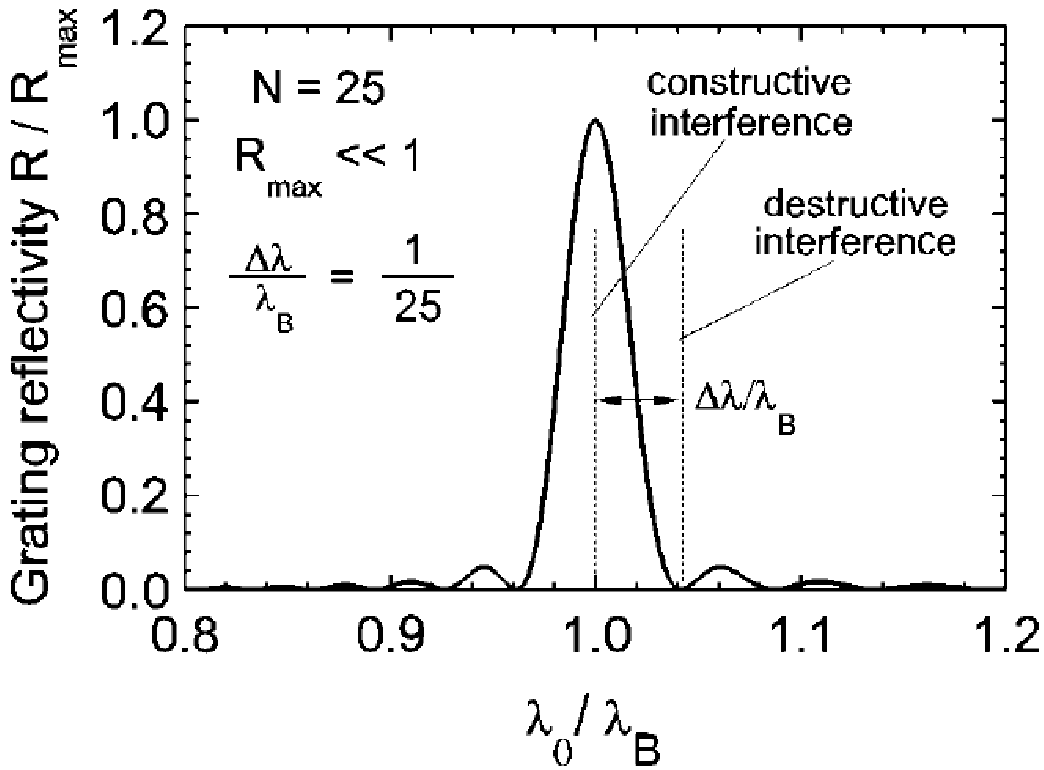
another zero $N\Delta\phi = 4\pi$



1D Photonic Crystals { Step-Index Grating (9) }

This pattern **continues** with increasing detuning

Oscillatory dependence of reflectivity on wavelength



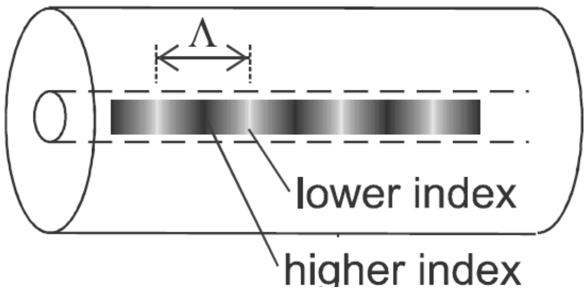
1D Photonic Crystals { Sinusoidal Index Grating (1) }

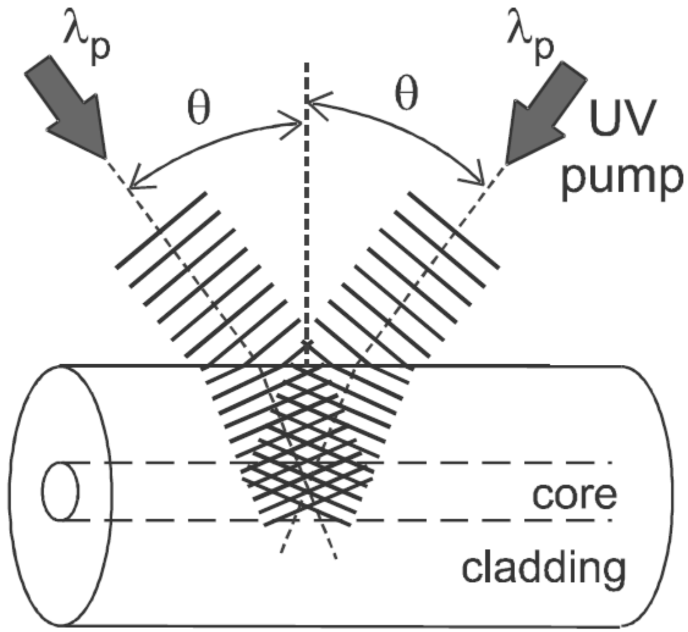
➢ The refractive index (**n**) in a Bragg grating often **varies** sinusoidally with position

fiber Bragg grating

$$\Lambda = \frac{\lambda_p}{2 \sin \theta}$$

Separation between planes of **maximum intensity**





1D Photonic Crystals { Sinusoidal Index Grating (2) }

❖ If **pump intensity** and **exposure time** is short enough

- Resulting index variation may be expressed by the sinusoidal form

$$\boxed{n(x) = \bar{n} + \Delta n \cos\left(\frac{2\pi x}{\Lambda}\right)}$$

average index

spacing between index maxima

$$\left\{\begin{array}{ll} \lambda_B = 2n\Lambda & \textit{Bragg reflection}\text{ wavelength} \\ \Delta\lambda = \dfrac{\lambda_0}{N} & \text{Resonance half-width} \\ R_{max} \approx (\kappa L)^2 & \text{Peak reflectivity} \end{array}\right.$$

One small difference

attenuation constant

sinusoidal grating $\longrightarrow$ $\kappa \equiv \dfrac{\pi \Delta n}{\lambda_B}$

step-index grating $\longrightarrow$ $\kappa \equiv 2\Delta n / \lambda_B$

1D Photonic Crystals { Sinusoidal Index Grating (2) }

Arbitrary κL *is of interest* $\xrightarrow{\text{coupled mode theory}}$

$$R_{\max} = \tanh^2(\kappa L)$$

(peak reflectivity, arbitrary *L*)

In the limit $\kappa L \ll 1 \longrightarrow R_{\max} \approx (\kappa L)^2$

In the limit $\kappa L \gg 1 \longrightarrow R_{\max} \to 1.$

$$N_{\text{eff}} = \frac{L_{\text{eff}}}{\Lambda} = \left(\frac{\lambda_B}{\Delta n}\right)\left(\frac{2n}{\lambda_B}\right) = \frac{2n}{\Delta n}$$

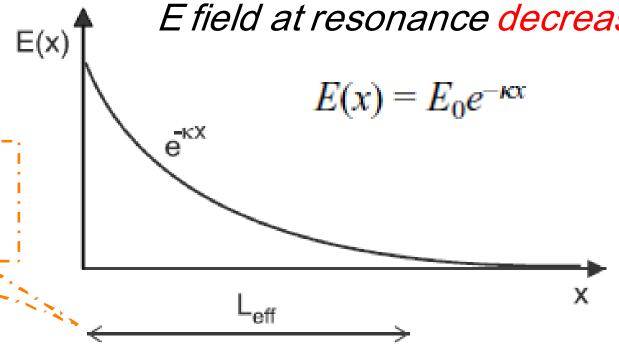
$$\downarrow$$

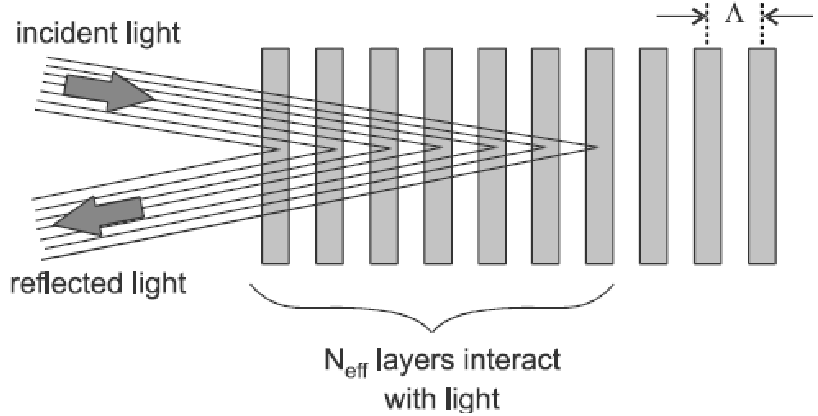
$$\frac{\Delta\lambda}{\lambda_0} = \frac{1}{N_{\text{eff}}} = \frac{\Delta n}{2n}$$

$$L_{\text{eff}} \equiv \pi/\kappa$$

E field at resonance decreases

$$E(x) = E_0 e^{-\kappa x}$$





$$\frac{\Delta\lambda}{\lambda_0} = \begin{cases} \lambda_0/(2nL) & \text{for } \kappa L \ll 1 \\ \Delta n/(2n) & \text{for } \kappa L \gg 1 \end{cases}$$

1D Photonic Crystals { Sinusoidal Index Grating (2) }

✓ Most important application of Bragg gratings → **mirrors in a laser cavity**

✓ High reflectivity that is obtained for $\kappa L \gg 1$ → Selectivity for the laser output

✓ Fiber Bragg grating as a sensor → physical parameters → Strain; Temperature change; Pressure change

1D Photonic Crystals { Photonic Band Gap (1) }

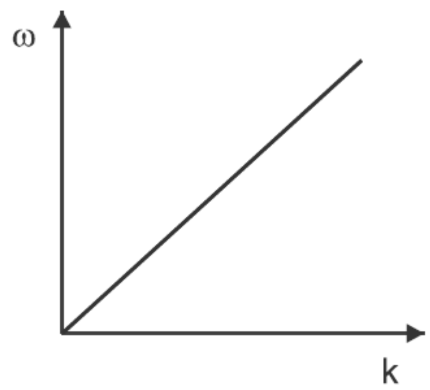
- One of the important characteristics of a Bragg grating

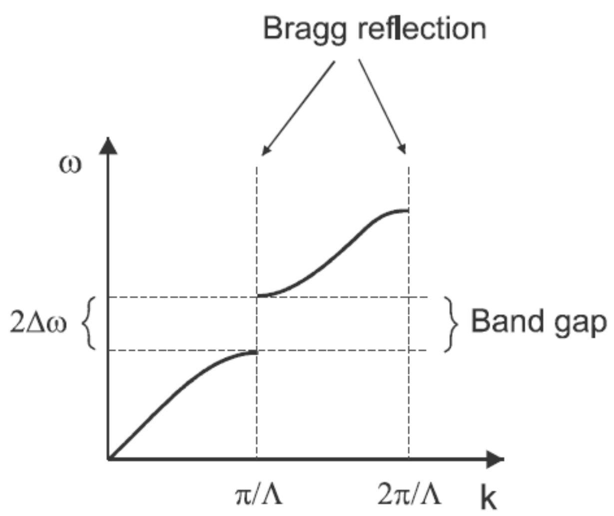
  → Exponential attenuation of light for wavelengths close to λ_B

  → For a sufficiently **long grating** → 100 % reflection

The range of wavelengths for which light is attenuated is referred to as the ***stop band***

A **frequency gap** in the **photon spectrum** is known as a ***photonic band gap***

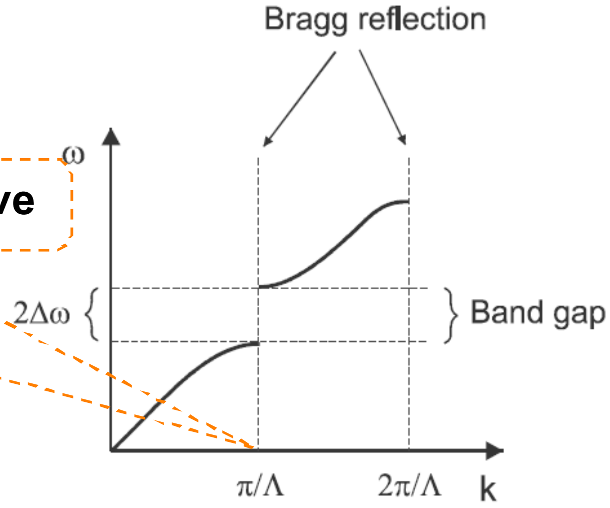




1D Photonic Crystals { Photonic Band Gap (1) }

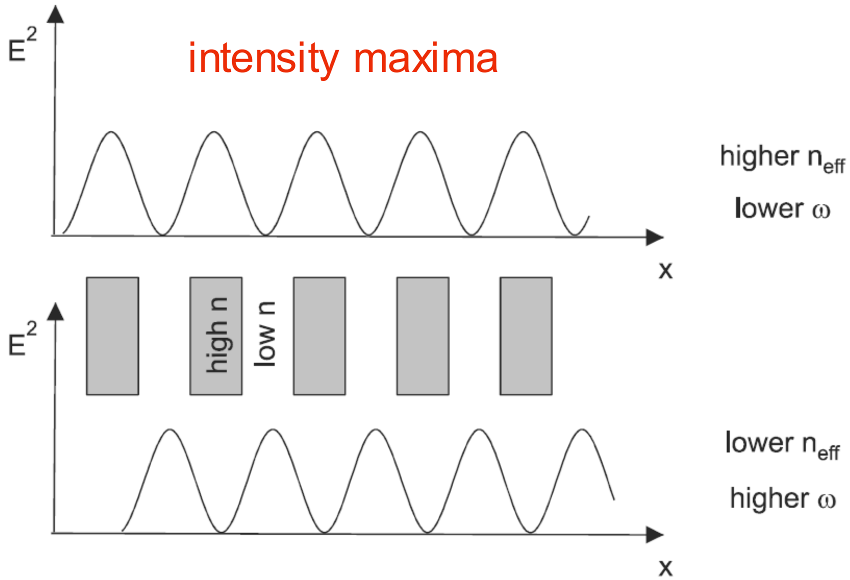
- There are two values of the light frequency $k = \frac{\pi}{\Lambda}$?

- The wave is not propagating, but is actually a **standing wave**



- Physical origin of two frequency ? intensity profiles

intensity maxima



effective index relates → frequency, wave vector

$$\omega = \left(\frac{c}{n_{eff}}\right) k = \left(\frac{c}{n_{eff}}\right)\frac{\pi}{\Lambda} \quad (23)$$

$\Delta n \uparrow \Rightarrow$ gap $\uparrow$

1D Photonic Crystals { Photonic Band Gap (2) }

✓ Calculating $\Delta\omega$

$$\omega = \frac{2\pi c}{\lambda_0} \longrightarrow \Delta\omega = -\frac{2\pi c}{\lambda_0^2}\Delta\lambda = -\frac{2\pi c}{\lambda_0}\cdot\frac{\Delta\lambda}{\lambda_0}$$

$$\frac{\Delta\lambda}{\lambda_0} = \begin{cases} \lambda_0/(2nL) & \text{for } \kappa L \ll 1 \\ \Delta n/(2n) & \text{for } \kappa L \gg 1 \end{cases}$$

$$\frac{\Delta\omega}{\omega} = \frac{\Delta\lambda}{\lambda_0} = \frac{\Delta n}{2n}$$

$$\boxed{\frac{\text{frequency gap}}{\text{center frequency}} = \frac{2\Delta\omega}{\omega} = \frac{\Delta n}{n}}$$

1D Photonic Crystals { Localized Modes (1) }

✓ If we have some frequencies within the photonic band gap

➡ Light wave's *E field are exponentially attenuated*

localized modes

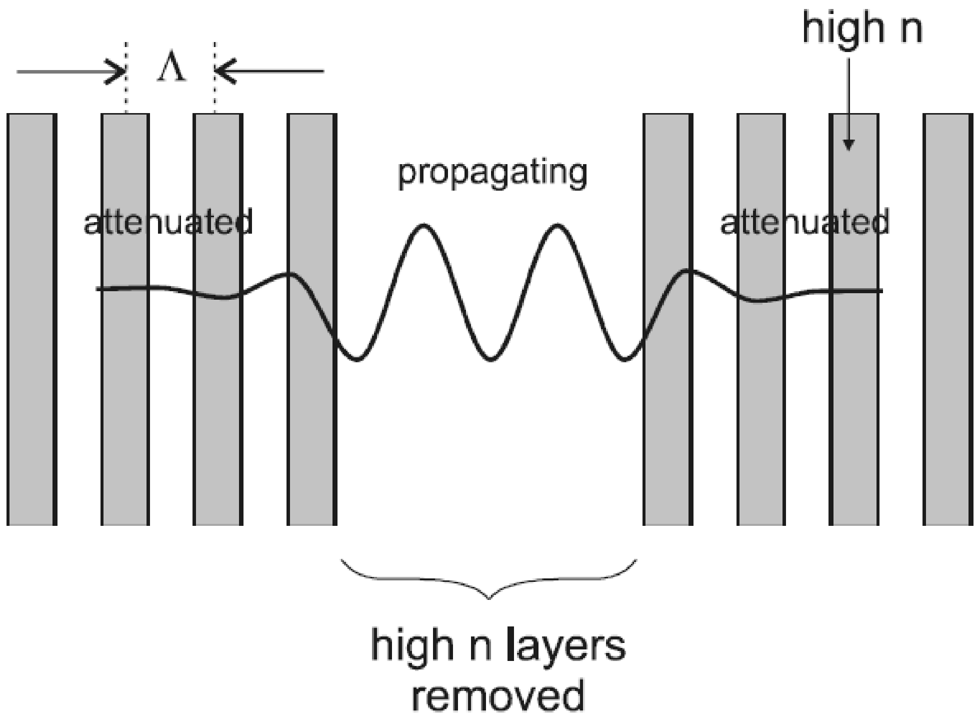
✓ If we create **discontinuity** or **defect** in the structure

➡ Light energy residing in **optical mode** confined to vicinity of the defect

❑ Simplest type of defect → edge / surface of a photonic crystal

❑ 1D Bragg grating defects →
- **begin point** of grating
- **removing** one or more of the high-index layers

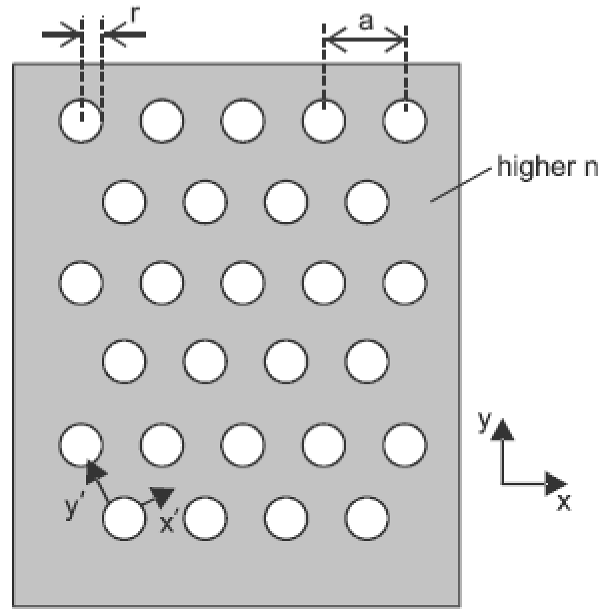
1D Photonic Crystals { Localized Modes (2) }



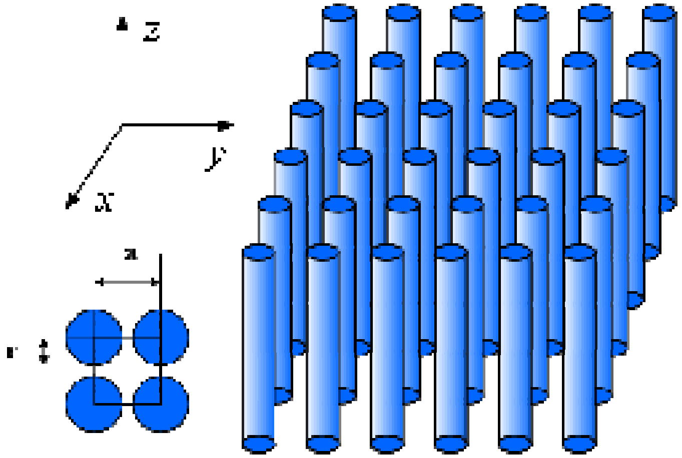
Layers are removed → The result is a localized mode

an **optical cavity** formed by two semiinfinite Bragg gratings

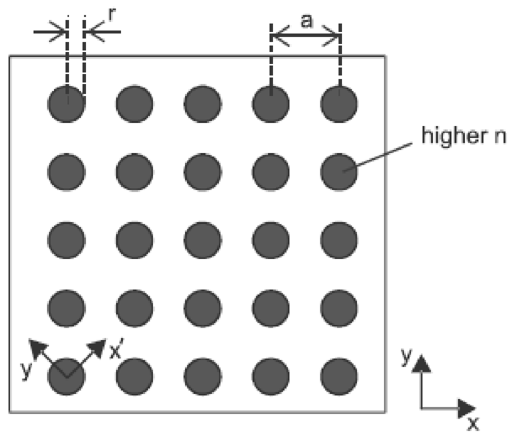
2D Photonic Crystals



2-D Triangular Lattice - Holes in a Substrate with higher n



2-D Square lattice - Rods in Air



Two symmetry directions *x and x' are shown.*

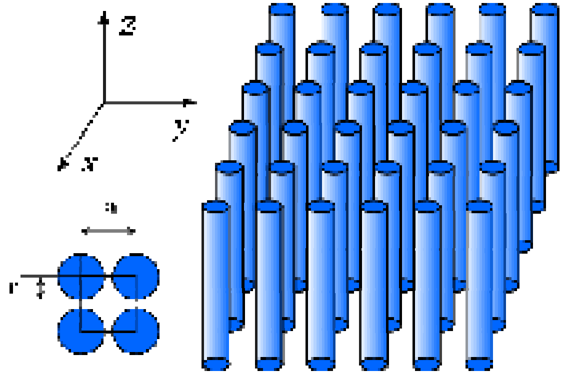
Planar Geometry (1)

❑ For 2D photonic crystals

1- Propagation mostly **in the *x–y plane***

2- Propagate mostly **along the *z* axis**

1) Propagation in the *x–y plane* and **Square Lattice**



➢ Propagating along **+*X* axis** → refractive index spatially **vary**

Like **1D Bragg grating**

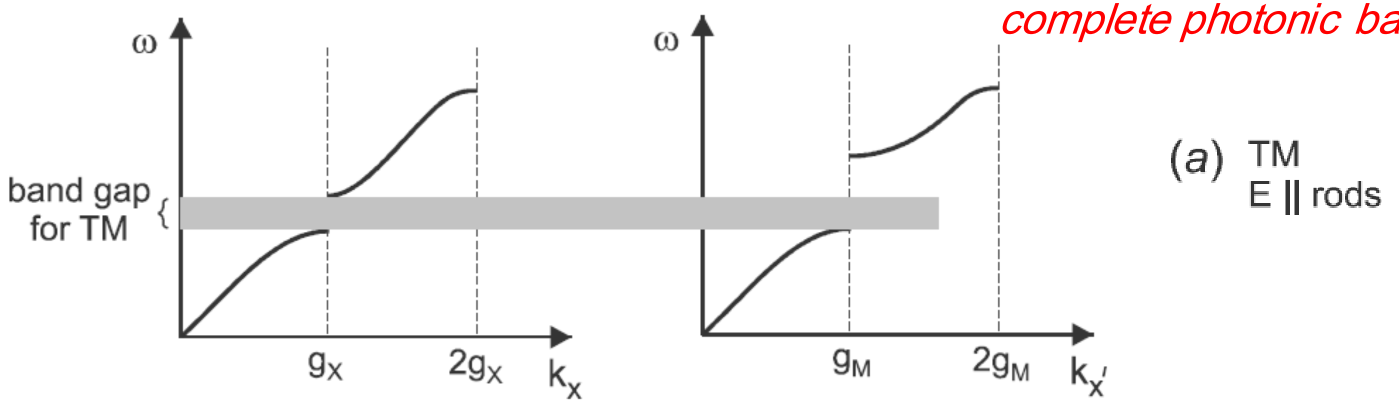
❑ Bragg condition ✓ → light will be strongly reflected, and gap opens in dispersion relation

❖ 2D Difference with **1D Bragg grating** ➡ 2D is **periodic** in 2 direction **x , x'**

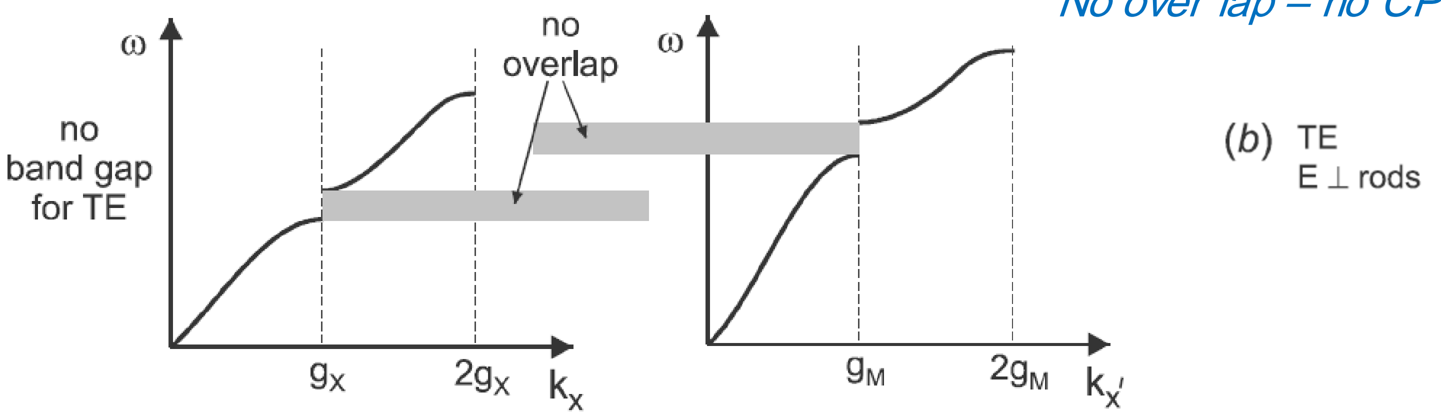
Planar Geometry (2)

2 directions → 2 Band gaps → Different position and width

complete photonic band gap



No over lap – no CPBG



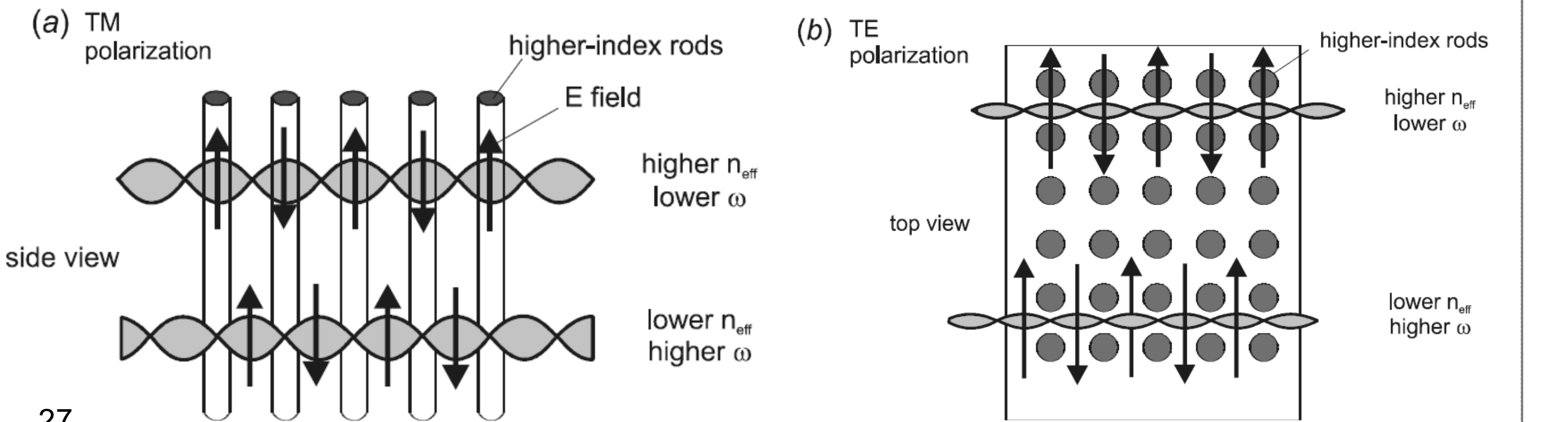
Planar Geometry (3)

TE polarization *E field **along y** and **perpendicular** to rods (with B along z)*

TM polarization *E* field **along *z*** *(with B along –y and **perpendicular** to the rods)*

- **Physical origin** of difference between TM and TE polarization

Intensity peaks in the standing waves



Planar Geometry (3)

confine light in two dimensions

band gaps for different directions and polarizations all **overlap** in some frequency range

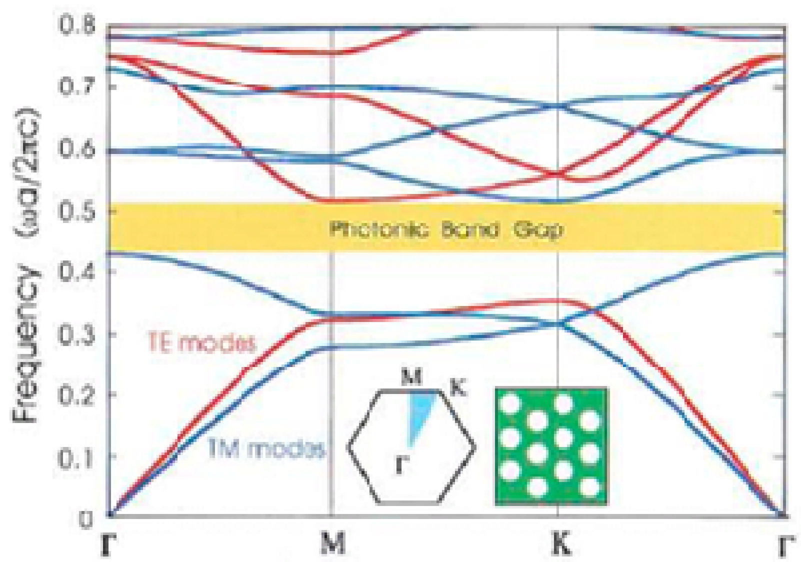
1. A square lattice of dielectric rods

   There is **PBG** for **both** TE and TM polarization

   ⇨ Does not overlap ⇨ No **CPBG**

2. A triangular lattice of air holes

   ⇨ Overlap ✓



2D Photonic Crystals - Fiber Geometry

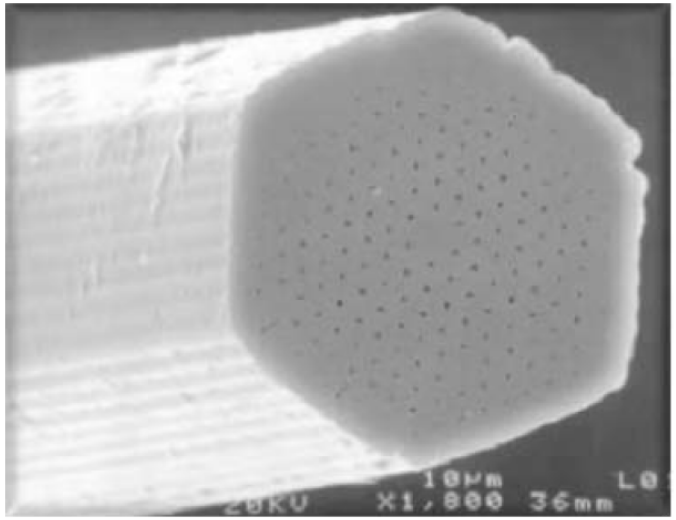
2. Light propagating mostly in the ***z direction***

component perpendicular to the **rods** or **air holes** are **Negligible**

Usage: **optical fiber**

- Guiding types in optical fibers
  - Total internal reflection (TIR)
  - Photonic band gap (PBG)

Guiding by Effective Index (1)



Early ***photonic crystal fiber*** called holey fiber

Single missing air hole, allows confinement of light by TIR

- Advantages
  - only one glass type
  - no doping

Cladding region being "doped" with air holes

Guiding by Effective Index (2)

- Light guidance of Photonic crystal fibers → Single mode / Multi mode

- Single-mode guidance of light in conventional step-index fiber →

$$V \equiv \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2}$$

$$V < 2.405$$

- Single-mode guidance for holey fiber →

$$V_{PCF} \equiv \frac{2\pi \Lambda}{\lambda}\sqrt{n_c^2 - n_{cl}^2(\lambda)}$$

$$V_{PCF} < \pi$$

- Effective cladding index is not a **constant** → varies strongly **with wavelength**
- Shorter wavelengths, light is **better confined** and **index is higher**
- Longer wavelengths, **diffraction** prevents the light from being well confined
- glass region between the **air holes** should be sufficiently wide → ✓ **diffraction**

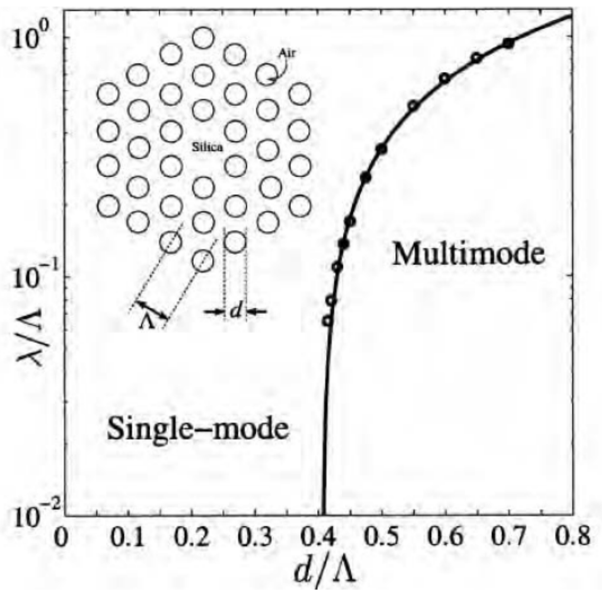
holey fiber *is also called* ***Endlessly single-mode***

Guiding by Effective Index (3)

❖ Boundaries between **single-mode** and **multimode**

$\frac{d}{\Lambda}$ increases

λ decreases



This fibers can be scaled up

Larger → high-power fiber lasers

Smaller → Frequency conversion

Guiding by Photonic Band Gap (1)

- Condition of Photonic band-gap in Crystal type (triangular lattice of air holes)

→ substrate material → $\varepsilon_r > 7.2$, $n > 2.7$

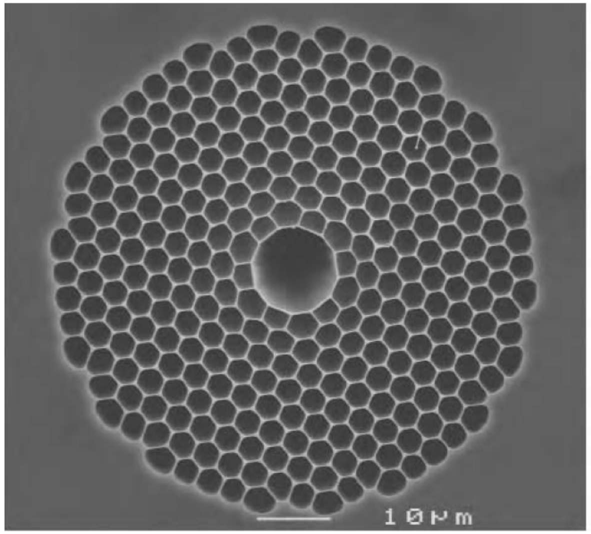
- (n) Silica glass <1.5 → **Seems** no Photonic band-gap guiding for holey fiber

- $\varepsilon_r > 7.2$ is only for light propagating perpendicular to the air holes (TM)

- For propagation parallel to holes (TE) if one use proper geometry of air holes

→ **Complete PBG**

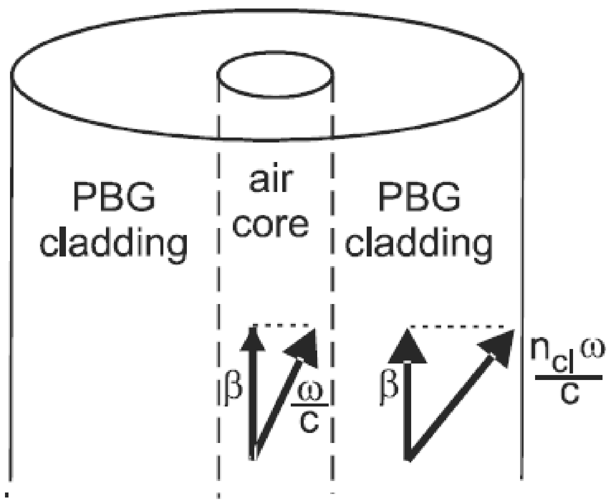
Hollow-core fiber

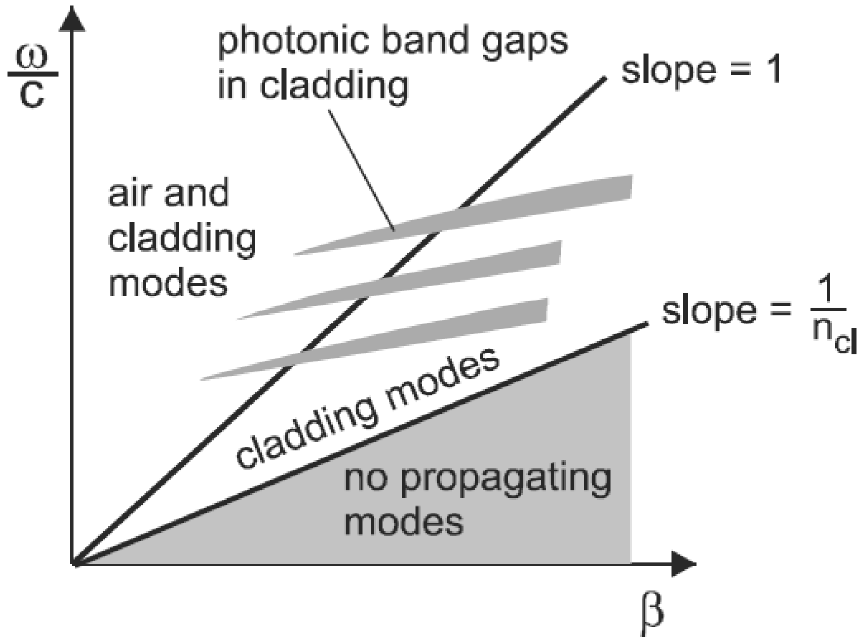


Guiding by Photonic Band Gap (2)

❖ PBG
- → Δn
- → Photonic crystal structure
- → Propagating direction of light → β axial wave vector

❑ Modes with larger β → directed more nearly down the fiber axis ⇐ same ω





Guiding by Photonic Band Gap (3)

❖ **Absorption** and **Rayleigh scattering** in the fiber core ⇒ much lower in hollow-core fiber

⇒ Less amplification for further distance

❖ Much higher optical powers

❖ Less dispersion ⇒ Very short optical pulses propagate without significant spreading in time

All because light is propagating mostly in **air**, rather than in the glass

3D Photonic Crystals (1)

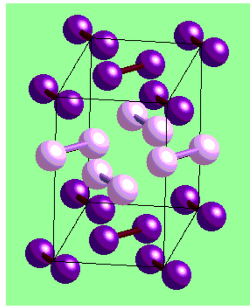
- In 1987 The concept of the 3-D photonic crystal introduced independently

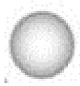
E. Yablonovitch — Improving the efficiency of semiconductor **laser** devices

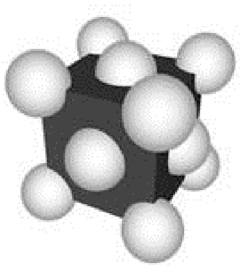
S. John — “**Localizing**” or confining light to a small region of space

a material with a **complete photonic band gap**

diamond-type
Need n > 1.87

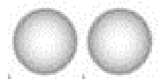


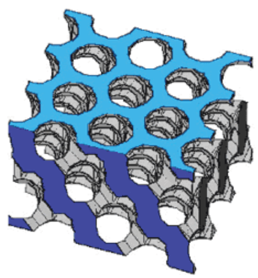




FCC cubic

Opal





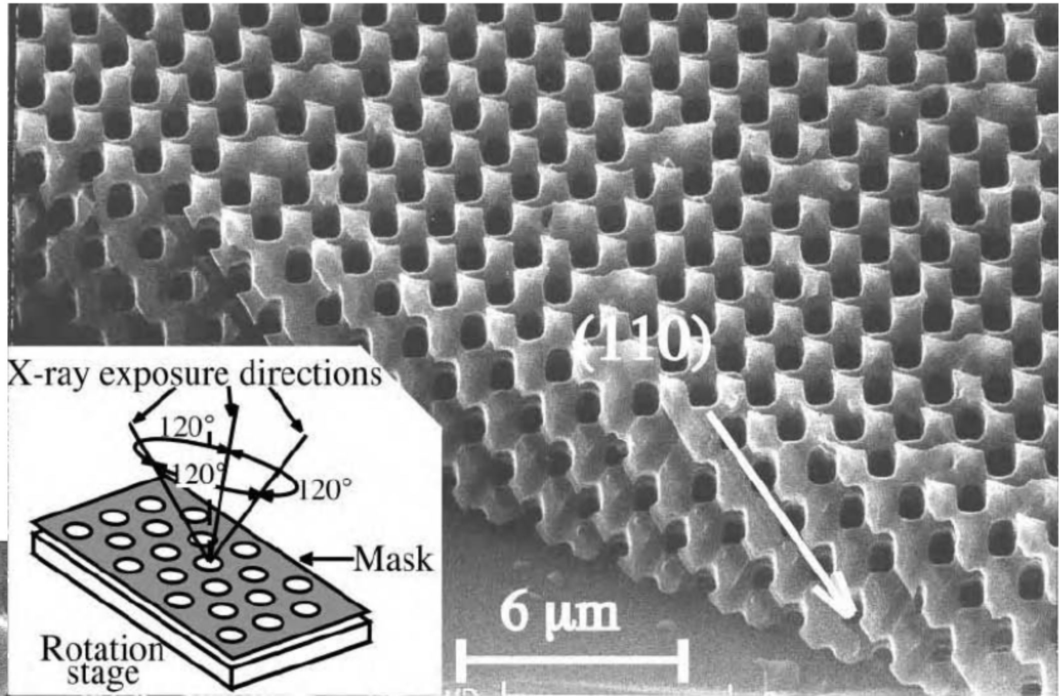
inverse opal
Need n > 2.8
Narrow band gap

3D Photonic Crystals (2)

- Creation Methods
  - ✓ "**Top-down**" approach

Yablonovite

Lincoln log



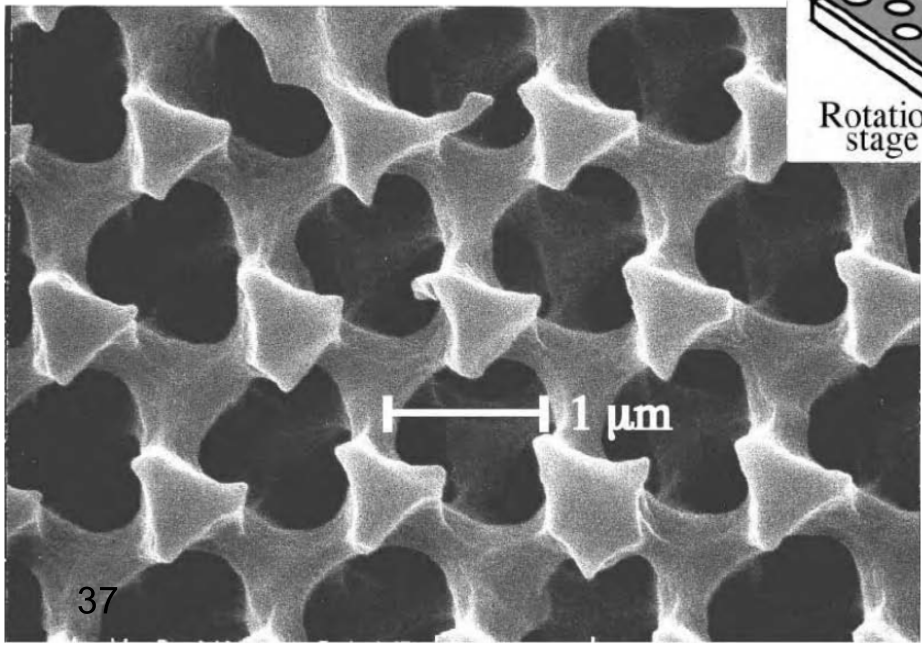
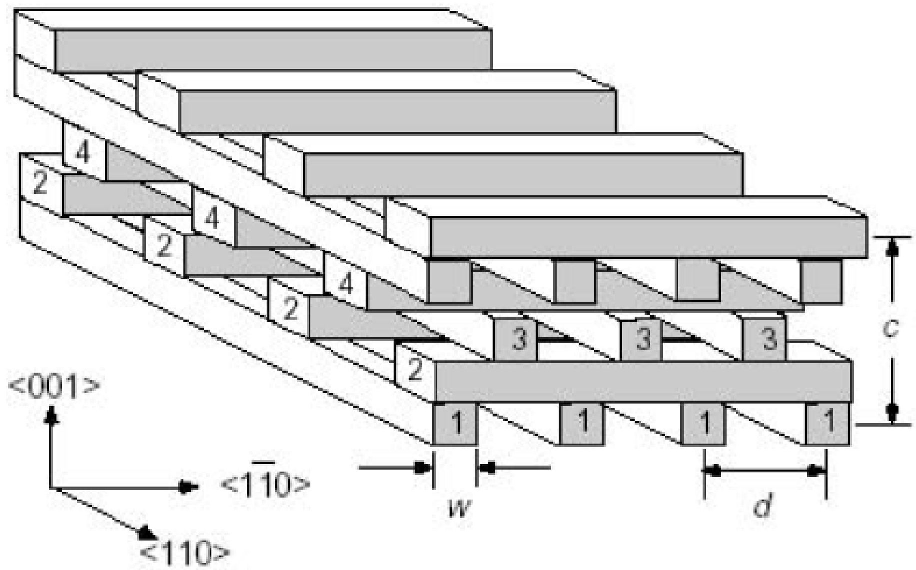


Figure 8-21 Yablonovite is formed by drilling holes into a dielectric at three precise angles, and results in a diamond - like structure. The holes can be created by exposure to X-rays through a mask

3D Photonic Crystals (3)

- Creation Methods
  - ✓ “**Top-down**” approach



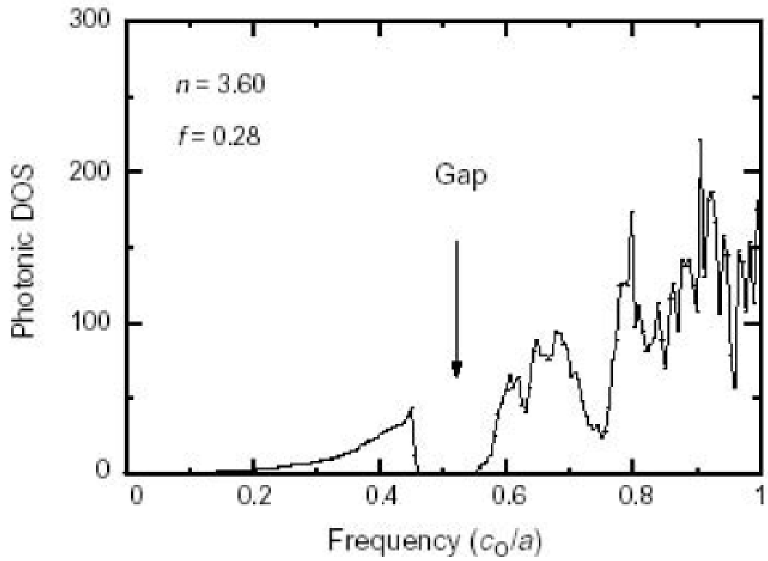


Figure 8-22 (a) The “Lincoln log” or “woodpile” structure has a diamond-type lattice symmetry, and exhibits a 3-D photonic band gap.
(b) Calculated density of states when *n* bars = *3.6 and fill 28% of the space.*

density of states (number of propagating modes per unit frequency interval)

3D Photonic Crystals (4)

- Creation Methods
  - ✓ "**bottom-up**" or "**self-assembly**" approaches

fossilization

self-assembling of SiO_2 Spheres with diameter of 100 to 1000 nm in the fcc structure

→

Injecting a high-index material between the spheres

→

selective etching of the original SiO_2 material

In "**Top-down**" approach for a given refractive index **wider band gap** can be achieved

In "**bottom-up**" approach better for mass-production levels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