

Module title: Theoretical Foundations of Numerical Analysis

Module Code: 28-10-232-01

Module Credit: 4

Term: First term 1404-1405

Lecturer: Dr. M. Arab-Ameri (arabameri@math.usb.ac.ir)

Lecturing time: Monday (7:30 – 9:30) and Tuesday (15:00 – 17:00)

Lecturing place: My room in the faculty of mathematics

Assessments: 30% mid-term 1 exam (1404/8/19)

30% mid-term 2 exam (1404/9/24)

40% final exam (1404/10/28)

Reference: Theoretical Numerical Analysis

(2th Edition,2007) Kendall Atkinson and Weimin Han

Module Subjects:

Linear Spaces

1th week: Linear spaces, Normed spaces

2th week: Convergence, Banach spaces, Completion of normed spaces

3th week: Inner product spaces, Orthogonality

4th week: Spaces of continuously differentiable functions, Holder spaces, L_p spaces, Compact sets

Linear Operators on Normed Spaces

5th week: Operators, Continuous linear, $L(V, W)$ as a Banach space

6th week: The geometric series theorem and its variants, Some more results on linear operators

7th week: Open mapping theorem, Principle of uniform boundedness, Convergence of numerical quadratures

8th week: Linear functionals, An extension theorem for linear functionals, The Riesz representation theorem

9th week: Types of convergence, Compact linear operators, Compact integral operators on $C(D)$

Approximation Theory

10th week: Approximation of continuous functions by polynomials, Interpolation theory, Lagrange polynomial interpolation, Hermite polynomial interpolation, Piecewise polynomial interpolation, Trigonometric interpolation

11th week: Best approximation, Convexity, lower semi-continuity, Some abstract existence results
Existence and Uniqueness of best approximation

12th week: Best approximations in inner product spaces, projection on closed convex sets, Best approximations in inner product spaces, Orthogonal polynomials, Projection operators, Interpolatory projections and their convergence

Fourier Analysis and Wavelets

13th week: Fourier series, Fourier transform, Discrete Fourier transform, Haar wavelets

Nonlinear Equations and Their Solution by Iteration

14th week: The Banach fixed-point theorem, Applications to iterative methods, Nonlinear equations

15th week: Linear and nonlinear integral equations, Ordinary differential equations in Banach spaces, Differential calculus for nonlinear operators

16th week: Fréchet and Gâteaux derivatives, Mean value theorems, Partial derivatives, The Gâteaux derivative and convex minimization, Newton's method in Banach spaces