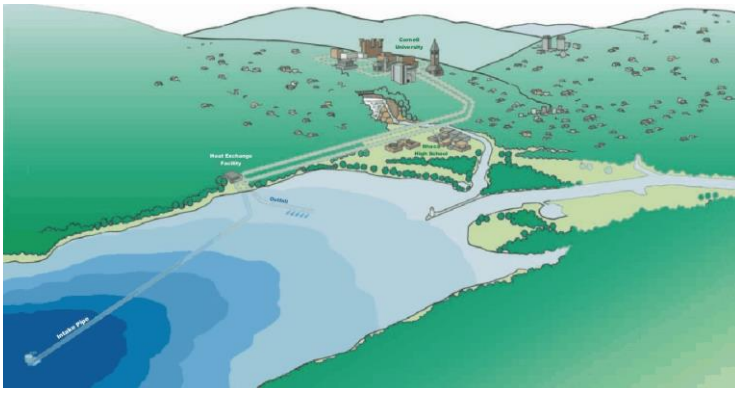
ماشینهای آبی
جریان ویسکوز در لوله ها



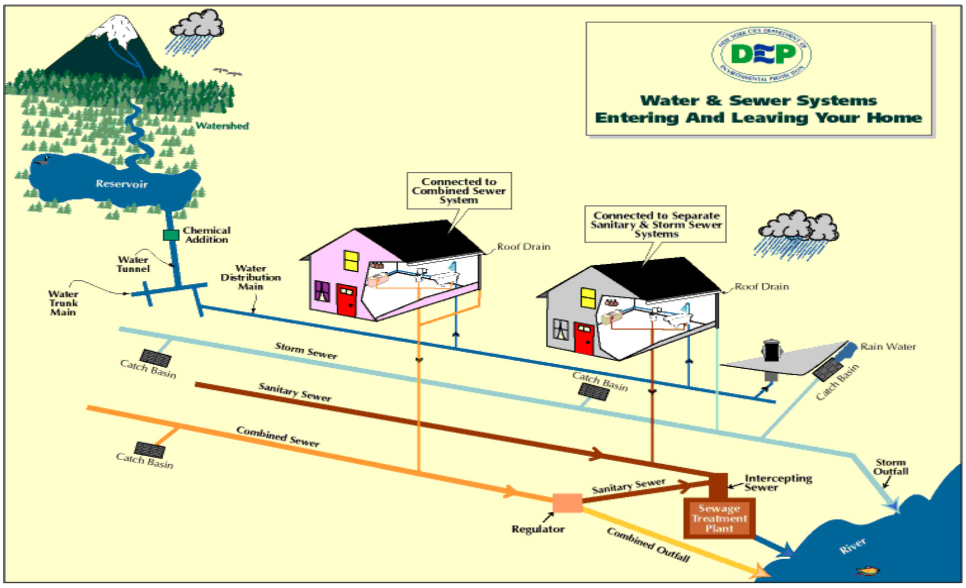
گروه مهندسی عمران
دانشگاه سیستان و بلوچستان

دکتر بهاره پیرزاده

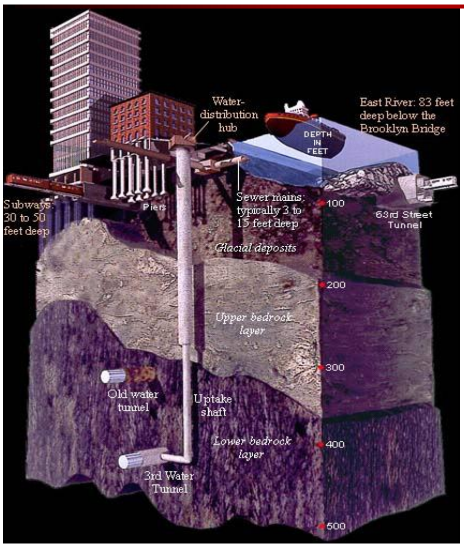
لوله ها در همه جا هستند!

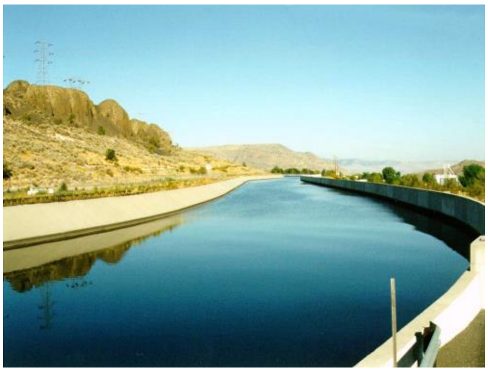
Owner: **City of Hammond, IN**
Project: **Water Main Relocation**
Pipe Size: **54"**

لوله ها در همه جا هستند!



لوله ها در همه جا هستند!



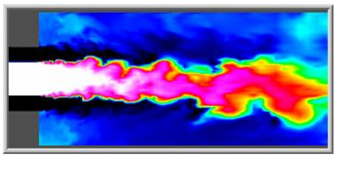


انواع مسایل در مهندسی

- تعیین اندازه قطر لوله برای حمل جریانی با دبی مشخص
- تعیین میزان فشار در سیستم توزیع آب با باز بودن شیر آتش نشانی
- آیا میتوان جریان عبوری از یک لوله قدیمی را با افزودن یک لوله صاف، افزایش داد؟

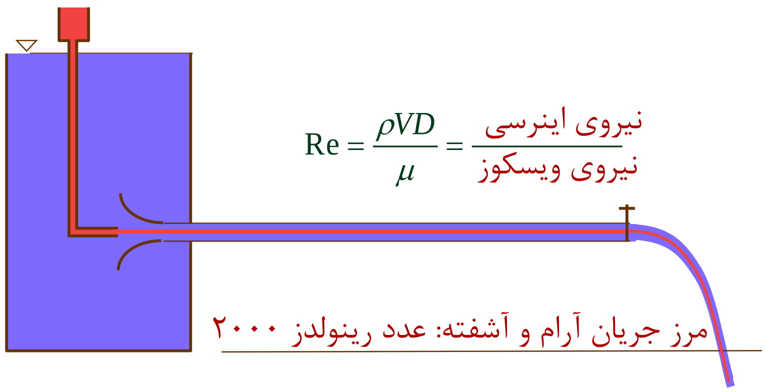
جریان ویسکوز در لوله ها (مرور)

- توسعه لایه مرزی
- آشفتگی
- توزیع سرعت
- افت انرژی (موضعی و اصلی)
- تکنیکهای حل مسایل



جریان آرام و آشفته

- آزمایش و عدد رینولدز



$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{\text{نیروی اینرسی}}{\text{نیروی ویسکوز}}$$

بار آبی (Head)

در دینامیک سیالات بار آبی (انرژی در واحد وزن با بعد طول) مفهومی است که انرژی سیال تراکم ناپذیر را به ارتفاع ستون استاتیک معادل آن سیال مرتبط می‌کند. بار آبی به اشکال زیر استفاده می‌شود:

Head	Terms
(elevation head) بار آبی ارتفاع	z
(pressure head) بار آبی فشار	P/γ
(velocity head) بار آبی سرعت	$v^2/2g$
(piezometric head/ hydraulic head) بار آبی پیزومتریک/هیدرولیک	$h = z + P/\gamma$
(Total head) بار آبی کل	$h = z + P/\gamma + v^2/2g$

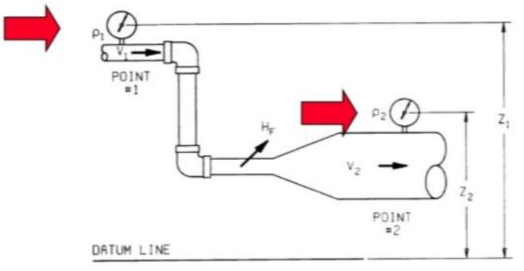
به طریق مشابه می‌توان فشار استاتیکی (یا فشار) را با P، فشار ناشی از ارتفاع را با γz و فشار پیزومتریک (piezometric pressure) را با $P + \gamma z$ نشان داد.

فشار دینامیکی (dynamic pressure) یا فشار سرعت (velocity pressure) در سیال تراکم ناپذیر نیز با رابطه زیر تعریف می‌شود:*

$$q = \frac{1}{2}\rho v^2$$

http://sahand.kntu.ac.ir/~soltanpour/

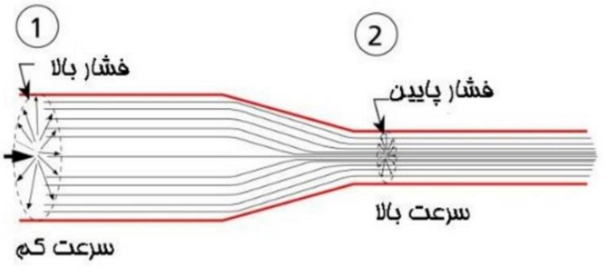
قانون برنولی (اصل بقای انرژی)



در یک سیستم سیال جاری و بدون اصطکاک، انرژی در نقاط مختلف برابر است.

انرژی در نقطه ۱ = انرژی در نقطه ۲ + افت‌های اصطکاکی

قانون برنولی (اصل بقای انرژی)



در یک لوله، هر چه سرعت زیاد شود فشار کم می شود.

قانون برنولی (اصل بقای انرژی)

انرژی سیال از سه بخش تشکیل شده است:

۱-انرژی پتانسیل (فشار)

۲-انرژی جنبشی (سرعت)

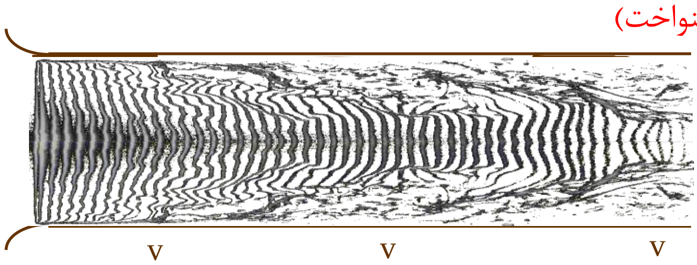
۳-انرژی ارتفاع

$$\frac{P}{\ } + \frac{V^{2}}{\ } + Z = \text{ثابت}$$

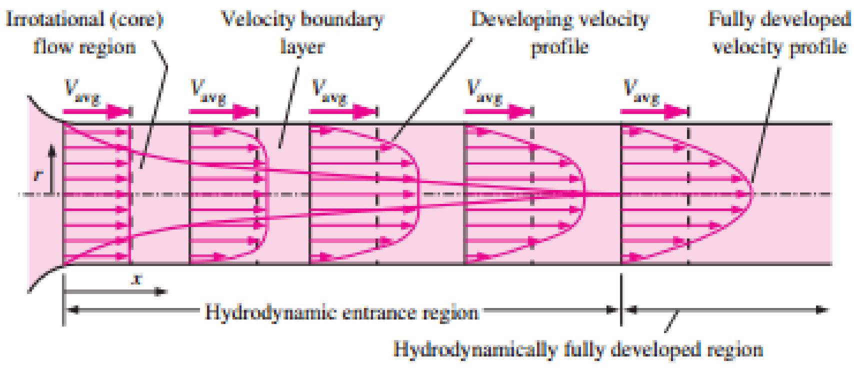
رشد لایه مرزی: طول ناحیه توسعه یافتگی جریان

جریان آب در یک لوله و در نزدیکی دیواره لوله، چه چیزی را تجربه میکند؟ (درگ یا برش)

چرا سرعت جریان در مرکز لوله، از جداره ها بیشتر است؟ (قانون بقای جرم و جریان غیریکنواخت)



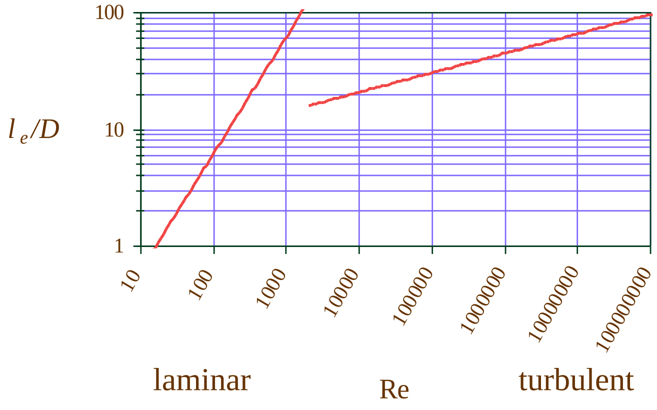
طول ناحیه توسعه یافتگی جریان (ادامه)



فاصله لازم برای پروفیل سرعت تا رسیدن به پروفیل توسعه یافته نسبت به پروفیل یکنواخت مقطع ورودی

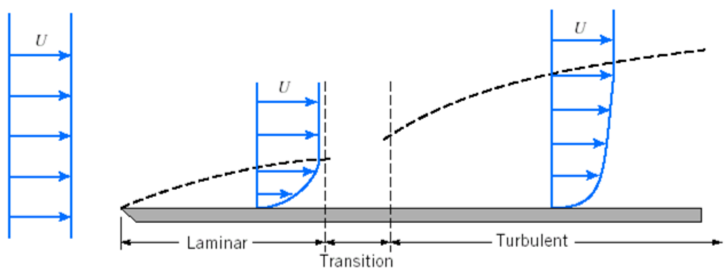
طول ناحیه توسعه یافتگی جریان

$$\frac{l_e}{D} = f\left(\text{Re}\right) \longrightarrow \frac{l_e}{D} = 0.06\,\text{Re} \qquad \frac{l_e}{D} = 4.4\left(\text{Re}\right)^{1/6}$$



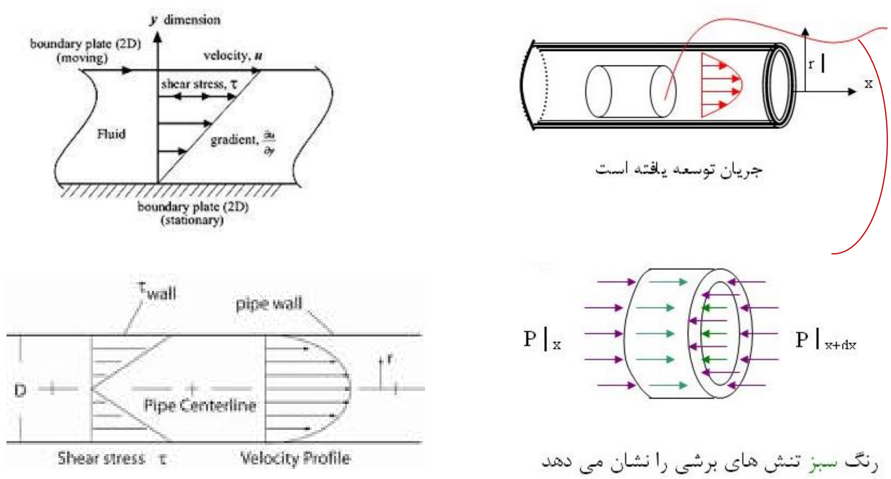
رشد لایه مرزی

برای سیالاتی که ویسکوزیته نسبتاً کمی دارند، اثر اصطکاک داخلی در سیال تنها در ناحیه باریکی از محیط که مرز سیال را تشکیل می‌دهد، قابل توجه است و لذا خارج از این ناحیه باریک در نزدیکی مرزهای جامد، باید جریان را ایده‌آل در نظر گرفت.



شکل رشد لایه و گذر از لایه مرزی آرام به متلاطم را نشان می‌دهد (البته در آن محور عمودی بزرگ شده است).

پروفیل تنش برشی در لوله



رنگ سبز تنش های برشی را نشان می دهد

رنگ بنفش فشار را نشان می دهد

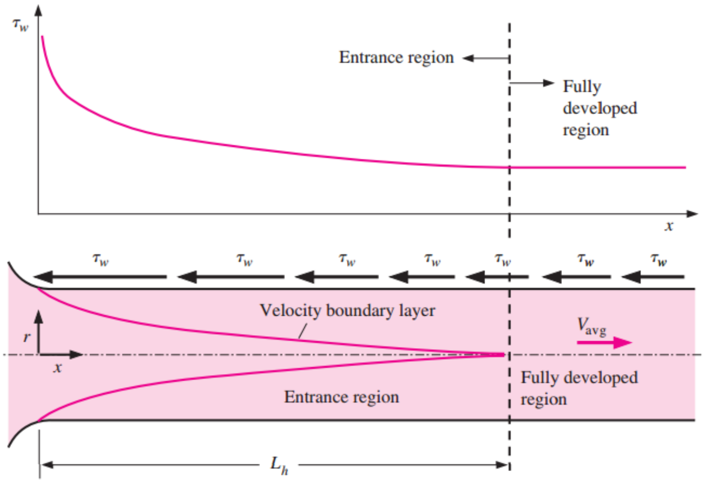
تغییرات تنش برشی در طول لوله

تنش برشی در جداره لوله متناسب با شیب پروفیل سرعت در نزدیکی دیواره (گرادیان سرعت در عمق) است. از آنجاییکه در ناحیه کاملا توسعه یافته، پروفیل سرعت ثابت می ماند، لذا تنش برشی دیواره در این ناحیه عدد ثابتی است.

میزان تنش برشی در ورودی لوله، یعنی درجاییکه ضخامت لایه مرزی حداقل است، حداکثر مقدار خود را دارد که به تدریج در طول لوله کاهش می یابد تا به مقدار ثابتی در ناحیه جریان توسعه یافته می رسد.

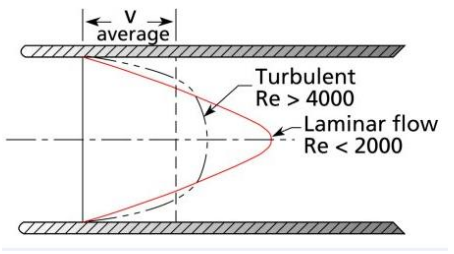
از آنجاییکه طبق رابطه مومنتوم در طول لوله (شکل اسلاید قبل)، فشار و تنش برشی با هم مرتبط هستند، افت فشار در مقطع ورودی لوله زیاد است و اثر مقطع ورود جریان، افزایش ضریب اصطکاک ورودی لوله میباشد.

تغییرات تنش برشی در طول لوله



پروفیل توزیع سرعت

- آشفتگی باعث انتقال مومنتوم از مرکز لوله به سمت جداره ها می شود.
- اختلاط سیال (تبادل مومنتوم) باعث ایجاد پروفیل سرعت نسبتا یکنواخت (در مقایسه با جریان آرام) می شود.
- در نزدیکی دیواره لوله، گردابه ها کوچک هستند (سایز آنها متناسب با فاصله از مرز می باشد.



قانون لگاریتمی سرعت

قانون لگاریتمی سرعت (باکمک آنالیز ابعادی و تجربه):

$$\frac{u}{u_*} = 2.5 \ln \frac{yu_*}{\nu} + 5.0$$

برای : $\frac{yu_*}{\nu} > 20$

سرعت برشی:

$$u_* = \sqrt{\frac{\tau_0}{\rho}}$$

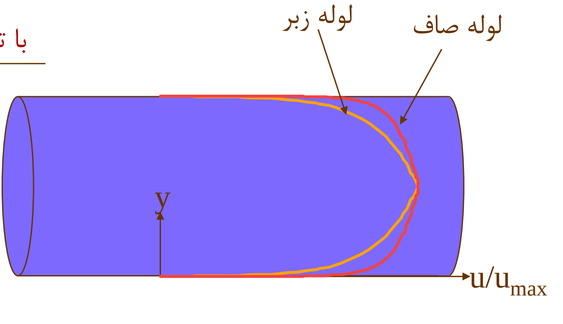
با توازن نیروها:

$$\tau_0 = \frac{\gamma h_f d}{4l}$$

$$u_* = \sqrt{\frac{g h_f d}{4l}}$$

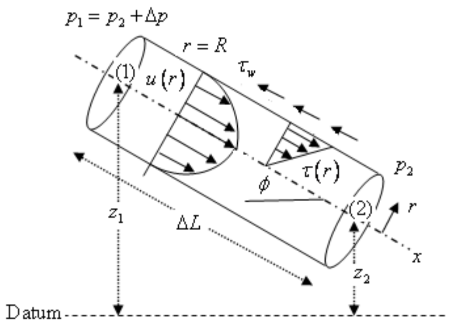
$$u_* = V\sqrt{\frac{f}{8}}$$

توزیع سرعت



مسایل جریان در لوله

- تعریف حجم کنترل و نوشتن معادلات برای جریان سیال
  در حجم کنترل مورد نظر در لوله
- تخمین میزان افت در لوله
- بکارگیری از آنالیز ابعادی



آنالیز ابعادی در جریان ویسکوز

- آیا آنالیز ابعادی را بخاطر دارید؟

$$C_p \frac{D}{l} = f\left(\frac{\varepsilon}{D}, \mathrm{Re}\right) \quad \text{که} \quad \mathrm{Re} = \frac{\rho V D}{\mu} \quad \text{و} \quad C_p = \frac{-2\Delta p}{\rho V^2}$$

ضریب فشار در جریان غیرقابل تراکم (در اعداد رینولدز بالا، این ضریب عدد ثابتی است)

- دو پارامتر بی بعد مهم در لوله ها
  - Re – مرز جریان آرام و آشفته
  - ε/D – زبری یا صافی دیواره

- تقسیم بندی جریان
  - جریان داخلی (Internal)(مانند رودخانه ها یا لوله ها) : جریانهای محصور به جداره :find C_p
  - جریان خارجی (External) یا (مانند هوای اطراف یک هواپیما یا جسم غوطه ور در آب) جریانهای دور از جداره : find C_d

افت انرژی در لوله ها

$$\mathrm{f} = \left(C_p \frac{D}{L}\right) = f\left(\frac{\varepsilon}{D}, \mathrm{Re}\right)$$

آنالیز ابعادی

$$\rho g h_l = -\Delta p - \rho g \Delta z$$

در حالت کلی

$$\mathrm{C}_p = \frac{-2\Delta p}{\rho V^2} \qquad \rho g h_l = -\Delta p \qquad \mathrm{C}_p = \frac{2 g h_l}{V^2}$$

با فرض جریان افقی

$$\mathrm{f} = \frac{2 g h_\mathrm{f}}{V^2}\frac{D}{L}$$

$$h_\mathrm{f} = \mathrm{f}\,\frac{L}{D}\frac{V^2}{2g}$$

برای هر دو نوع جریان آشفته و آرام برقرار است

معادله دارسی وایسباخ Darcy-Weisbach

$$\mathrm{f} = 8\frac{u_*^2}{V^2} \qquad h_\mathrm{f} = 8\frac{L}{D}\frac{u_*^2}{2g}$$

افت در لوله ها؛ افتهای اصلی

- جریان آرام
- جریان آشفته
- فرمول کولبرووک Colebrook Formula
- دیاگرام مودی Moody diagram
- Swamee-Jain

Laminar Flow Friction Factor

$$V = \frac{\rho g D^2}{32\mu}\frac{h_l}{L}$$

Hagen-Poiseuille

$$h_f = \frac{32\mu L V}{\rho g D^2}$$

$h_f \propto V$

$$Q = \frac{\pi D^4}{128\mu}\frac{\rho g h_l}{l}$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

Darcy-Weisbach

$$\frac{32\mu L V}{\rho g D^2} = f\frac{L}{D}\frac{V^2}{2g}$$

f مستقل از زبری است

$$f = \frac{64\mu}{\rho V D} = \frac{64}{\text{Re}}$$

جریان آشفته

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

- جریان در لوله (از نظر هیدرولیکی) صاف (von Karman, 1930)

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{\text{Re}\sqrt{f}}{2.51}\right)$$

- جریان در لوله زبر (von Karman, 1930)

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{3.7D}{\varepsilon}\right)$$

- برای هر دو نوع لوله صاف و زبر (Colebrook)

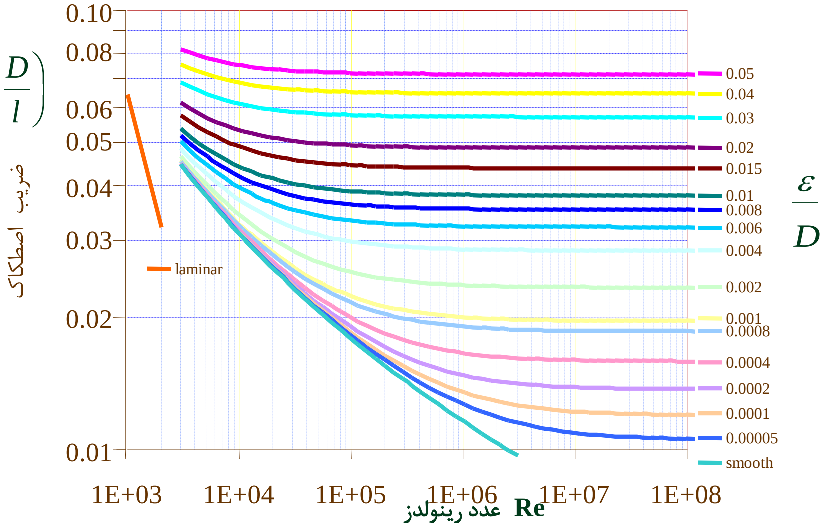
$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$u_* = V\sqrt{\frac{f}{8}}$$

(با استفاده از دیاگرام مودی)

Moody Diagram

$$f = \left(C_p \frac{D}{l} \right)$$



Swamee-Jain

- 1976
- محدودیتها:
  - $\varepsilon/D < 2 \times 10^{-2}$
  - Re $> 3 \times 10^3$
  - کمتر از ۳٪ اختلاف با نتایج حاصل از دیاگرام مودی
- مناسب برای برنامه نویسی کامپیوتری

$$f = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}} \right) \right]^2}$$

no f

$$Q = \frac{-\pi}{\sqrt{2}} D^{5/2} \sqrt{\frac{g h_f}{L}} \log\left(\frac{\varepsilon}{3.7D} + 2.51\nu \sqrt{\frac{L}{2 g h_f D^3}} \right)$$

Colebrook

$$D = 0.66 \left[\varepsilon^{1.25} \left(\frac{L Q^2}{g h_f} \right)^{4.75} + \nu Q^{9.4} \left(\frac{L}{g h_f} \right)^{5.2} \right]^{0.04}$$

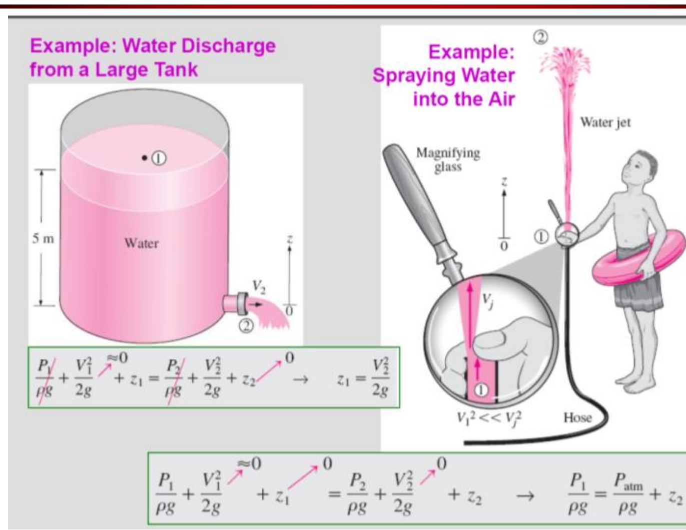
دقت کنید که هر معادله دو ترم دارد. چرا؟

زبری لوله (بر اساس جنس لوله)

pipe material	pipe roughness ε (mm)
glass, drawn brass, copper	0.0015
commercial steel or wrought iron	0.045
asphalted cast iron	0.12
galvanized iron	0.15
cast iron	0.26
concrete	0.18-0.6
rivet steel	0.9-9.0
corrugated metal	45
PVC	0.12

$\frac{\varepsilon}{d}$ باید بی بعد باشد

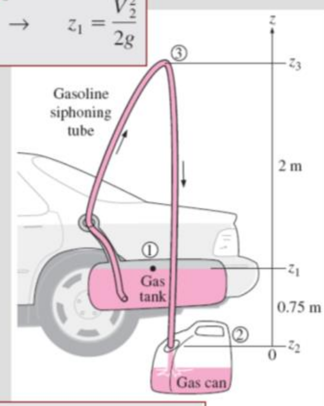
تکنیکهای حل مسایل



تکنیکهای حل مسایل

$$\frac{\cancel{P_1}}{\rho g}+\frac{V_1^2}{2g}^{\approx 0}+z_1=\frac{\cancel{P_2}}{\rho g}+\frac{V_2^2}{2g}+z_2^{\,0}\ \rightarrow\ z_1=\frac{V_2^2}{2g}$$

Example: Siphoning Out Gasoline from a Fuel Tank



$$\frac{P_2}{\rho g}+\cancel{\frac{V_2^2}{2g}}+\cancel{z_2}^{\,0}=\frac{P_3}{\rho g}+\cancel{\frac{V_3^2}{2g}}+z_3\ \rightarrow\ \frac{P_{atm}}{\rho g}=\frac{P_3}{\rho g}+z_3$$

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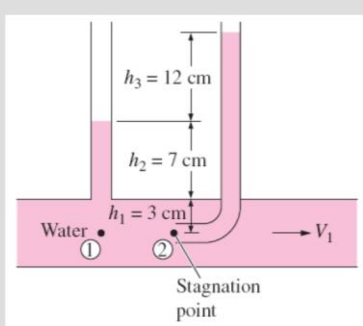
تکنیکهای حل مسایل

Example: Velocity Measurement by a Pitot Tube



$$P_1=\rho g(h_1+h_2)$$
$$P_2=\rho g(h_1+h_2+h_3)$$

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+\cancel{z_1}=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}^{\,0}+\cancel{z_2}\ \rightarrow\ \frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g}$$

تکنیکهای حل مسایل

●محاسبه افت با دانستن دبی، جنس و قطر لوله (Q, D, ε):

$$\mathrm{Re} = \frac{4Q}{\pi D \nu} \qquad f = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad h_f = f\,\frac{8}{\pi^2 g}\,\frac{LQ^2}{D^5}$$

●محاسبه دبی با دانستن هد افت، جنس، قطر و طول لوله (h_f, D, L, ε):

$$Q = \frac{-\pi}{\sqrt{2}} D^{5/2} \sqrt{\frac{g h_f}{L}} \log\left(\frac{\varepsilon}{3.7D} + 2.51\nu \sqrt{\frac{L}{2 g h_f D^3}}\right)$$

●محاسبه قطر لوله با دانستن هد افت، دبی، جنس و طول لوله (h_f, Q, L, ε):

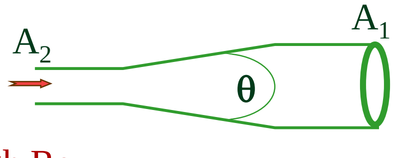
$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{LQ^2}{g h_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{g h_f}\right)^{5.2}\right]^{0.04}$$

اگر افتهای موضعی کمتر از ۵ درصد افتهای اصلی باشند، قابل صرفنظر کردن هستند.

افتهای موضعی؛انبساط(Expansion)

- در درس مکانیک سیالات، با کمک معادلات جرم، مومنتوم و انرژی، میازن افت در یک انبساط را محاسبه کرده اید.
- افتهای موضعی بطور تحلیلی قابل محاسبه نیستند و باید اندازه گیری شوند.
- افتهای موضعی عموما بصورت حاصلضرب k (ضریب افت) در هد سرعت بیان می شوند.

انبساط مخروطی شکل



$$h_{ex} = K\frac{V^2}{2g}$$

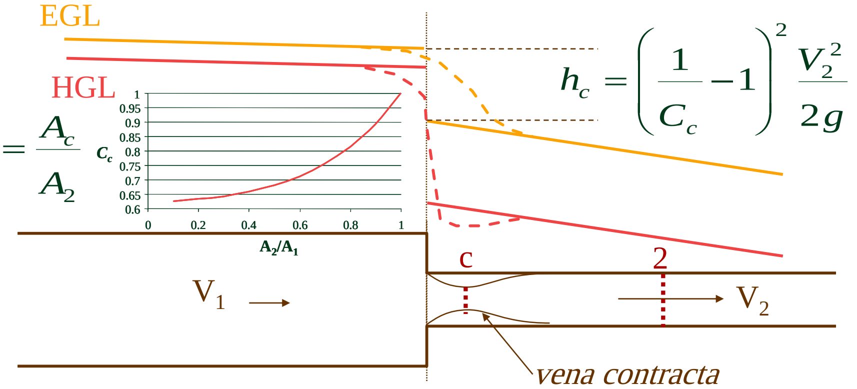
High Re

$$C_p = f\left(\text{geometry}, \cancel{\mathrm{Re}}\right) \qquad C_p = \frac{-2\Delta p}{\rho V^2} \qquad C_p = \frac{2 g h_{ex}}{V^2} \qquad h_{ex} = C_p \frac{V^2}{2g}$$

$$h_{ex} = \frac{V_{in}^2}{2g}\left(1 - \frac{A_{in}}{A_{out}}\right)^2$$

انقباض ناگهانی (Sudden Contraction)

$$C_c = \frac{A_c}{A_2}$$



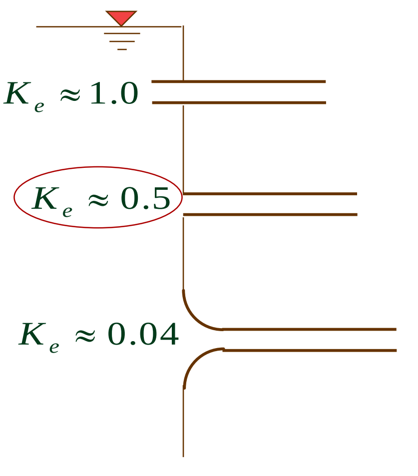
$$h_c = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$$

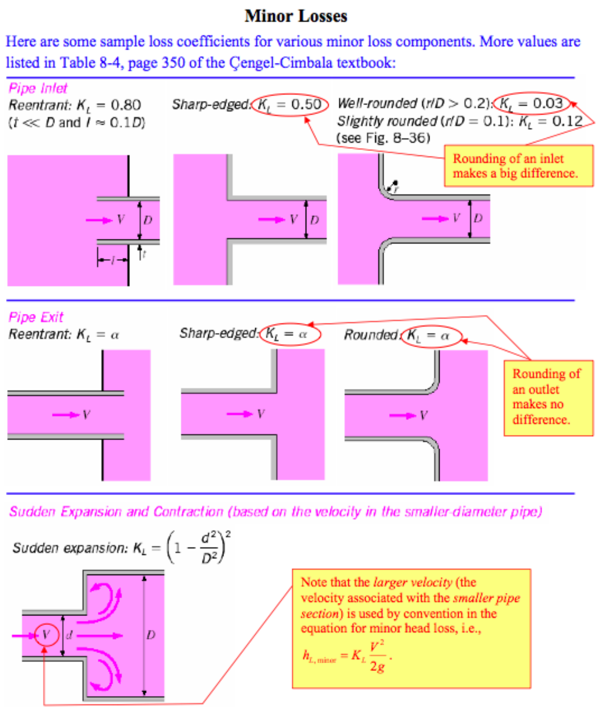
- در انقباض تدریجی، میزان افت کمتر از انقباض ناگهانی است.
- رابطه مربوط به انبساط ناگهانی نیز، مشابه میباشد.

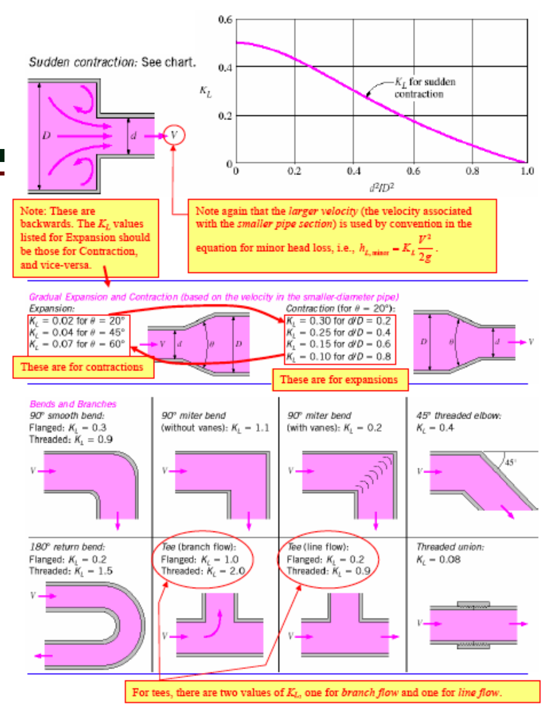
افتهای ورودی (Entrance)

افت ورودی با تغییر شکل تدریجی مقطع ورودی، کاهش می یابد. این تغییر شکل تدریجی، منجر به حذف ناحیه vena contracta می شود.

$$h_e = K_e \frac{V^2}{2g}$$



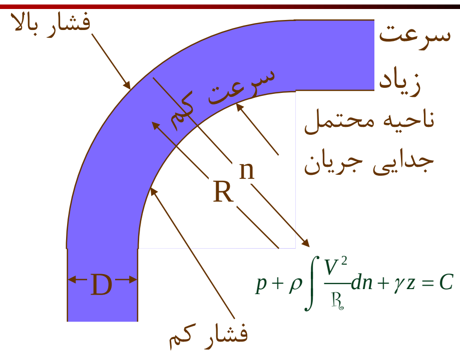

Minor Losses
Here are some sample loss coefficients for various minor loss components. More values are listed in Table 8-4, page 350 of the Çengel-Cimbala textbook:
Pipe Inlet
Reentrant: $K_L = 0.80$
($t \ll D$ and $l \approx 0.1D$)
Sharp-edged: $K_L = 0.50$
Well-rounded ($r/D > 0.2$): $K_L = 0.03$
Slightly rounded ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 8–36)
Rounding of an inlet makes a big difference.
Pipe Exit
Reentrant: $K_L = \alpha$
Sharp-edged: $K_L = \alpha$
Rounded: $K_L = \alpha$
Rounding of an outlet makes no difference.
Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)
Sudden expansion: $K_L = \left(1 - \frac{d^2}{D^2}\right)^2$
Note that the larger velocity (the velocity associated with the smaller pipe section) is used by convention in the equation for minor head loss, i.e., $h_{L,\text{minor}} = K_L \frac{V^2}{2g}$.


Sudden contraction: See chart.
K_L for sudden contraction
d^2/D^2
Note: These are backwards. The K_L values listed for Expansion should be those for Contraction, and vice-versa.
Note again that the larger velocity (the velocity associated with the smaller pipe section) is used by convention in the equation for minor head loss, i.e., $h_{L,\text{minor}} = K_L \frac{V^2}{2g}$.
Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)
Expansion:
$K_L = 0.02$ for $\theta = 20°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$
These are for contractions
Contraction (for $\theta = 20°$):
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$
These are for expansions
Bends and Branches
90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$
90° miter bend (without vanes): $K_L = 1.1$
90° miter bend (with vanes): $K_L = 0.2$
45° threaded elbow: $K_L = 0.4$
180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$
Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$
Threaded union:
$K_L = 0.08$
For tees, there are two values of K_L, one for branch flow and one for line flow.

افت هد در خم ها

➢افت هد تابعی از نسبت شعاع خم به قطر لوله است (R/D)

➢بعد از خم، در فاصله در حدود چندبرابر قطر لوله، پروفیل سرعت به حالت نرمال خود در قبل از خم برمی گردد.



$$h_b = K_b \frac{V^2}{2g}$$

$K_b \sim 0.6 - 0.9$

افت هد در شیرها

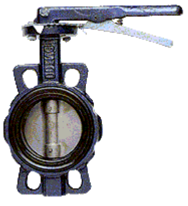
➢افت تابعی از نوع و موقعیت شیر است.

➢جریان پیچیده درون شیر میتواند منجر به افت زیاد شود.

What is V?

$$h_v = K_v \frac{V^2}{2g}$$

$$h_v = K_v \frac{8Q^2}{g\pi^2 D^4}$$



- ضریب افت شیر (K_v) می تواند بزرگتر از ۱ باشد.

ضرایب افت اصطکاکی برای انواع شیرها

Valve	K	L_{eq}/D
Gate valve, wide open	0.15	7
Gate valve, 3/4 open	0.85	40
Gate valve, 1/2 open	4.4	200
Gate valve, 1/4 open	20	900
Globe valve, wide open	7.5	350

تکنیکهای حل مسایل

- صرفنظر از افتهای موضعی
- ~~طول لوله معادل~~
- روشهای سعی و خطا
  - بکار بردن معادله Swamee-Jain برای D یا Q
  - بکار بردن معادله Swamee-Jain برای افت هد
  - فرض کردن یک ضریب اصطکاک
- استفاده از نرم افزارهای شبیه سازی شبکه لوله

تکنیک حل: محاسبه افت هد

- این نوع مسایل، بطور صریح قابل حل می باشند.

$$h_{minor} = \sum K \frac{V^2}{2g} \qquad h_{minor} = \frac{8Q^2}{g\pi^2}\sum \frac{K}{D^4}$$

$$\mathrm{Re} = \frac{4Q}{\pi D \nu} \qquad f = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad h_f = f\frac{8}{g\pi^2}\frac{LQ^2}{D^5}$$

$$h_l = \sum h_f + \sum h_{minor}$$

تکنیک حل: سعی و خطا در محاسبه دبی یا قطر

- کلیه افتها را بعنوان افت هد اصلی درنظر بگیرید.
- باکمک معادله Swamee-Jain قطر یا دبی را تعیین کنید.
- افتهای موضعی را محاسبه کنید.

$$h_{ex} = K\frac{8Q^2}{g\pi^2 D^4}$$

- افتهای اصلی جدید را با کم کردن افتهای مضعی از کل افت محاسبه کنید.

$$h_f = h_l - \sum h_{ex}$$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04}$$

$$Q = \frac{-\pi}{\sqrt{2}} D^{5/2}\sqrt{\frac{gh_f}{L}}\log\left(\frac{\varepsilon}{3.7D} + 2.51\nu\sqrt{\frac{L}{2gh_f D^3}}\right)$$

تکنیک حل: بکارگیری نرم افزار در محاسبه دبی یا قطر

- روش سعی و خطا
- معادلات زیر حل می شوند.

$$\mathrm{Re} = \frac{4Q}{\pi D \nu} \qquad \mathrm{f} = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad h_\mathrm{f} = \mathrm{f}\,\frac{8}{g\pi^2}\frac{LQ^2}{D^5}$$

$$h_{minor} = K\frac{8Q^2}{g\pi^2 D^4}$$

$$h_l = \sum h_\mathrm{f} + \sum h_{minor}$$

با کمک نرم افزار، محاسبات را تا جایی ادامه دهید که افت محاسبه شده با افت داده شده برابر شود.

تکنیک حل: بکارگیری نرم افزار در محاسبه دبی یا قطر با فرض اولیه برای f

- ضریب اصطکاک دارای تغییرات شدیدی نیست.
- اگر دبی معین است، f را ۰,۰۲ فرض کنید و اگر قطر لوله معین است، با فرض لوله زبر، f را از این رابطه بیابید.

$$\frac{1}{\sqrt{\mathrm{f}}} = 2\log\left(\frac{3.7D}{\varepsilon}\right)$$

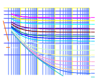
- محاسبه افتهای موضعی با کمک معادله دارسی وایسباخ
- محاسبه دبی یا قطر
- محاسبه Re و ε/D
- تعیین f جدید از دیاگرام مودی
- انجام سعی و خطا تا رسیدن به جواب نهایی

مثال: محاسبه افتهای اصلی و فرعی در مساله زیر

- حداکثر جریان قابل حمل بین دو مخزن، برای آب با دمای بین ۴ تا ۲۰ درجه را بیابید.

مثال (ادامه)

- عدد رینولدز در دو لوله چقدر است؟ 90,000 & 125,000
- در دیاگرام مودی : ε/D= 0.0006, 0.0008



- اثر دما چیست؟
- چرا اثر دما بسیار کم است؟
- مقدار K شیر چقدر باید باشد تا دبی ۵۰٪ کاهش یابد؟

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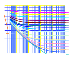
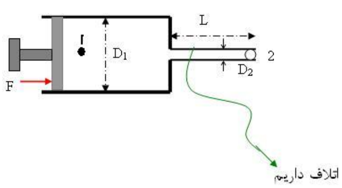
مثال (ادامه)

- آیا افتهای موضعی قابل صرفنظرند؟ بله
- دقت محاسبات افت چقدر است؟ ٪۵
- چه اتفاقی می افتد اگر زبری ۱۰ برابر شود؟ f از ۰,۰۲ به ۰,۰۳۵ می رسد.



- برای افزایش ٪۳۰ دبی جریان، چه میتوان کرد؟ میتوان لوله ای با قطرکمتر بکار برد.

مثال ۲



$D_2 = 1$ mm
$D_1 = 1$ cm
$\rho = 1000$ kg/m^3
$m^*_{out} = 2$ gr/s
$\mu = 10^{-2}$ pa.s
$L = 2$ cm
$V_1 = 2.5 \times 10^{-2}$ m/s

$$find: F$$

$$\left(\frac{p}{\rho g}+\frac{v^2}{2g}+z\right)_1-\left(\frac{p}{\rho g}+\frac{v^2}{2g}+z\right)_2=h_{loss}$$

با فرض جریان آرام:

$$f=\frac{64}{re}=\frac{64\mu}{\rho v D}$$

با صرف نظر از اصطکاک در اتصالات:

$$h_m=0$$

$$h_{loss}=h_m+h_f=h_f$$

$$h_f=f\frac{l}{D}\frac{v^2}{2g}\rightarrow h_{f1}+h_{f2}=\frac{32\mu_1 l_1 v_1}{\rho g {D^2}_1}+\frac{32\mu_2 l_2 v_2}{\rho g {D^2}_2}$$

مثال۲ (ادامه)

$$f_1 = 0 \to h_{f1} = 0 \to h_f = h_{f2} = \frac{32\mu_2 l_2 v_2}{\rho g {D^2}_2} = \frac{32 \times 10^{-2} \times 2 \times 10^{-2} \times 2.5}{1000 \times 10 \times 10^{-6}} = 1.6$$

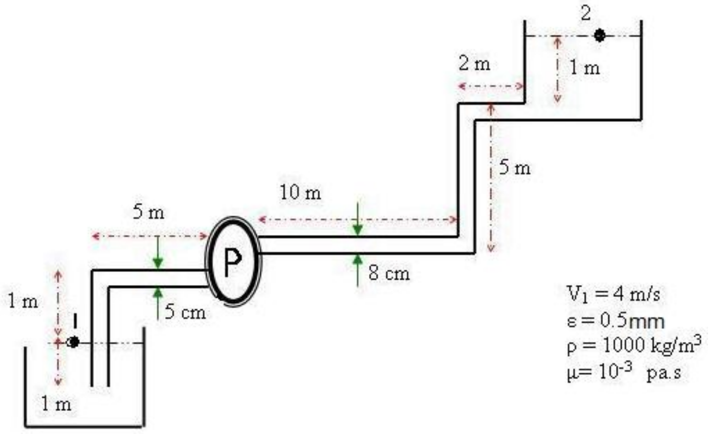
$$Q_{in} = Q_{out} \to A_1 v_1 = A_2 v_2 \to v_2 = 2.5ms^{-1}$$

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} - \frac{{v_2}^2}{2g} = 1.6 \to p_1 = 1.9 \times 10^4 pa$$

$$F = p_1 A_1 = \frac{\pi}{4}(10^{-2})^2 \times 1.9 \times 10^4 = 1.4$$

$$\dot{w} = F.v = 1.4 \times 2.5 \times 10^{-2}$$

مثال ۳



$find : \dot{w_{pump}}$

مثال ۳ (ادامه)

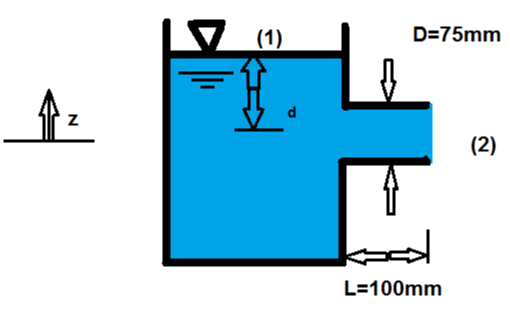
$$\left(\frac{p}{\rho g}+\frac{v^2}{2g}+z\right)_1-\left(\frac{p}{\rho g}+\frac{v^2}{2g}+z\right)_2=h_{loss}-h_p$$
$$\dot{w}_{pump}=\rho g h_p Q \rightarrow h_p=h_{loss}+7$$

صرف نظر از اصطکاک در اتصالات:

$$h_{loss}=h_{f1}+h_{f2}=\frac{32\mu_1 l_1 v_1}{\rho g {D^2}_1}+\frac{32\mu_2 l_2 v_2}{\rho g {D^2}_2}$$
$$h_{f1}=\frac{7}{5\times10^{-2}}f_1\frac{16}{2\times10}$$
$$\mathrm{Re}_{D1}=\frac{\rho v D}{\mu}=\frac{1000\times10^{-2}\times4\times5}{10^{-3}}=2\times10^5>2300$$
$$\frac{\varepsilon}{D}=0.01\xrightarrow{curve}f_1=0.04\rightarrow h_{f1}=4.48m$$
$$h_{f2}=\frac{17}{8\times10^{-2}}f2\frac{(1.4)^2}{2\times10}$$
$$\mathrm{Re}_{D2}=\frac{\rho v D}{\mu}=\frac{1000\times10^{-2}\times1.4\times8}{10^{-3}}=12.5\times10^4>2300$$
$$\dot{w}_{pump}=\rho g h_p Q=1000\times10\times18\times25\times10^{-4}\times4\times\frac{\pi}{4}=1414.7w$$

مثال ۴

جریان آب باآهنگ Q در لوله ی متصل به مخزن با سطح ثابت داریم. ورودی چهار گوش است. با فرض جریان smooth وضریب افت جزیی k=0.5، عمق آب در مخزن را بیابید.



$$Q=0.01\frac{m^3}{s}$$
$$D=75mm$$
$$L=100m$$
$$\rho_{water}=999\frac{kg}{m^3}$$
$$\mu=10^{-3}\frac{kg}{m.s}$$

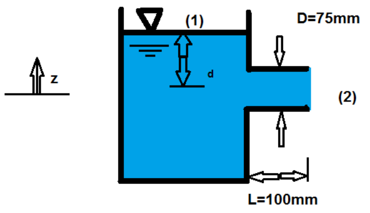
مثال ۴ (ادامه)

بین نقطه 1و2 رابطه ی بقای انرژی می نویسیم

$$(\frac{p}{\rho g}+\frac{V^2}{2g}+Z)_1-(\frac{p}{\rho g}+\frac{V^2}{2g}+Z)_2=h_{l_T}=h_l+h_{l_m}$$
$$h_l=f\frac{L}{D}\frac{V^2}{2g}\ \ \&\ \ h_{l_m}=k\frac{V^2}{2g}$$

در این مثال

$$p_1=p_2=p_{atm}$$
$$z_1=d$$
$$z_2=0$$



$$d-\frac{V^2}{2g}=f\frac{L}{D}\frac{V^2}{2g}+k\frac{V^2}{2g}$$
$$\rightarrow (f\frac{L}{D}\frac{V^2}{2g}+k\frac{V^2}{2g}+\frac{V^2}{2g})=\frac{V^2}{2g}(f\frac{L}{D}+k+1)$$
$$d=\frac{8Q^2}{\pi^2D^4g}(f\frac{L}{D}+k+1)$$

مثال ۴ (ادامه)

$$R_e=\frac{\rho VD}{\mu}=\frac{4\rho Q}{\pi\mu D}$$
$$=\frac{4}{\pi}\times 999\frac{kg}{m^3}\times 0.01\frac{m^3}{s}\times\frac{m.s}{1\times10^{-3}kg}\times\frac{1}{0.075m}=1.7\times10^5$$

از نمودار داریم

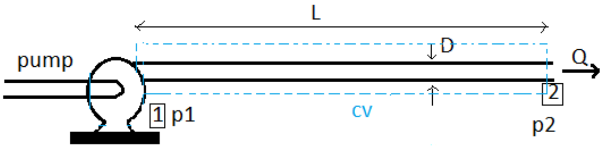
$f=0.017$ وبرای

$k=0.5$ لذا:

$$d=\frac{8}{\pi^2}\times(0.01)^2\frac{m^6}{s^2}\times\frac{1}{(0.075)^4m^4}\times\frac{s^2}{9.81m}\left[(0.017)\frac{100m}{0.075m}+0.5+1\right]=6.31m$$

مثال۵

سیستم تامین آب مطابق شکل موجوداست. کوچکترین قطراستانداردD راحساب کنید.



$$\varepsilon = 5 \times 10^{-6} ft$$
$$p_2 \geq 30 psig$$
$$p_1 \leq 65 psig$$
$$L = 500 ft$$
$$Q = 1500 gpm$$

مثال۵ (ادامه)

حل) برای تعیین کمترین قطر استانداردی که افت فشار دلخواه را در آهنگ جریان برقرار کند تکرار لازم است. ماکزیمم افت فشارمجاز در طول L چنین است:

$$\Delta p_{\max} = p_{1\max} - p_{2\min} = (65 - 30)\, psi = 35 psi$$

$$\left(\frac{p}{\rho} + \alpha\frac{V^2}{2} + gz\right)_2 - \left(\frac{p}{\rho} + \alpha\frac{V^2}{2} + gz\right)_1 = h_{l_T} = h_l + h_{l_m} = f\frac{L}{D}\frac{V_2^2}{2}$$

فرض) جریان پایاو تراکم پذیر است و

$$h_{l_m} = 0$$
$$z_1 = z_2$$
$$V_1 = V_2 = V$$
$$\alpha_1 \approx \alpha_2$$

لذا:

$$\Delta p = p_1 - p_2 = f\frac{L}{D}\frac{V^2}{2}$$

مثال ۵ (ادامه)

حل D ازاین معادله مشکل است. زیرا v و f به D بستگی دارند؛ لذا از روش تکرار دستی استفاده میکنیم. ابتدا معادله ی بالا و عدد رینولدز را بر حسب Q می نویسیم.

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$
$$\rightarrow \Delta p = \frac{8fL\rho Q^2}{\pi^2 D^5}$$
$$\rightarrow R_{ed} = \frac{\rho V D}{\mu} = \frac{4Q}{\pi V D}$$

Q رابه فوت مکعب بر ثانیه تبدیل میکنیم

$$Q = 1500\frac{gal}{\min} \times \frac{\min}{60s} \times \frac{ft^3}{7.48gal} = 3.34\frac{ft^3}{s}$$

به عنوان حدس اول مقدار اسمی 4 اینچ (قطر داخلی واقعی 4.026 اینچ) رادر نظر میگیریم:

$$\rightarrow R_{ed} = \frac{4Q}{\pi \nu D} = \frac{4}{\pi} \times 3.34\frac{ft^3}{s} \times \frac{s}{1.21 \times 10^{-5} ft^2} \times \frac{1}{4.026in} \times 12\frac{in}{ft} = 1.06 \times 10^6$$

$$\frac{\varepsilon}{D} = 1.5 \times 10^{-5}$$

مثال ۵ (ادامه)

از جدول f-زبری موجود در همین صفحه مقدار f را مییابیم:

$$f \approx 0.012$$
$$\rightarrow \Delta p = \frac{8fL\rho Q^2}{\pi^2 D^5} = \frac{8}{\pi^2} \times 0.012 \times 500ft \times 1.94\frac{slug}{ft^3} \times (3.34)^2\frac{ft^6}{s^2} \times \frac{1}{(4.026)^5 in^5} \times 1728\frac{in^3}{ft^3} \times \frac{lbf.s^2}{slug.ft}$$
$$= 172\frac{lbf}{in^2} > \Delta p_{\max}$$

چون این افت فشار خیلی زیاد است مقدار اسمی 6 اینچ (قطر داخلی واقعی 6.065 اینچ) رادر نظر میگیریم:

$$\rightarrow R_{ed} = \frac{4Q}{\pi \nu D} = 6.95 \times 10^5$$
$$\frac{\varepsilon}{D} = 1.5 \times 10^{-5}$$
$$\rightarrow f \approx 0.013$$
$$\rightarrow \Delta p = \frac{8fL\rho Q^2}{\pi^2 D^5} = 24\frac{lbf}{in^2} < \Delta p_{\max}$$

مثال۵ (ادامه)

چون این مقدار کمترازافت فشارمجازاست مقدار اسمی 5 اینچ (قطر داخلی واقعی 5.047 اینچ) رادر نظر میگیریم:

$$\rightarrow R_{ed} = \frac{4Q}{\pi\nu D} = 8.36 \times 10^5$$

$$\frac{\varepsilon}{D} = 1.2 \times 10^{-5}$$

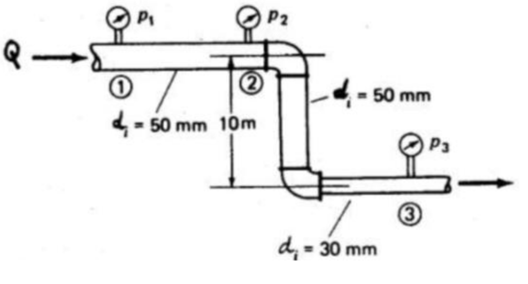
$$\rightarrow f \approx 0.0122$$

$$\rightarrow \Delta p = \frac{8fL\rho Q^2}{\pi^2 D^5} = 56.4\frac{lbf}{in^2} > \Delta p_{\max}$$

لذا فشار قابل قبول باقطر اسمی 6اینچ حاصل میشود.

مثال۶

۱- دبی جریان در لولهٔ نشان داده شده ۵ L/s است. اگر فشار نسبی در نقاط ۱، ۲ و ۳ به ترتیب ۱۲/۵ kPa، ۱۱/۵ kPa و ۱۰/۳ kPa باشد،افت هد بین نقاط ۱ و ۲ و نقاط ۱ و ۳ را بیابید.



مثال۶ (ادامه)

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + (h_L)_{1-2}$

$12.5/9.79 + v_1^2/2g + 10 = 11.5/9.79 + v_2^2/2g + 10 + (h_L)_{1-2}$

$v_1^2/2g = v_2^2/2g \qquad (h_L)_{1-2} = 0.1021\text{ m}$

$p_1/\gamma + v_1^2/2g + z_1 = p_3/\gamma + v_3^2/2g + z_3 + (h_L)_{1-3}$

$v_1 = Q/A_1 = (5\times10^{-3})/[(\pi)(0.050)^2/4] = 2.546\text{ m/s}$

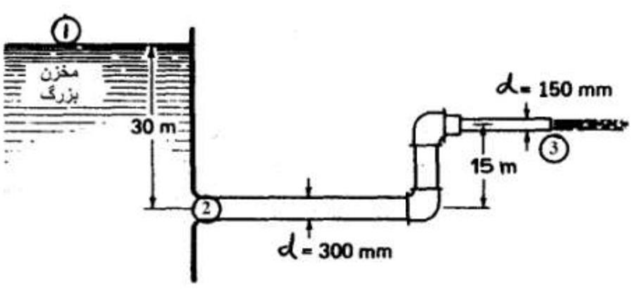
$v_3 = Q/A_3 = (5\times10^{-3})/[(\pi)(0.030)^2/4] = 7.074\text{ m/s}$

$12.5/9.79 + 2.546^2/[(2(9.807)] + 10 = 10.3/9.79 + 7.074^2/[(2)(9.807)] + 0 + (h_L)_{1-3}$

$(h_L)_{1-3} = 8.00\text{ m}$

مثال۷

در شکل زیر دبی آب ۱۴۰ L/s است. افت هد بین نقاط ۲ و ۳ را بیابید.



مثال۷ (ادامه)

$$p_2/\gamma + v_2^2/2g + z_2 = p_3/\gamma + v_3^2/2g + z_3 + h_L$$

$$v_2 = Q/A_2 = (140 \times 10^{-3})/[(\pi)(0.300)^2/4] = 1.981 \text{ m/s}$$

$$v_3 = Q/A_3 = (140 \times 10^{-3})/[(\pi)(0.150)^2/4] = 7.922 \text{ m/s}$$

$$p_2/9.79 + 1.981^2/[(2)(9.807)] + 0 = 0 + 7.922^2/[(2)(9.807)] + 15 + h_L$$

$$h_L = p_2/9.79 - 18.00$$

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$0 + 0 + 30 = p_2/9.79 + 1.981^2/[(2)(9.807)] + 0 + 0$$

$$p_2/9.79 = 29.80 \text{ m}$$

$$h_L = 29.80 - 18.00 = 11.80 \text{ m}$$

مثال۸

۳- آب در یک لولهٔ چدنی با سرعت ۴/۲ m/s حرکت می کند. طول لوله ۴۰۰ m و قطر آن ۱۵۰ mm است. افت هد اصطکاکی را بیابید. (ε=۰/۰۰۰۲۶ m)

$$h_f = (f)(L/d)(v^2/2g)$$

$$\text{Re} = dv/\nu = (\tfrac{150}{1000})(4.2)/(1.02 \times 10^{-6}) = 6.18 \times 10^5$$

$$\epsilon/d = 0.00026/0.150 = 0.00173$$

از نمودار مودی $\Longrightarrow f = 0.0226$

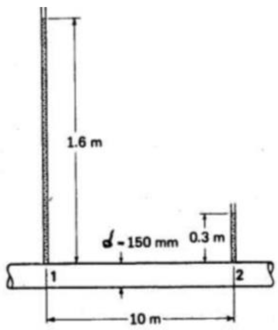
$$h_f = 0.0226[400/(\tfrac{150}{1000})]\{4.2^2/[(2)(9.807)]\} = 54.20 \text{ m}$$

مثال۹

دبی جریان در لولهٔ شکل زیر چقدر است؟

$\epsilon = 0.0000442\ \text{m}$

$\nu = 0.114 \times 10^{-5}\ \text{m}^2/\text{s}$



فشار در نقطهٔ 1 و نقطهٔ 2 از رابطه

$p = \rho g h$

حساب می شود که ارتفاع در نقطه 1 برابر 1/6 متر و در نقطه 2 برابر 0/3 متر است

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$1.6 + v_1^2/2g + 0 = 0.3 + v_2^2/2g + 0 + h_f$$

$$v_1^2/2g = v_2^2/2g$$

$$h_f = 1.3\ \text{m} = (f)(L/d)(v^2/2g)$$

حدس $\Longrightarrow f = 0.015$

$$1.3 = 0.015[10/(0.150)]\{v^2/[(2)(9.807)]\} \qquad v = 5.050\ \text{m/s}$$

$$\text{Re} = dv/\nu = (0.150)(5.050)/(0.114 \times 10^{-5}) = 6.64 \times 10^5$$

$$\epsilon/d = 0.0000442/(0.150) = 0.000295$$

از نمودار مودی $\Longrightarrow f = 0.016$

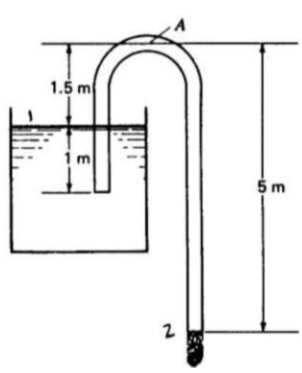
$$1.3 = 0.016[10/(0.150)]\{v^2/[(2)(9.807)]\} \qquad v = 4.889\ \text{m/s}$$

$$M = \rho A v = 1000[(\pi)(0.150)^2/4](4.889) = 86.4\ \text{kg/s}$$

مثال ۱۰

۵- سیالی با چگالی نسبی ۰/۶ از طریق لوله ای با قطر ۲۵ mm از تانک تخلیه می شود. دبی جریان و کمترین فشار را در لوله بیابید. کل طول لوله ۹ m و طول لوله از تانک تا نقطهٔ A ۳/۲۵ m است. از افت های فرعی صرف نظر کنید.

$\varepsilon/d = ۰/۰۰۰۴$ $\qquad \nu = ۵\times ۱۰^{-۷}\ \text{m}^{۲}/\text{s}$



مثال ۱۰ (ادامه)

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$0 + 0 + (5 - 1.5) = 0 + v_2^2/[(2)(9.807)] + 0 + h_f$

$h_f = 3.5 - 0.05098 v_2^2 = (f)(L/d)(v^2/2g)$

حدس $\Rightarrow f = 0.016$

$h_f = 0.016[9/(0.025)]\{v_2^2/[(2)(9.807)]\} = 0.2937 v_2^2$

$0.2937 v_2^2 = 3.5 - 0.05098 v_2^2 \qquad v = 3.187 \text{ m/s}$

$\text{Re} = dv/\nu = (0.025)(3.187)/(5.0 \times 10^{-7}) = 1.59 \times 10^5$

$\varepsilon/d=$۰/۰۰۰۰۴ از نمودار مودی با داشتن $\Rightarrow f = 0.019$.

$h_f = 0.019[9/(0.025)]\{v_2^2/[(2)(9.807)]\} = 0.3487 v_2^2$

$0.3487 v_2^2 = 3.5 - 0.05098 v_2^2 \qquad v = 2.959 \text{ m/s}$

$\text{Re} = dv/\nu = (0.025)(2.959)/(5.0 \times 10^{-7}) = 1.48 \times 10^5$

$\Rightarrow f = 0.019$

$Q = Av = [(\pi)(0.025)^2/4](2.959) = 1.45 \text{ L/s}$

$p_1/\gamma + v_1^2/2g + z_1 = p_A/\gamma + v_A^2/2g + z_A + h_L$

$0 + 0 + (5 - 1.5) = p_A/[(0.6)(9.79)] + 2.959^2/[(2)(9.807)] + 5 + h_f$

$p_A = -11.43 - 5.874 h_f$

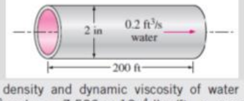
$h_f = 0.019[3.25/(0.025)]\{2.959^2/[(2)(9.807)]\} = 1.103 \text{ m}$

$p_A = -11.43 - (5.874)(1.103) = -17.91 \text{ kPa}$

مثال ۱۱

EXAMPLE 8–3 Determining the Head Loss in a Water Pipe

Water at 60°F (ρ = 62.36 lbm/ft^3 and μ = 7.536 × 10^{-4} lbm/ft · s] is flowing steadily in a 2-in-diameter horizontal pipe made of stainless steel at a rate of 0.2 ft^3/s (Fig. 8–30). Determine the pressure drop, the head loss, and the required pumping power input for flow over a 200-ft-long section of the pipe.



Properties The density and dynamic viscosity of water are given to be ρ = 62.36 lbm/ft^3 and μ = 7.536 × 10^{-4} lbm/ft · s, respectively.

Analysis We recognize this as a problem of the first type, since flow rate, pipe length, and pipe diameter are known. First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2 \text{ ft}^3/\text{s}}{\pi(2/12 \text{ ft})^2/4} = 9.17 \text{ ft/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(62.36 \text{ lbm/ft}^3)(9.17 \text{ ft/s})(2/12 \text{ ft})}{7.536 \times 10^{-4} \text{ lbm/ft} \cdot \text{s}} = 126{,}400$$

Since Re is greater than 4000, the flow is turbulent. The relative roughness of the pipe is estimated using Table 8–2

$$\varepsilon/D = \frac{0.000007 \text{ ft}}{2/12 \text{ ft}} = 0.000042$$

The friction factor corresponding to this relative roughness and Reynolds number is determined from the Moody chart. To avoid any reading error, we determine f from the Colebrook equation on which the moody chart is based:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0.000042}{3.7} + \frac{2.51}{126{,}400\sqrt{f}}\right)$$

Using an equation solver or an iterative scheme, the friction factor is determined to be f = 0.0174. Then the pressure drop (which is equivalent to pressure loss in this case), head loss, and the required power input become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.0174 \frac{200 \text{ ft}}{2/12 \text{ ft}} \frac{(62.36 \text{ lbm/ft}^3)(9.17 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)$$

$$= 1700 \text{ lbf/ft}^2 = 11.8 \text{ psi}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.0174 \frac{200 \text{ ft}}{2/12 \text{ ft}} \frac{(9.17 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 27.3 \text{ ft}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.2 \text{ ft}^3/\text{s})(1700 \text{ lbf/ft}^2)\left(\frac{1 \text{ W}}{0.737 \text{ lbf} \cdot \text{ft/s}}\right) = 461 \text{ W}$$

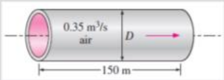
Therefore, power input in the amount of 461 W is needed to overcome the frictional losses in the pipe.

Discussion It is common practice to write our final answers to three significant digits, even though we know that the results are accurate to at most two significant digits because of inherent inaccuracies in the Colebrook equation, as discussed previously. The friction factor could also be determined easily from the explicit Haaland relation (Eq. 8–51). It would give f = 0.0172, which is sufficiently close to 0.0174. Also, the friction factor corresponding to ε = 0 in this case is 0.0171, which indicates that this stainless-steel pipe can be approximated as smooth with negligible error.

مثال ۱۲

EXAMPLE 8–4 Determining the Diameter of an Air Duct

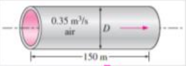
Heated air at 1 atm and 35°C is to be transported in a 150-m-long circular plastic duct at a rate of 0.35 m^3/s (Fig. 8–31). If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.



Properties The density, dynamic viscosity, and kinematic viscosity of air at 35°C are $\rho = 1.145$ kg/m³, $\mu = 1.895 \times 10^{-5}$ kg/m · s, and $\nu = 1.655 \times 10^{-5}$ m²/s.

Analysis This is a problem of the third type since it involves the determination of diameter for specified flow rate and head loss. We can solve this problem by three different approaches: (1) an iterative approach by assuming a pipe diameter, calculating the head loss, comparing the result to the specified head loss, and repeating calculations until the calculated head loss matches the specified value; (2) writing all the relevant equations (leaving the diameter as an unknown) and solving them simultaneously using an equation solver; and (3) using the third Swamee–Jain formula. We will demonstrate the use of the last two approaches.

The average velocity, the Reynolds number, the friction factor, and the head loss relations are expressed as (D is in m, V is in m/s, and Re and f are dimensionless)



$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.35 \text{ m}^3/\text{s}}{\pi D^2/4}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{VD}{1.655 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0 \log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \quad \rightarrow \quad 20 \text{ m} = f\frac{150 \text{ m}}{D}\frac{V^2}{2(9.81 \text{ m/s}^2)}$$

The roughness is approximately zero for a plastic pipe (Table 8–2). Therefore, this is a set of four equations and four unknowns, and solving them with an equation solver such as EES gives

$$D = \mathbf{0.267 \text{ m}}, \quad f = 0.0180, \quad V = 6.24 \text{ m/s}, \quad \text{and} \quad \text{Re} = 100{,}800$$

Therefore, the diameter of the duct should be more than 26.7 cm if the head loss is not to exceed 20 m. Note that Re > 4000, and thus the turbulent flow assumption is verified.

The diameter can also be determined directly from the third Swamee–Jain formula to be

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

$$= 0.66\left[0 + (1.655 \times 10^{-5} \text{ m}^2/\text{s})(0.35 \text{ m}^3/\text{s})^{9.4}\left(\frac{150 \text{ m}}{(9.81 \text{ m/s}^2)(20 \text{ m})}\right)^{5.2}\right]^{0.04}$$

$$= \mathbf{0.271 \text{ m}}$$

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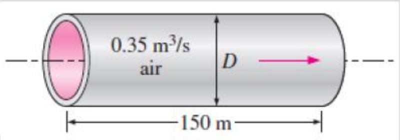
مثال ۱۳

EXAMPLE 8–5 Determining the Flow Rate of Air in a Duct

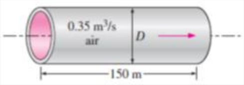
Reconsider Example 8–4. Now the duct length is doubled while its diameter is maintained constant. If the total head loss is to remain constant, determine the drop in the flow rate through the duct.



مثال ۱۳ (ادامه)

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and head loss. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

The average velocity, Reynolds number, friction factor, and the head loss relations are expressed as (D is in m, V is in m/s, and Re and f are dimensionless)



$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi (0.267 \text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.267 \text{ m})}{1.655 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \quad \rightarrow \quad 20 \text{ m} = f\frac{300 \text{ m}}{0.267 \text{ m}}\frac{V^2}{2(9.81 \text{ m/s}^2)}$$

This is a set of four equations in four unknowns and solving them with an equation solver such as EES gives

$$\dot{V} = 0.24 \text{ m}^3/\text{s}, \quad f = 0.0195, \quad V = 4.23 \text{ m/s}, \quad \text{and} \quad \text{Re} = 68{,}300$$

Then the drop in the flow rate becomes

$$\dot{V}_{\text{drop}} = \dot{V}_{\text{old}} - \dot{V}_{\text{new}} = 0.35 - 0.24 = \mathbf{0.11 \text{ m}^3/\text{s}} \quad \text{(a drop of 31 percent)}$$

Therefore, for a specified head loss (or available head or fan pumping power), the flow rate drops by about 31 percent from 0.35 to 0.24 m³/s when the duct length doubles.

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مثال ۱۳ (ادامه)

Alternative Solution If a computer is not available (as in an exam situation), another option is to set up a *manual iteration loop*. We have found that the best convergence is usually realized by first guessing the friction factor f, and then solving for the velocity V. The equation for V as a function of f is

Average velocity through the pipe: $$V = \sqrt{\frac{2gh_L}{fL/D}}$$

Once V is calculated, the Reynolds number can be calculated, from which a *corrected* friction factor is obtained from the Moody chart or the Colebrook equation. We repeat the calculations with the corrected value of f until convergence. We guess $f = 0.04$ for illustration:

Iteration	f (guess)	V, m/s	Re	Corrected f
1	0.04	2.955	4.724×10^4	0.0212
2	0.0212	4.059	5.489×10^4	0.01973
3	0.01973	4.207	5.727×10^4	0.01957
4	0.01957	4.224	5.754×10^4	0.01956
5	0.01956	4.225	5.756×10^4	0.01956

Notice that the iteration has converged to three digits in only three iterations and to four digits in only four iterations. The final results are identical to those obtained with EES, yet do not require a computer.

Discussion The new flow rate can also be determined directly from the second Swamee Jain formula to be

$$\dot{V} = -0.965\left(\frac{gD^5h_L}{L}\right)^{0.5} \ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{gD^3h_L}\right)^{0.5}\right]$$

$$= -0.965\left(\frac{(9.81 \text{ m/s}^2)(0.267 \text{ m})^5(20 \text{ m})}{300 \text{ m}}\right)^{0.5}$$

$$\times \ln\left[0 + \left(\frac{3.17(1.655 \times 10^{-5} \text{ m}^2/\text{s})^2(300 \text{ m})}{(9.81 \text{ m/s}^2)(0.267 \text{ m})^3(20 \text{ m})}\right)^{0.5}\right]$$

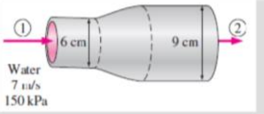
$$= 0.24 \text{ m}^3/\text{s}$$

Note that the result from the Swamee–Jain relation is the same (to two significant digits) as that obtained with the Colebrook equation using EES or using our manual iteration technique. Therefore, the simple Swamee–Jain relation can be used with confidence.

مثال ۱۴

EXAMPLE 8–6 **Head Loss and Pressure Rise during Gradual Expansion**

A 6-cm-diameter horizontal water pipe expands gradually to a 9-cm-diameter pipe (Fig. 8–40). The walls of the expansion section are angled 10° from the axis. The average velocity and pressure of water before the expansion section are 7 m/s and 150 kPa, respectively. Determine the head loss in the expansion section and the pressure in the larger-diameter pipe.



Assumptions **1** The flow is steady and incompressible. **2** The flow at sections 1 and 2 is fully developed and turbulent with $\alpha_1 = \alpha_2 \cong 1.06$.
Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$. The loss coefficient for a gradual expansion of total included angle $\theta = 20°$ and diameter ratio $d/D = 6/9$ is $K_L = 0.133$ (by interpolation using Table 8–4).
Analysis Noting that the density of water remains constant, the downstream velocity of water is determined from conservation of mass to be

$$\dot m_1 = \dot m_2 \rightarrow \rho V_1 A_1 = \rho V_2 A_2 \rightarrow V_2 = \frac{A_1}{A_2}V_1 = \frac{D_1^2}{D_2^2}V_1$$

$$V_2 = \frac{(0.06\ \text{m})^2}{(0.09\ \text{m})^2}(7\ \text{m/s}) = 3.11\ \text{m/s}$$

مثال ۱۴ (ادامه)

Then the irreversible head loss in the expansion section becomes

$$h_L = K_L\frac{V_1^2}{2g} = (0.133)\frac{(7\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 0.333\ \text{m}$$

Noting that $z_1 = z_2$ and there are no pumps or turbines involved, the energy equation for the expansion section is expressed in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + \cancel{z_1} + \cancelto{0}{h_{\text{pump, }u}} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + \cancel{z_2} + \cancelto{0}{h_{\text{turbine, }e}} + h_L$$

or

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + h_L$$

Solving for P_2 and substituting,

$$P_2 = P_1 + \rho\left\{\frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2} - gh_L\right\} = (150\ \text{kPa}) + (1000\ \text{kg/m}^3)$$

$$\times\left\{\frac{1.06(7\ \text{m/s})^2 - 1.06(3.11\ \text{m/s})^2}{2} - (9.81\ \text{m/s}^2)(0.333\ \text{m})\right\}$$

$$\times\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)$$

$$= 168\ \text{kPa}$$

Therefore, despite the head (and pressure) loss, the pressure *increases* from 150 to 168 kPa after the expansion. This is due to the conversion of dynamic pressure to static pressure when the average flow velocity is decreased in the larger pipe.

مثال ۱۵

Example 1. *Oil, with* $\rho = 900 kg/m3$ *and kinematic coefficient of viscosity* $\nu = 0,00001 m2/s$, *flows at* $q_v = 0,2 m3/s$ *through 500 m of 200-mm diameter cast-iron pipe. Determine (a) the head loss and (b) the pressure drop if the pipe slopes down at 10 in the flow direction.*
Solution. *First we compute the Reynolds number* $Re = V \cdot d/\nu = 4 \cdot q_v/(\pi d \nu) = 4 \cdot 0,2/(3,14 \cdot 0,2 \cdot 0,00001) = 128000$. *Velocity is equal to* $v = q_v/(\frac{\pi d^2}{4}) = 6.4\ m/s$. *Absolute roughness for iron-cast pipe is* $\varepsilon = 0.26$ *mm. So relative roughness is* $\varepsilon = \frac{\varepsilon}{d} = 0,0013$. *Now we are able to calculate using the formula(10) or Moody chart, friction factor* $f = 0,0225$. *Then the head loss is*

$$h_f = f\frac{L}{d}\frac{v^2}{2g} = 0,0225\frac{500}{0.2}\frac{6,4^2}{2\cdot 9,81} = 117\ m$$

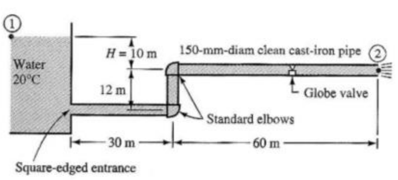
For inclined pipe the head loss is

$$h_f = \frac{\Delta p}{\rho g} + z_1 - z_2 = \frac{\Delta p}{\rho g} + L\sin 10^o.$$

So pressure drop is

$$\Delta p = \rho g(h_f - 500\cdot\sin 10^o) = 900\cdot 9,81\cdot(117-87) = 265\cdot 10^3.$$

مثال ۱۶



Example 3. *Find (see figure 5.)*

1. *the discharge through the pipeline as in figure for H=10 m*
2. *determine the head loss* H_1 *for* $q_v = 60\ l/s$.

$D = 0.15m$, $\frac{\varepsilon}{D} = 0,0017$, $\nu = 1,01\cdot 10^{-6}$.
Solution. *The energy equation applied between points 1 and 2 including all the losses, can be written as*

$$H_1 = \frac{v^2}{2g} + f\,\frac{L}{D}\,\frac{v^2}{2g} + (K_1+K_2+K_3+K_4)\,\frac{v^2}{2g}$$

مثال۱۶ (ادامه)

where $K_1 = 0,5$ is entrance loss coefficient, $K_2, K_3 = 0,9$ as a standard elbow, and $K_4 = 10$ for globe valve fully open. Putting the values tot the above formula one obtain

$$H_1 = \frac{v^2}{2g}(13,3 + 680\ f)$$

To start we need to guess f. A good first guess is the "fully rough" value (wholly turbulent) for $\varepsilon/d = 0,0017$ from Moody chart. It is $f \approx 0,022$. Now from Darcy-Weisbach formula (4) we have

$$10 = \frac{v^2}{2g}(13,3 + 680\ 0,022) \qquad \Rightarrow \quad v_2 = 2.63, \quad Re = 391000$$

$$\begin{array}{lll} f \approx 0,023 & v = 2,6\ m/s & Re = vd/\nu \approx 380000 \\ f_{new}(380000) = 0,023 & v = 2,6\ m/s & Re = vd/\nu \approx 380000 \end{array}$$

We accepted $v = 2,60\ m/s$, $q_v = v(\frac{\pi d^2}{4}) = 45,9 \cdot 10^-3\ m^3/s$.
For the second part, with q_v known, the solution is straightforward:

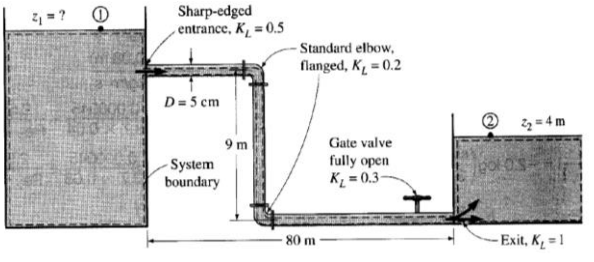
$$v_2 = \frac{q_v}{A} = \frac{0.06}{\frac{\pi 0.15^2}{4}} = 3.40\ m/s, \qquad Re = 505000, \qquad f = 0.023$$

and

$$H_1 = \frac{3,4^2}{2 \cdot 9,81}(13,3 + 680\ 0,023) = 17,06\ m$$

مثال ۱۷

Example 4. ***Flow between two reservoirs.*** *Water at 10^oC flow from a large reservoir to a small one through a 5-cm-diameter cast iron piping system shown in figure (6). Determine the elevation z_1 for the flow rate of $6l/s$.*



(Answer. 27,9 m)

مثال ۱۸

Find the head loss due to the flow of 1,500 gpm of oil ($\nu = 1.15\times10^{-4}\ ft^2/s$) through 1,600 feet of 8" diameter cast iron pipe. If the density of the oil $\rho = 1.75\ slug/ft^3$, what is the power to be supplied by a pump to the fluid? Find the BHP of the pump if its efficiency is 0.85.

Solution

We have the following information.

$\rho = 1.75\ slug/ft^3$ $\qquad \nu = 1.15\times10^{-4}\ ft^2/s$ $\qquad D = 0.667\ ft$

Therefore, the cross-sectional area is

$$A = \pi D^2/4 = \pi\times(0.667\ ft)^2/4 = \boxed{0.349\ ft^2}$$

$$Q = 1500\,(gpm)\times\frac{1\,(ft^3/s)}{448.8\,(gpm)} = \boxed{3.34\ \frac{ft^3}{s}}$$

Therefore, the average velocity through the pipe is $V = \frac{Q}{A} = \frac{3.34\,(ft^3/s)}{0.349\,(ft^2)} = \boxed{9.58\ \frac{ft}{s}}$

We can calculate the Reynolds number.

مثال۱۸ (ادامه)

$$\mathrm{Re} = \frac{DV}{\nu} = \frac{0.667\,(ft)\times9.58\,(ft/s)}{1.15\times10^{-4}\,(ft^2/s)} = \boxed{5.55\times10^4}$$ Therefore, the flow is turbulent.

For cast iron, $\varepsilon = 8.5\times10^{-4}\ ft$. Therefore, the relative roughness is

$$\frac{\varepsilon}{D} = \frac{8.5\times10^{-4}\,(ft)}{0.667\,(ft)} = \boxed{1.27\times10^{-3}}$$

Because we have the values of both the Reynolds number and the relative roughness, it is efficient to use the Zigrang-Sylvester equation for a once-through calculation of the turbulent flow friction factor.

$$\frac{1}{\sqrt{f}} = -4.0\log_{10}\left[\frac{\varepsilon/D}{3.7} - \frac{5.02}{\mathrm{Re}}\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{13}{\mathrm{Re}}\right)\right]$$

$$= -4.0\log_{10}\left[\frac{1.27\times10^{-3}}{3.7} - \frac{5.02}{5.55\times10^4}\log_{10}\left(\frac{1.27\times10^{-3}}{3.7} + \frac{13}{5.55\times10^4}\right)\right] = 12.8$$

which yields $\boxed{f = 0.00612}$

مثال۱۸ (ادامه)

The head loss is obtained by using

$$h_f = 2f\left(\frac{L}{D}\right)\left(\frac{V^2}{g}\right) = 2\times 0.00612\times\frac{1,600\,(ft)}{0.667\,(ft)}\times\frac{(9.58\ ft/s)^2}{32.2\left(ft/s^2\right)} = \boxed{83.7\ ft}$$

The mass flow rate is $\dot{m} = \rho Q = 1.75\left(\frac{slug}{ft^3}\right)\times 3.34\left(\frac{ft^3}{s}\right) = \boxed{5.85\ \frac{slug}{s}}$

The power supplied to the fluid is calculated from

$$\text{Power to Fluid} = \dot{m}\,h_f g = 5.85\left(\frac{slug}{s}\right)\times 83.7\,(ft)\times 32.2\left(\frac{ft}{s^2}\right) = \boxed{1.58\times 10^4\ \frac{ft\bullet lb_f}{s}}$$

We know that $1\ \text{HorsePower} = 550\,\frac{ft\bullet lb_f}{s}$. Therefore, $\text{Power to Fluid} = \boxed{28.7\ hp}$

The efficiency of the pump $\eta = 0.85$. Therefore,

$$\text{Brake Horse Power} = \frac{\text{Power to Fluid}}{\eta} = \frac{28.7\,(hp)}{0.85} = \boxed{33.7\ hp}$$

مثال ۱۹

Water at $15^\circ C$ flows through a $25-cm$ diameter riveted steel pipe of length $450\ m$ and roughness $\varepsilon = 3.2\ mm$. The head loss is known to be $7.30\ m$. Find the volumetric flow rate of water in the pipe.

Solution

For water at $15^\circ C$, $\rho = 999\ kg/m^3$ $\mu = 1.16\times 10^{-3}\ Pa\bullet s$ so that the kinematic viscosity can be calculated as $\nu = \mu/\rho = \boxed{1.16\times 10^{-6}\ m^2/s}$

The pipe diameter is given as $D = 0.25\ m$, so that the cross-sectional area is

$A = \pi D^2/4 = \pi\times(0.25\ m)^2/4 = \boxed{4.91\times 10^{-2}\ m^2}$

The length of the pipe is given as $\boxed{L = 450\ m}$

We do not know the velocity of water in the pipe, but we can express the Reynolds number in terms of the unknown velocity.

$\text{Re} = \frac{DV}{\nu} = \frac{0.25\,(m)\times V}{1.16\times 10^{-6}\left(m^2/s\right)} = \boxed{2.16\times 10^5\ V}$ where V must be in m/s.

At this point, we do not know whether the flow is laminar or turbulent. Given the size of the pipe and the head loss, it is reasonable to assume turbulent flow and proceed. In the end, we need to check whether this assumption is correct.

مثال ۱۹ (ادامه)

Now, we are given the head loss h_f. Let us write the result for h_f in terms of the friction factor.

$h_f = 2f\left(\frac{L}{D}\right)\left(\frac{V^2}{g}\right)$ Substitute the values of known entities in this equation.

$7.30\,(m) = 2\,f \times \left(\frac{450\,(m)}{0.25\,(m)}\right) \times \left(\frac{V^2}{9.81\left(m/s^2\right)}\right)$ This can be rearranged to yield

$\boxed{fV^2 = 1.99\times10^{-2}\,\frac{m^2}{s^2}}$ where V must be in m/s.

Taking the square root, we find $\boxed{\sqrt{f} = \frac{0.141}{V}}$

We can see that the product $\mathrm{Re}\sqrt{f}$ can be calculated, even though we do not know the velocity V.

$\mathrm{Re}\sqrt{f} = 2.16\times10^5\,V \times \frac{0.141}{V} = \boxed{3.05\times10^4}$

Given $\varepsilon = 3.2\,mm$, the relative roughness is

$\frac{\varepsilon}{D} = \frac{3.2\times10^{-3}\,(m)}{0.25\,(m)} = \boxed{1.28\times10^{-2}}$

مثال ۱۹ (ادامه)

Therefore, the entire right side in the Colebrook Equation for the friction factor is known. We can use the Colebrook Equation to evaluate the friction factor in an once-through calculation.

$$\frac{1}{\sqrt{f}} = -4.0\log_{10}\left[\frac{\varepsilon/D}{3.7} + \frac{1.26}{\mathrm{Re}\sqrt{f}}\right] = -4.0\log_{10}\left[\frac{1.28\times10^{-2}}{3.7} + \frac{1.26}{3.05\times10^4}\right] = \boxed{9.82}$$

Therefore, the friction factor is $\boxed{f = 0.0104}$

Using $\sqrt{f} = \frac{0.141}{V}$, we can evaluate the velocity as

$V = \frac{0.141\,(m/s)}{\sqrt{f}} = \frac{0.141\,(m/s)}{0.102} = \boxed{1.39\,m/s}$ so that the volumetric flow rate is obtained as

$Q = VA = 1.39\,(m/s)\,\times\,4.91\times10^{-2}\,\left(m^2\right) = \boxed{6.80\times10^{-2}\,\frac{m^3}{s}}$

We must check the Reynolds number. $\mathrm{Re} = 2.16\times10^5\,V = \boxed{3.00\times10^5}$. This is well over $4{,}000$ so that we can conclude that the assumption of turbulent flow is correct.

مثال ۲۰

Determine the size of smooth 14-gage BWG copper tubing needed to convey 10 gpm of a process liquid of kinematic viscosity $\nu = 2.40\times10^{-5}\ ft^2/s$ over a distance of 133 ft at ground level using a storage tank at an elevation of 20 ft. You can assume minor losses from fittings in the line to account for 5 ft of head.

In this problem, we are asked to calculate the diameter D of the tube. We are given $\boxed{L = 150\ ft}$ and $Q = 10\ gpm \times \dfrac{1\left(ft^3/s\right)}{448.8\left(gpm\right)} = \boxed{2.23\times10^{-2}\ \dfrac{ft^3}{s}}$. Given that the storage tank is located at an elevation of 20 ft above ground, we can infer that the available head loss for friction in the flow through the tube is $h_f = (20-5)\ ft = \boxed{15\ ft}$.

The diameter appears in both the Reynolds number and the result for the head loss in terms of the friction factor. Let us begin with the head loss and write it in terms of the volumetric flow rate, which is known.

$$h_f = 2f\left(\frac{L}{D}\right)\left(\frac{V^2}{g}\right) = 2f\left(\frac{L}{D}\right)\left(\frac{\left(4Q/\pi D^2\right)^2}{g}\right) = f\times\frac{32\,LQ^2}{\pi^2 g D^5}$$

Substituting known entities in this equation, we obtain

مثال ۲۰ (ادامه)

$$15\ ft = f\times\frac{32\times133\,(ft)\times\left(2.23\times10^{-2}\ ft^3/s\right)^2}{\pi^2\times32.2\left(ft/s^2\right)\times D^5} = 6.65\times10^{-3}\ \frac{f}{D^5}\quad \text{so that}\quad \boxed{f = 2.26\times10^3\ D^5}$$

where D must be in feet.

The Reynolds number can be written as

$$\mathrm{Re} = \frac{4Q}{\pi\nu D} = \frac{4\times2.23\times10^{-2}\left(ft^3/s\right)}{\pi\times2.40\times10^{-5}\left(ft^2/s\right)\times D} = \boxed{\frac{1.18\times10^3}{D}}\ \text{where } D \text{ must be in feet.}$$

We can make further progress if we assume the type of flow, so that we can use a correlation for the friction factor. It is reasonable in process situations with this flow rate to assume turbulent flow. So, we shall proceed with that assumption, to be verified later when we can calculate the Reynolds number.

It does not matter which correlation we use, because we must solve an implicit equation for the diameter in either case. So, let us use the Colebrook equation because it is simpler. For a smooth tube, the roughness , $\varepsilon = 0$, so that we can set the relative roughness $\varepsilon/D = 0$ in the Colebrook equation to obtain

مثال ۲۰ (ادامه)

$$\frac{1}{\sqrt{f}} = -4.0\log_{10}\left[\frac{1.26}{\mathrm{Re}\sqrt{f}}\right]$$

In this equation, substitute for both the friction factor and the Reynolds number in terms of the diameter, to obtain

$$\frac{1}{47.5D^{5/2}} = -4.0\log_{10}\left[\frac{1.26}{\left(1.18\times10^3/D\right)\times\left(47.5\,D^{5/2}\right)}\right] = -4.0\log_{10}\left[\frac{2.25\times10^{-5}}{D^{3/2}}\right]$$

or

$$D^{-5/2} = -190\log_{10}\left[2.25\times10^{-5}\,D^{-3/2}\right]$$

Solving this equation, we obtain $D = 7.91\times10^{-2}\,ft = \boxed{0.949"}$

مثال ۲۰ (ادامه)

A table of standard tubing dimensions for specified nominal diameters and Birmingham Wire Gage (BWG) values can be found in many places. The textbook by Welty et al. provides it as Appendix N. From the table, we find that for 14-gage tubing with an outside diameter of 1", the inside diameter is 0.834". The next higher outside diameter available is $1\frac{1}{4}$ – inch, and for this OD, 14-gage tubing comes with an inside diameter of 1.084". Therefore, we must select one of these two tubes. If we want to be sure to obtain the desired flow rate, we must choose the value that is larger than 0.949". You may wonder why. Here is an approximate answer.

In turbulent flow, the friction factor $f \propto V^{-a}$, where $0 \le a < 1$. In laminar flow, $f \propto V^{-1}$. In both cases, we can write $fV^2 \propto V^b$ where $b > 0$. Therefore, the head loss from pipe flow friction $h_f = \frac{1}{D} fV^2\left(\frac{2L}{g}\right) \propto \frac{1}{D} fV^2 \propto \boxed{\frac{1}{D}V^b}$

For a fixed volumetric flow rate, as the diameter is increased, V^b decreases and $1/D$ also decreases. Therefore, the head loss decreases for a given volumetric flow rate as the diameter is increased. This means that with a fixed head loss available, we can comfortably achieve the desired flow rate using a suitable valve. On the other hand, if we choose a diameter that is smaller than the calculated value, we would need a larger head available for driving the flow than is available.

Now, let us use the actual inside diameter of the selected tube, $D = 1.084" = 9.03\times10^{-2}\,ft$ to evaluate the Reynolds number of the flow.

مثال ۲۰ (ادامه)

$\text{Re} = \frac{4Q}{\pi \nu D} = \frac{4 \times 2.23 \times 10^{-2}\,(ft^3/s)}{\pi \times 2.40 \times 10^{-5}\,(ft^2/s) \times 9.03 \times 10^{-2}\,(ft)} = \boxed{1.31 \times 10^4}$ Therefore, the flow is turbulent as assumed.

The actual friction factor can be calculated from the Zigrang-Sylvester equation.

$$\frac{1}{\sqrt{f}} = -4.0 \log_{10}\left[\frac{\varepsilon/D}{3.7} - \frac{5.02}{\text{Re}}\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{13}{\text{Re}}\right)\right]$$

$$= -4.0 \log_{10}\left[0 - \frac{5.02}{1.31 \times 10^4}\log_{10}\left(0 + \frac{13}{1.31 \times 10^4}\right)\right] = 11.8$$

yielding $\boxed{f = 0.00724}$

The actual head loss for the desired volumetric flow rate will be

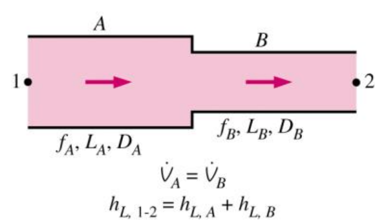
$$h_f = f \times \frac{32\,LQ^2}{\pi^2 g D^5} = 0.00724 \times \frac{32 \times 133\,(ft) \times \left(2.23 \times 10^{-2}\ ft^3/s\right)^2}{\pi^2 \times 32.2\,(ft/s^2) \times \left(9.03 \times 10^{-2}\ ft\right)^5} = \boxed{8.03\ ft}$$

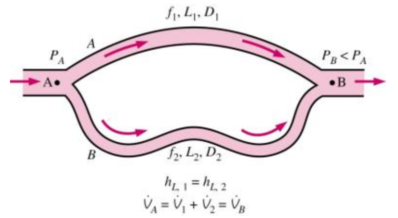
which is less than available head of 15 ft.

Therefore, we must specify 14-gage, $1\frac{1}{4}$ – inch tubing for this application.

شبکه لوله و انتخاب پمپ

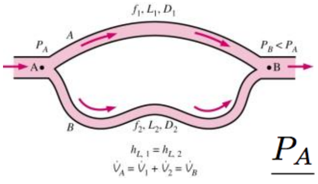
- دو نوع شبکه داریم:
  - لوله های سری
    - دبی جریان ثابت است.
    - افت کلی، برابر مجموع افت در لوله هاست.
  - لوله های موازی
    - دبی کلی، برابر مجموع دبی در اجزا است.
    - افت فشار در هر شاخه انشعاب یکسان است.





شبکه لوله و انتخاب پمپ

- برای لوله های موازی، حجم کنترل را بین دو نقطه A و B تعریف می کنیم:



$$V_A = V_B$$

$$\frac{P_A}{\rho g} + \alpha_1 \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \alpha_2 \frac{V_B^2}{2g} + z_B + h_L$$

$$h_L = \frac{\Delta P}{\rho g}$$

- چون ΔP در انشعابات یکسان است، افت هد در انشعابات برابر می باشد:

$$h_{L,1} = h_{L,2} \longrightarrow f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

شبکه لوله و انتخاب پمپ

- رابطه افت هد در انشعابات، منجر به روابط زیر می شود:

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2} \right)^{\frac{1}{2}} \qquad \frac{\dot{\mathcal{V}}_1}{\dot{\mathcal{V}}_2} = \frac{D_1^2}{D_2^2} \left(\frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2} \right)^{\frac{1}{2}}$$

- بر اساس معادلات فوق، رابطه بین سرعتها، غیر خطی است.
- مشابه این روابط در مدارهای الکتریکی هم وجود دارد:
  - دبی جریان (VA) : جریان در مدار (I)
  - گرادیان فشار (Δp): پتانسیل الکتریکی (V)
  - افت هد (h_L): مقاومت جریان (R) (هرچند که افت هد، کاملا غیرخطی است.)

شبکه لوله و انتخاب پمپ

- وقتی در یک شبکه لوله، پمپ یا توربین داریم؛ هد آنها نیز در معادله ظاهر می شود:

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L$$

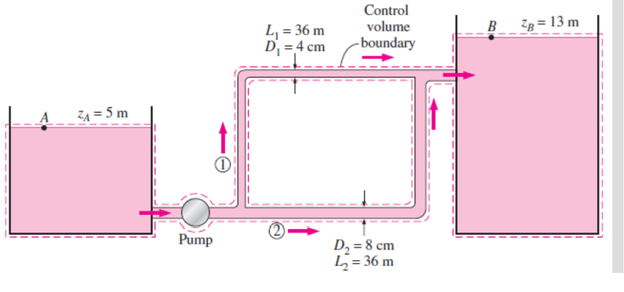
- هد موثر پمپ و توربین، تابع دبی جریان عبوری از آنها هستند و لذا اعداد ثابتی نیستند.

- نقطه کار یک سیستم جایی است که سیستم در توازن است؛ یعنی، هد پمپ با هد افت برابر است.

مثال ۲۱

EXAMPLE 8–7 Pumping Water through Two Parallel Pipes

Water at 20°C is to be pumped from a reservoir (z_A = 5 m) to another reservoir at a higher elevation (z_B = 13 m) through two 36-m-long pipes connected in parallel, as shown in Fig. 8–47. The pipes are made of commercial steel, and the diameters of the two pipes are 4 and 8 cm. Water is to be pumped by a 70 percent efficient motor–pump combination that draws 8 kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rate through each of the parallel pipes.



مثال ۲۱ (ادامه)

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998\ \text{kg/m}^3$ and $\mu = 1.002 \times 10^{-3}$ kg/m · s. The roughness of commercial steel pipe is $\varepsilon = 0.000045$ m (Table 8–2).

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Therefore, we would normally use a trial-and-error approach here. However, equation solvers such as EES are widely available, and thus, we simply set up the equations to be solved by an equation solver. The useful head supplied by the pump to the fluid is determined from

$$\dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump},u}}{\eta_{\text{pump-motor}}} \quad \rightarrow \quad 8000\ \text{W} = \frac{(998\ \text{kg/m}^3)\dot{V}(9.81\ \text{m/s}^2)h_{\text{pump},u}}{0.70} \qquad (1)$$

We choose points A and B at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_A = P_B = P_{\text{atm}}$) and that the fluid velocities at both points are nearly zero ($V_A = V_B = 0$) since the reservoirs are large, the energy equation for a control volume between these two points simplifies to

$$\cancel{\frac{P_A}{\rho g}} + \alpha_A \overbrace{\frac{V_A^2}{2g}}^{0} + z_A + h_{\text{pump},u} = \cancel{\frac{P_B}{\rho g}} + \alpha_B \overbrace{\frac{V_B^2}{2g}}^{0} + z_B + h_L$$

or

$$h_{\text{pump},u} = (z_B - z_A) + h_L$$

or

$$h_{\text{pump},u} = (13\ \text{m} - 5\ \text{m}) + h_L \qquad (2)$$

where

$$h_L = h_{L,1} = h_{L,2} \qquad (3)(4)$$

مثال ۲۱ (ادامه)

We designate the 4-cm-diameter pipe by 1 and the 8-cm-diameter pipe by 2. Equations for the average velocity, the Reynolds number, the friction factor, and the head loss in each pipe are

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad V_1 = \frac{\dot{V}_1}{\pi(0.04\ \text{m})^2/4} \qquad (5)$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad V_2 = \frac{\dot{V}_2}{\pi(0.08\ \text{m})^2/4} \qquad (6)$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(998\ \text{kg/m}^3)V_1(0.04\ \text{m})}{1.002 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} \qquad (7)$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(998\ \text{kg/m}^3)V_2(0.08\ \text{m})}{1.002 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} \qquad (8)$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.04} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad (9)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.08} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad (10)$$

$$h_{L,1} = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1 \frac{36\ \text{m}}{0.04\ \text{m}}\frac{V_1^2}{2(9.81\ \text{m/s}^2)} \qquad (11)$$

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2 \frac{36\ \text{m}}{0.08\ \text{m}}\frac{V_2^2}{2(9.81\ \text{m/s}^2)} \qquad (12)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \qquad (13)$$

This is a system of 13 equations in 13 unknowns, and their simultaneous solution by an equation solver gives

$$\dot{V} = \mathbf{0.0300\ m^3/s}, \quad \dot{V}_1 = \mathbf{0.00415\ m^3/s}, \quad \dot{V}_2 = \mathbf{0.0259\ m^3/s}$$

$$V_1 = 3.30\ \text{m/s}, \quad V_2 = 5.15\ \text{m/s}, \quad h_L = h_{L,1} = h_{L,2} = 11.1\ \text{m}, \quad h_{\text{pump}} = 19.1\ \text{m}$$

$$\text{Re}_1 = 131{,}600, \quad \text{Re}_2 = 410{,}000, \quad f_1 = 0.0221, \quad f_2 = 0.0182$$

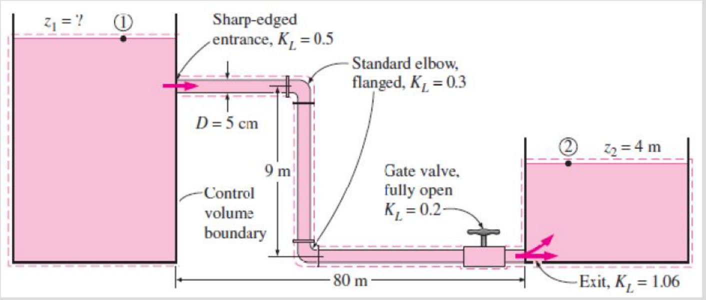
Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion The two parallel pipes have the same length and roughness, but the diameter of the first pipe is half the diameter of the second one. Yet only 14 percent of the water flows through the first pipe. This shows the strong dependence of the flow rate on diameter. Also, it can be shown that if the free surfaces of the two reservoirs were at the same elevation (and thus $z_A = z_B$), the flow rate would increase by 20 percent from 0.0300 to 0.0361 m³/s. Alternately, if the reservoirs were as given but the irreversible head losses were negligible, the flow rate would become 0.0715 m³/s (an increase of 138 percent).

مثال ٢٢

EXAMPLE 8–8 Gravity-Driven Water Flow in a Pipe

Water at 10°C flows from a large reservoir to a smaller one through a 5-cm-diameter cast iron piping system, as shown in Fig. 8–48. Determine the elevation z_1 for a flow rate of 6 L/s.



Properties The density and dynamic viscosity of water at 10°C are $\rho = 999.7\ \text{kg/m}^3$ and $\mu = 1.307 \times 10^{-3}$ kg/m · s. The roughness of cast iron pipe is $\varepsilon = 0.00026$ m (Table 8–2).

مثال ٢٢ (ادامه)

Analysis The piping system involves 89 m of piping, a sharp-edged entrance ($K_L = 0.5$), two standard flanged elbows ($K_L = 0.3$ each), a fully open gate valve ($K_L = 0.2$), and a submerged exit ($K_L = 1.06$). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are nearly zero ($V_1 \approx V_2 \approx 0$), the energy equation for a control volume between these two points simplifies to

$$\frac{\cancel{P_1}}{\cancel{\rho g}} + \alpha_1 \cancelto{0}{\frac{V_1^2}{2g}} + z_1 = \frac{\cancel{P_2}}{\cancel{\rho g}} + \alpha_2 \cancelto{0}{\frac{V_2^2}{2g}} + z_2 + h_L \quad \rightarrow \quad z_1 = z_2 + h_L$$

where

$$h_L = h_{L,\text{ total}} = h_{L,\text{ major}} + h_{L,\text{ minor}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006\ \text{m}^3/\text{s}}{\pi(0.05\ \text{m})^2/4} = 3.06\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(3.06\ \text{m/s})(0.05\ \text{m})}{1.307 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} = 117{,}000$$

The flow is turbulent since Re > 4000. Noting that $\varepsilon/D = 0.00026/0.05 = 0.0052$, the friction factor is determined from the Colebrook equation (or the Moody chart),

مثال۲۳ (ادامه)

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.0052}{3.7} + \frac{2.51}{117{,}000\sqrt{f}}\right)$$

It gives $f = 0.0315$. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}} + 2K_{L,\text{elbow}} + K_{L,\text{valve}} + K_{L,\text{exit}}$$
$$= 0.5 + 2 \times 0.3 + 0.2 + 1.06 = 2.36$$

Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} = \left(0.0315\,\frac{89\text{ m}}{0.05\text{ m}} + 2.36\right)\frac{(3.06\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 27.9\text{ m}$$

$$z_1 = z_2 + h_L = 4 + 27.9 = \mathbf{31.9\ m}$$

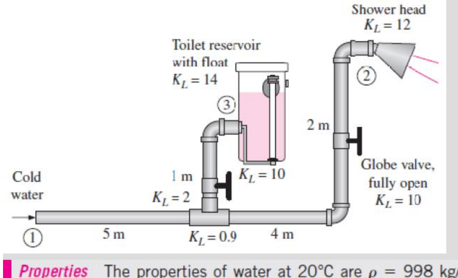
Therefore, the free surface of the first reservoir must be 31.9 m above the ground level to ensure water flow between the two reservoirs at the specified rate.

Discussion Note that $fL/D = 56.1$ in this case, which is about 24 times the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in about 4 percent error. It can be shown that at the same flow rate, the total head loss would be 35.9 m (instead of 27.9 m) if the valve were three-fourths closed, and it would drop to 24.8 m if the pipe between the two reservoirs were straight at the ground level (thus eliminating the elbows and the vertical section of the pipe). The head loss could be reduced further (from 24.8 to 24.6 m) by rounding the entrance. The head loss can be reduced significantly (from 27.9 to 16.0 m) by replacing the cast iron pipes by smooth pipes such as those made of plastic.

مثال ۲۴

EXAMPLE 8–9 **Effect of Flushing on Flow Rate from a Shower**

The bathroom plumbing of a building consists of 1.5-cm-diameter copper pipes with threaded connectors, as shown in Fig. 8–49. (*a*) If the gage pressure at the inlet of the system is 200 kPa during a shower and the toilet reservoir is full (no flow in that branch), determine the flow rate of water through the shower head. (*b*) Determine the effect of flushing of the toilet on the flow rate through the shower head. Take the loss coefficients of the shower head and the reservoir to be 12 and 14, respectively.



Properties The properties of water at 20°C are $\rho = 998$ kg/m^3, $\mu = 1.002 \times 10^{-3}$ kg/m · s, and $\nu = \mu/\rho = 1.004 \times 10^{-6}$ m^2/s. The roughness of copper pipes is $\varepsilon = 1.5 \times 10^{-6}$ m.

مثال ۲۴ (ادامه)

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and pressure drop. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

(*a*) The piping system of the shower alone involves 11 m of piping, a tee with line flow ($K_L = 0.9$), two standard elbows ($K_L = 0.9$ each), a fully open globe valve ($K_L = 10$), and a shower head ($K_L = 12$). Therefore, $\Sigma K_L = 0.9 + 2 \times 0.9 + 10 + 12 = 24.7$. Noting that the shower head is open to the atmosphere, and the velocity heads are negligible, the energy equation for a control volume between points 1 and 2 simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

$$\rightarrow \quad \frac{P_{1,\text{gage}}}{\rho g} = (z_2 - z_1) + h_L$$

Therefore, the head loss is

$$h_L = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 2 \text{ m} = 18.4 \text{ m}$$

Also,

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} \quad \rightarrow \quad 18.4 = \left(f\frac{11 \text{ m}}{0.015 \text{ m}} + 24.7\right)\frac{V^2}{2(9.81 \text{ m/s}^2)}$$

مثال ۲۴ (ادامه)

since the diameter of the piping system is constant. Equations for the average velocity in the pipe, the Reynolds number, and the friction factor are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi (0.015 \text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

This is a set of four equations with four unknowns, and solving them with an equation solver such as EES gives

$$\dot{V} = 0.00053 \text{ m}^3/\text{s}, \quad f = 0.0218, \quad V = 2.98 \text{ m/s}, \quad \text{and} \quad \text{Re} = 44{,}550$$

Therefore, the flow rate of water through the shower head is **0.53 L/s.**

(*b*) When the toilet is flushed, the float moves and opens the valve. The discharged water starts to refill the reservoir, resulting in parallel flow after the tee connection. The head loss and minor loss coefficients for the shower branch were determined in (*a*) to be $h_{L,2} = 18.4$ m and $\Sigma K_{L,2} = 24.7$, respectively. The corresponding quantities for the reservoir branch can be determined similarly to be

مثال ۲۴ (ادامه)

$$h_{L,3} = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 1 \text{ m} = 19.4 \text{ m}$$

$$\Sigma K_{L,3} = 2 + 10 + 0.9 + 14 = 26.9$$

The relevant equations in this case are

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$h_{L,2} = f_1 \frac{5 \text{ m}}{0.015 \text{ m}} \frac{V_1^2}{2(9.81 \text{ m/s}^2)} + \left(f_2 \frac{6 \text{ m}}{0.015 \text{ m}} + 24.7\right) \frac{V_2^2}{2(9.81 \text{ m/s}^2)} = 18.4$$

$$h_{L,3} = f_1 \frac{5 \text{ m}}{0.015 \text{ m}} \frac{V_1^2}{2(9.81 \text{ m/s}^2)} + \left(f_3 \frac{1 \text{ m}}{0.015 \text{ m}} + 26.9\right) \frac{V_3^2}{2(9.81 \text{ m/s}^2)} = 19.4$$

$$V_1 = \frac{\dot{V}_1}{\pi(0.015 \text{ m})^2/4}, \quad V_2 = \frac{\dot{V}_2}{\pi(0.015 \text{ m})^2/4}, \quad V_3 = \frac{\dot{V}_3}{\pi(0.015 \text{ m})^2/4}$$

$$\text{Re}_1 = \frac{V_1(0.015 \text{ m})}{1.004 \times 10^{-6} \text{m}^2/\text{s}}, \quad \text{Re}_2 = \frac{V_2(0.015 \text{ m})}{1.004 \times 10^{-6} \text{m}^2/\text{s}}, \quad \text{Re}_3 = \frac{V_3(0.015 \text{ m})}{1.004 \times 10^{-6} \text{m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}_1 \sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}_2 \sqrt{f_2}}\right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}_3 \sqrt{f_3}}\right)$$

مثال ۲۴ (ادامه)

Solving these 12 equations in 12 unknowns simultaneously using an equation solver, the flow rates are determined to be

$$\dot{V}_1 = 0.00090 \text{ m}^3/\text{s}, \quad \dot{V}_2 = 0.00042 \text{ m}^3/\text{s}, \quad \text{and} \quad \dot{V}_3 = 0.00048 \text{ m}^3/\text{s}$$

Therefore, the flushing of the toilet **reduces the flow rate of cold water through the shower by 21 percent** from 0.53 to 0.42 L/s, causing the shower water to suddenly get very hot (Fig. 8–50).

Discussion If the velocity heads were considered, the flow rate through the shower would be 0.43 instead of 0.42 L/s. Therefore, the assumption of negligible velocity heads is reasonable in this case. Note that a leak in a piping system would cause the same effect, and thus an unexplained drop in flow rate at an end point may signal a leak in the system.