جلسه ۹

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Titels

2.6.4.3 Analysis of non-degenerately doped semiconductors

- Non-degenerate semiconductors are defined as semiconductors for which the Fermi energy is at least 3kT away from either band edge.

- The reason we restrict ourselves to non-degenerate semiconductors is that this definition allows the Fermi function to be replaced with a simple exponential function, i.e. the Maxwell-Boltzmann distribution function. The carrier density integral can then be solved analytically yielding:

$$n_o \cong \int_{E_c}^{\infty} \frac{8\pi\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E-E_c}\, e^{\frac{E_F-E}{kT}} dE = N_c e^{\frac{E_F-E_c}{kT}} \tag{2.6.13}$$

with

$$N_c = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} \tag{2.6.14}$$

where N_c is the *effective density of states* in the *conduction band*. The Fermi energy, E_F, is obtained from:

$$E_F = E_c + kT \ln \frac{n_o}{N_c} \tag{2.6.15}$$

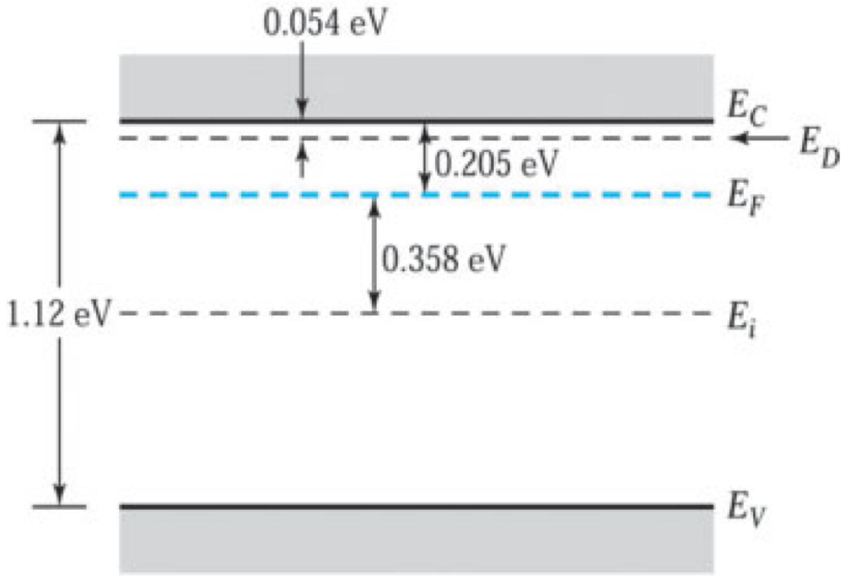

Fig. 23 Band diagram showing Fermi level E_F and intrinsic Fermi level E_i.

- The calculation of the electron density starts by assuming that the semiconductor is neutral, so that there is no net charge in the material.
- The charge density in a semiconductor depends on the free electron and hole density and on the ionized impurity densities.
- Ionized donors, which have given off an electron, are positively charged.
- Ionized acceptors, which have accepted an electron, are negatively charged.
- The total charge density is therefore given by:

$$\rho = q(p_o - n_o + N_d^+ - N_a^-) = 0 \tag{2.6.35}$$

- The hole concentration in thermal equilibrium can be written as a function of the electron density by using the mass action law(قانون اثر جرم) (2.6.23). This yields the following relation between the electron density and the ionized impurity densities:

$$n_o \cdot p_o = N_c N_v e^{(E_v - E_c)/kT} = n_i^2 \qquad (2.6.23)$$

$$n_o = \frac{n_i^2}{n_o} + N_d^+ - N_a^- \qquad (2.6.36)$$

- Solving this quadratic equation yields a solution for the electron density, namely:

$$n_o = \frac{N_d^+ - N_a^-}{2} + \sqrt{\left(\frac{N_d^+ - N_a^-}{2}\right)^2 + n_i^2} \qquad (2.6.37)$$

- The same derivation can be repeated for holes, yielding:

$$p_o = \frac{N_a^- - N_d^+}{2} + \sqrt{\left(\frac{N_a^- - N_d^+}{2}\right)^2 + n_i^2} \qquad (2.6.38)$$

- The above expressions provide the free carrier densities for compensated semiconductors assuming that all donors and acceptors are ionized.

- From the carrier densities, one then obtains the Fermi energies using equations (2.6.28) and (2.6.29) which are repeated below:

$$E_F = E_i + kT \ln \frac{n_o}{n_i} \qquad (2.6.28)$$

or

$$E_F = E_i - kT \ln \frac{p_o}{n_i} \qquad (2.6.29)$$

$p_o + N_d^+ = n_o + N_a^-$

Example 2.6a	A germanium wafer is doped with a shallow donor density of $3n_i/2$. Calculate the electron and hole density.

Example 2.6b	A silicon wafer is doped with a shallow acceptor doping of 10^{16} cm^{-3}. Calculate the electron and hole density.

2.6.4.4 General analysis

- A more general analysis takes also into account the fact that the ionization of the impurities is not 100%, but instead is given by the impurity distribution functions provided in section 2.5.3.

$$f_{donor}(E_d) = \frac{1}{1 + \frac{1}{2} e^{(E_d - E_F)/kT}} \quad (2.5.2)$$

$$f_{acceptor}(E_a) = \frac{1}{1 + 4e^{(E_a - E_F)/kT}} \tag{2.5.3}$$

- The analysis again assumes that there is no net charge in the semiconductor (charge neutrality). This also means that the total density of positively charged particles (holes and ionized donors) must equals the total density of negatively charged particles (electrons and ionized acceptors) yielding:

$$p_o + N_d^+ = n_o + N_a^- \tag{2.6.39}$$

2.6.5. Non-equilibrium carrier densities

- Up until now, we have only considered the thermal equilibrium carrier densities, n_o and p_o.

- However most devices of interest are not in thermal equilibrium. Keep in mind that a constant ambient(محیطی) temperature is not a sufficient condition for thermal equilibrium.

- In fact, applying a non-zero voltage to a device or illuminating it with light will cause a non-equilibrium condition, even if the temperature is constant.

- As a result the electron density can still be calculated using the Fermi-Dirac distribution function, but with a different value for the Fermi energy. The total carrier density for a non-degenerate semiconductor is then described by:

$$n = n_o + \delta n = n_i \exp\left(\frac{F_n - E_i}{kT}\right) \qquad (2.6.44)$$

- Where σn is the *excess electron density* and F_n is the *quasi-Fermi energy* for the electrons. Similarly, the hole density can be expressed as:

$$p = p_o + \delta p = n_i \exp\left(\frac{E_i - F_p}{kT}\right) \qquad (2.6.45)$$

Where *σp* is the *excess hole density* and F_p is the *quasi-Fermi energy* for the holes.

Example 2.7	A piece of germanium doped with 10^{16} cm^{-3} shallow donors is illuminated with light generating 10^{15} cm^{-3} excess electrons and holes. Calculate the quasi-Fermi energies relative to the intrinsic energy and compare it to the Fermi energy in the absence of illumination.

$$E_F = E_i + kT \ln \frac{n_o}{n_i} \qquad (2.6.28)$$

$$E_F = E_i - kT \ln \frac{p_o}{n_i} \qquad (2.6.29)$$

Solution	The carrier densities when illuminating the semiconductor are: $$n = n_o + \delta n = 10^{16} + 10^{15} = 1.1\times 10^{16}\ \text{cm}^{-3}$$ $$p = p_o + \delta p \cong 10^{15}\ \text{cm}^{-3}$$ and the quasi-Fermi energies are: $$F_n - E_i = kT\ln\frac{n}{n_i} = 0.0259\times\ln\frac{1.1\times 10^{16}}{2\times 10^{13}} = 163\,\text{meV}$$ $$F_p - E_i = -kT\ln\frac{p}{n_i} = 0.0259\times\ln\frac{1\times 10^{15}}{2\times 10^{13}} = -101\,\text{meV}$$ For comparison, the Fermi energy in the absence of light equals $$E_F - E_i = kT\ln\frac{n_o}{n_i} = 0.0259\times\ln\frac{10^{16}}{2\times 10^{13}} = 161\,\text{meV}$$ which is very close to the quasi-Fermi energy of the majority carriers.