OPTOELECTRONICS (I)

Chapter 4:
I. Planar Waveguides

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3-1 Waveguide Modes

$\theta_c = \sin^{-1}(n_2/n_1)$.

Because of diffraction, any light beam of **finite width** inside a waveguide that starts out at a particular angle θ_1 will spread out into other angles, and the angular distribution will change as the light propagates down the waveguide. What we would like to find is a **pattern of light distribution** that remains constant along the waveguide. **Such a pattern is referred to as a *mode.***

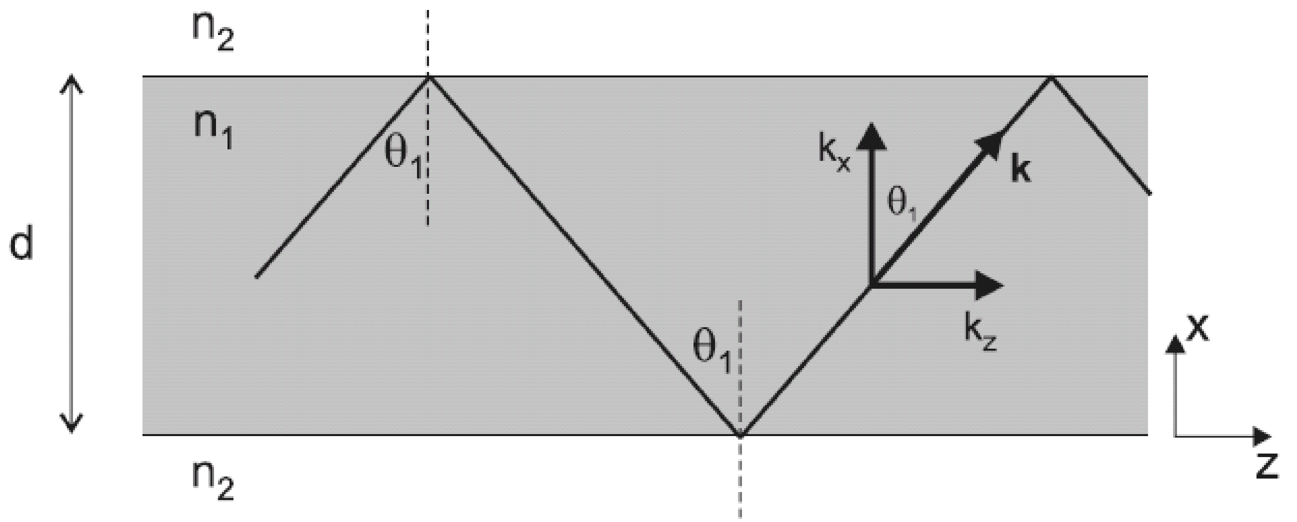


Figure 3-1 A **single ray** propagating down a planar waveguide. Superposition of two such rays with opposite k_x *constitutes a **waveguide mode**.*

3-1 Waveguide Modes

❖ The essential feature of a mode is that there is a pattern that is stable in time.

❖ To find a **mode** for the **waveguide**:

If we let the vectors $\mathbf{k_1}=k_x\hat{i} + k_z\hat{k}$ *and* $\mathbf{k_2}= - k_x\hat{i} + k_z\hat{k}$ *be* the propagation vectors for these two waves, the **total electric field** inside the waveguide can be written as:

$$E_{\text{mode}} = E_0 e^{i(\omega t - \mathbf{k}_1 \cdot \mathbf{r})} + E_0 e^{i(\omega t - \mathbf{k}_2 \cdot \mathbf{r})}$$

(1)

$$E_{\text{mode}} = E_0 e^{i(\omega t - k_x x - k_z z)} + E_0 e^{i(\omega t + k_x x - k_z z)}$$

(3)

$$\boxed{E_{\text{mode}} = E_0\, g(x, y) e^{i(\omega t - \beta z)}}$$

$$E_{\text{mode}} = E_0 e^{i(\omega t - k_z z)}[e^{ik_x x} + e^{-ik_x x}]$$

$$E_{\text{mode}} = 2E_0 \cos(k_x x) e^{i(\omega t - k_z z)} \qquad (2)$$

β: ***propagation constant*** *for the* ***waveguide mode***.

3-1 Waveguide Modes

- In order for the **waves** to **reinforce** each other after **many reflections**, the total round-trip phase change for propagation in the transverse (x) direction must be an integer multiple of 2π.

- For a waveguide of **thickness** d, *the total roundtrip* distance in the x *direction is* $2d$, *resulting in a* ***phase shift*** *of* $-k_x(2d)$.

- If the *phase shift* upon reflection is $\Phi(r)$, there is an *additional contribution of* $2\Phi(r)$ to the total round-trip phase shift. The condition for **self-reinforcing** fields then becomes

$$\boxed{-k_x(2d) + 2\phi_r = \pm m2\pi} \qquad (4)$$

$m = 0, 1, 2, 3 \ldots, p$ label the different modes allowed in the waveguide.

3-1 Waveguide Modes

$$-k_x(2d) + 2\phi_r = \pm m2\pi \qquad (4)$$

$$k_x = k_1 \cos\theta \quad \Rightarrow \quad k_x = \frac{2\pi}{\lambda}\cos\theta = \frac{2\pi n_1}{\lambda_0}\cos\theta \qquad (5)$$

From 4,5:

$$\boxed{\cos\theta_m = \frac{(\pm m\pi + \phi_r)\lambda_0}{2\pi n_1 d}} \qquad (6)$$

θ_m : *angle for **mode number** m.*

condition that **total internal r**eflection be satisfied, $\theta_m > \theta_c$, $\boxed{\sin\theta_c = n_2/n_1}$

maximum mode number, $\theta_p = \theta_c$

$$\cos\theta_p = \sqrt{1-\sin^2\theta_c} = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

3-1 Waveguide Modes

$$\cos\theta_m = \frac{(\pm m\pi + \phi_r)\lambda_0}{2\pi n_1 d}$$

$$\cos\theta_p = \sqrt{1 - \sin^2\theta_c} = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$\Rightarrow \quad \frac{2\pi d}{\lambda_0}\sqrt{n_1^2 - n_2^2} = p\pi + \phi_r \tag{7}$$

Maximum mode number p for a waveguide of given **thickness** and refractive index.

It is convenient to define:

normalized film thickness

$$V_p \equiv \frac{2\pi d}{\lambda_0}\sqrt{n_1^2 - n_2^2} \tag{8}$$

dimensionless parameter

$\Rightarrow$ **Maximum number** of modes in a **planar waveguide** $= p + 1$

where

$$p = \mathrm{int}\left(\frac{V_p}{\pi} - \frac{\phi_r}{\pi}\right) \tag{9}$$

3-1 Waveguide Modes

For the *pth* ***mode,*** *the* ***waveguide angle*** *is near the* ***critical angle****, which results in Φ(r) = 0*

$$\text{number of modes} = p + 1 = \text{int}\left(\frac{V_p}{\pi}\right) + 1 \quad (10)$$

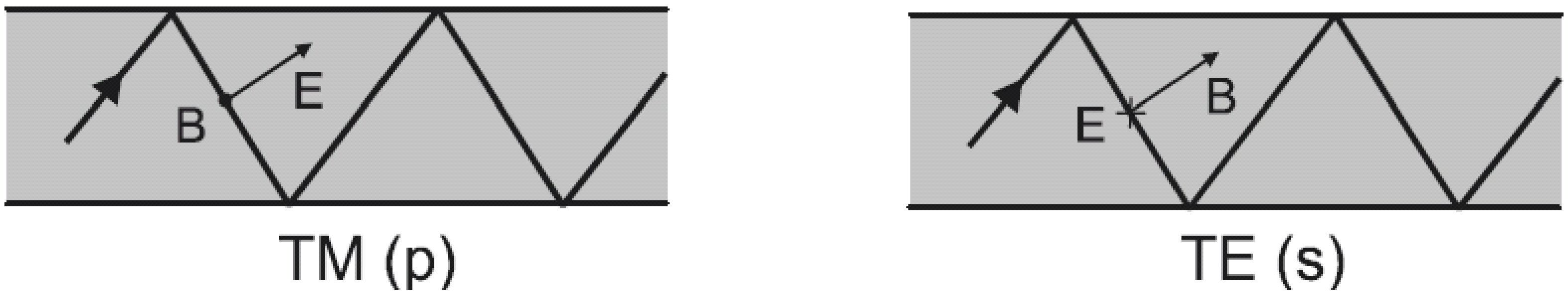
$$V_p \equiv \frac{2\pi d}{\lambda_0}\sqrt{n_1^2 - n_2^2}$$

For **each of these** modes, there are **two possible polarizations**

Total number of modes in the planar waveguide is = $2\,V_p/\pi$.

The **allowed values** of k_x *for the thick waveguide modes* can be approximated by:

$$k_x \simeq \frac{m\pi}{d} \quad (11)$$



EXAMPLE 3-1

❖Assuming light of **free-space** wavelength 1μm, determine the **number of modes**

(a) in a **microscope slide** of thickness 1 mm, immersed in **water**, and

(b) in a soap film in air of thickness 2 μm.

Take the **refractive index** of glass as 1.5 and that of water as 1.33.

Solution:

(a) The normalized film thickness is:

$$V_p = \frac{2\pi(1 \times 10^{-3})}{1 \times 10^{-6}} \sqrt{(1.5)^2 - (1.33)^2}$$

$$V_p = 4.7 \times 10^3$$

$$\boxed{V_p \equiv \frac{2\pi d}{\lambda_0} \sqrt{n_1^2 - n_2^2}}$$

$$\boxed{\text{number of modes} = p + 1 = \text{int}\left(\frac{V_p}{\pi}\right) + 1}$$

The **number** of waveguide modes is then $= 4700/\pi = 1500$, not including different polarizations. When including different polarizations, there are about 3000 modes.

EXAMPLE 3-1

- (b) in a soap film in air of thickness 2 μm.

Solution:

(b) The soap film is mostly water, with **refractive index** 1.33, so

$$\boxed{V_p \equiv \frac{2\pi d}{\lambda_0}\sqrt{n_1^2 - n_2^2}}$$

$$V_p = \frac{2\pi(2 \times 10^{-6})}{1 \times 10^{-6}}\sqrt{(1.33)^2 - 1^2}$$

$$V_p = 11$$

The number of **waveguide modes** is then int(11/π) + 1= int(3.5) + 1 = 4, not including **different polarizations**.

When **including different pol**arizations, there are 8 modes.

Effective Index

There is also the longitudinal wave vector component k_z, which is similarly related to by

k_x, k_z, θ

$$k_z = k_1 \sin\theta = \frac{2\pi n_1}{\lambda_0} \sin\theta \qquad (12) \qquad k_x^2 + k_z^2 = k_1^2$$

A **fourth parameter** that is often used to **specify** the **mode** is the ***effective index of refraction***, defined by

$$k_z \equiv \frac{2\pi n_{\mathrm{eff}}}{\lambda_0} \qquad (13) \qquad k = (2\pi n)/\lambda_0$$

The guided wave propagates at the **phase velocity** given by *$\omega t - k_z z = constant$, or*

$$\boxed{v_p = \frac{\omega}{k_z} = \frac{2\pi c}{\lambda_0 k_z} = \frac{c}{n_{\mathrm{eff}}}} \qquad (14)$$

The **propagation constant** k_z is commonly denoted as β in **optical fibers**, *and* referred to as the ***axial wave vector***.

Mode Velocities (1)

For a **planar waveguide**, the **effective** index of refraction can be written as

$$\boxed{n_{\text{eff}} = n_1 \sin\theta} \qquad (15)$$

phase velocity of the **waveguide mode:**

$$\boxed{v_p = \frac{\omega}{k_z} = \frac{2\pi c}{\lambda_0 k_z} = \frac{c}{n_{\text{eff}}}} \qquad (16) \qquad \Rightarrow \qquad v_p = \frac{c}{n_1 \sin\theta} = \frac{v_1}{\sin\theta}$$

phase velocity of a **plane wave** in medium 1: $v_1 = c/n_1$

Since $\sin\theta_c < \sin\theta < 1$ and $\sin\theta_c = n_2/n_1$

$$\Rightarrow \boxed{n_2 < n_{\text{eff}} < n_1} \quad (17) \qquad \text{and} \qquad \boxed{v_2 > v_p > v_1} \quad (18)$$

Mode Velocities (2)

➢ The resolution of this apparent paradox can be found in the distinction between phase and group **velocities**,

$$\boxed{v_2 > v_p > v_1}$$

$$\boxed{v_g = \frac{d\omega}{d\beta}} \quad (19)$$

$$\beta \equiv k_z.$$

$$\boxed{(n_1 k_0)^2 = k_x^2 + \beta^2} \quad (20)$$

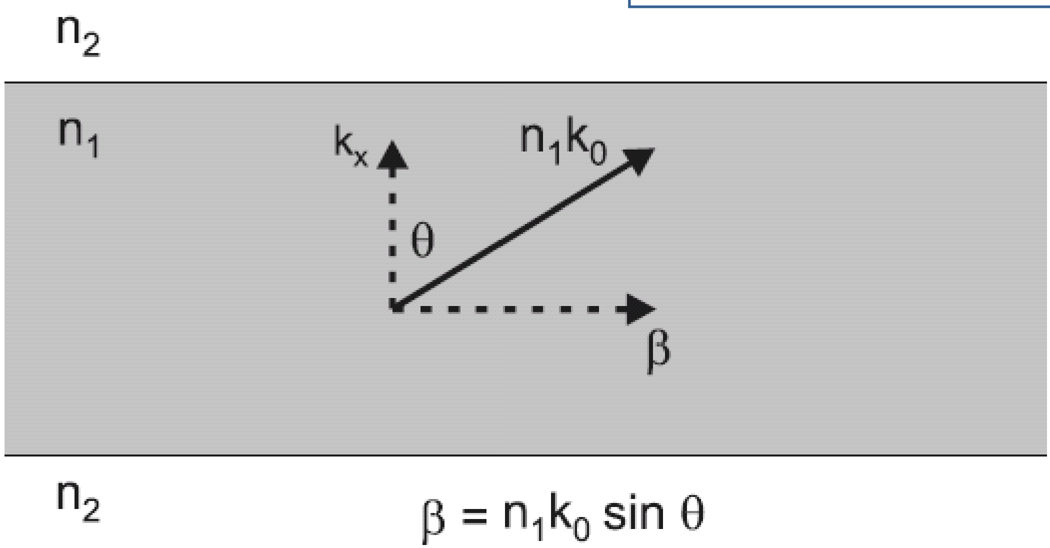


Figure 3-3 The **wave vector** for the ray of magnitude n_1k_0 can be broken down into its longitudinal and transverse components, β and k_x.

Mode Velocities (3)

$$\boxed{(n_1 k_0)^2 = k_x^2 + \beta^2}$$

$$\boxed{k_x \simeq \frac{m\pi}{d}}$$

$$k_1 = n_1 k_0 = n_1 2\pi/\lambda_0 \Rightarrow \left(\frac{n_1}{c}\right)^2 \omega^2 = \left(\frac{m\pi}{d}\right)^2 + \beta^2 \qquad (21)$$

$\boxed{\omega = c k_0}$ $\Rightarrow$ Taking the derivative with respect to β:

$$2\omega \frac{d\omega}{d\beta} = \left(\frac{c}{n_1}\right)^2 2\beta \qquad (22)$$

$$\boxed{v_p = \frac{\omega}{k_z} = \frac{2\pi c}{\lambda_0 k_z} = \frac{c}{n_{\mathrm{eff}}}}$$

and

$$\boxed{v_g = \frac{d\omega}{d\beta}} \Rightarrow \boxed{v_g v_p = v_1^2}$$

$$\boxed{v_p = \frac{c}{n_1 \sin\theta} = \frac{v_1}{\sin\theta}} \Rightarrow \boxed{v_g = v_1 \sin\theta} \qquad (23)$$

$$v_g < v_1$$

Mode Velocities (4)

❖ **What is the physical interpretation of the phase velocity?**

➢ Two **wave fronts** (1 and 2) propagating down the +z axis to form a waveguide mode.

➢ Plane wave 1 is propagating up and to the right, and plane wave 2 is propagating down and to the right.

➢ The intersection point can move down the waveguide faster than the **speed** of either wave individually. ($v_p > v_1$).

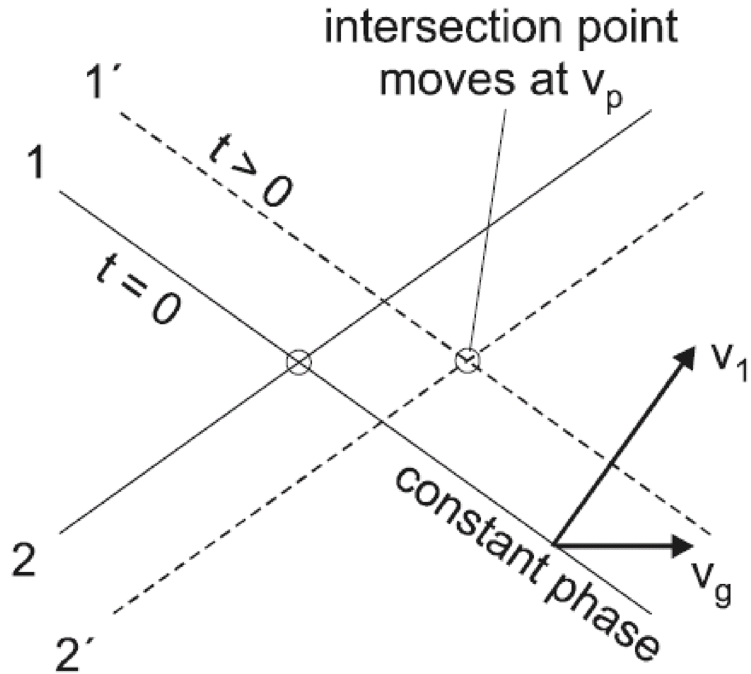


Figure 3-4 The motion of **wave fronts** for two component rays with opposite k_x *illustrates the difference* between group and phase **velocity**.

3-2. MODE CHART (1)

❖ A waveguide mode can be specified by any one of the parameters n_{eff}, θ, k_x, or $k_z = \beta$.

❖ *For* a **waveguide** of given thickness *d, an approximate expression for the allowed mode* ***angles*** θ_m *can be obtained* :

$$\cos\theta_m = \frac{(\pm m\pi + \phi_r)\lambda_0}{2\pi n_1 d} \quad (6)$$

$$m\pi \gg \phi_r$$

$$\cos\theta_m \simeq \frac{m\lambda_0}{2\, n_1 d} \quad (24)$$

$$k_x = \frac{2\pi}{\lambda}\cos\theta = \frac{2\pi n_1}{\lambda_0}\cos\theta \quad (5)$$

$$k_z = k_1 \sin\theta = \frac{2\pi n_1}{\lambda_0}\sin\theta \quad (12)$$

$$n_{\text{eff}} = n_1 \sin\theta \quad (15)$$

3-2. MODE CHART (1)

$$\boxed{\cos\theta_m = \frac{(\pm m\pi + \phi_r)\lambda_0}{2\pi n_1 d}} \quad (6)$$

Solving Eq. (3-6) for *Φ(r) and substituting into Eq.* (2-23),

for TE polarization:

$$\tan\frac{\phi_r}{2} = \frac{\sqrt{\sin^2\theta_1 - (n_2/n_1)^2}}{\cos\theta_1} \quad (2\text{-}23)$$

for TM polarization:

$$\tan\frac{\phi_r}{2} = \left(\frac{n_1}{n_2}\right)^2 \frac{\sqrt{\sin^2\theta_1 - (n_2/n_1)^2}}{\cos\theta_1} \quad (2\text{-}24)$$

for TE polarization:

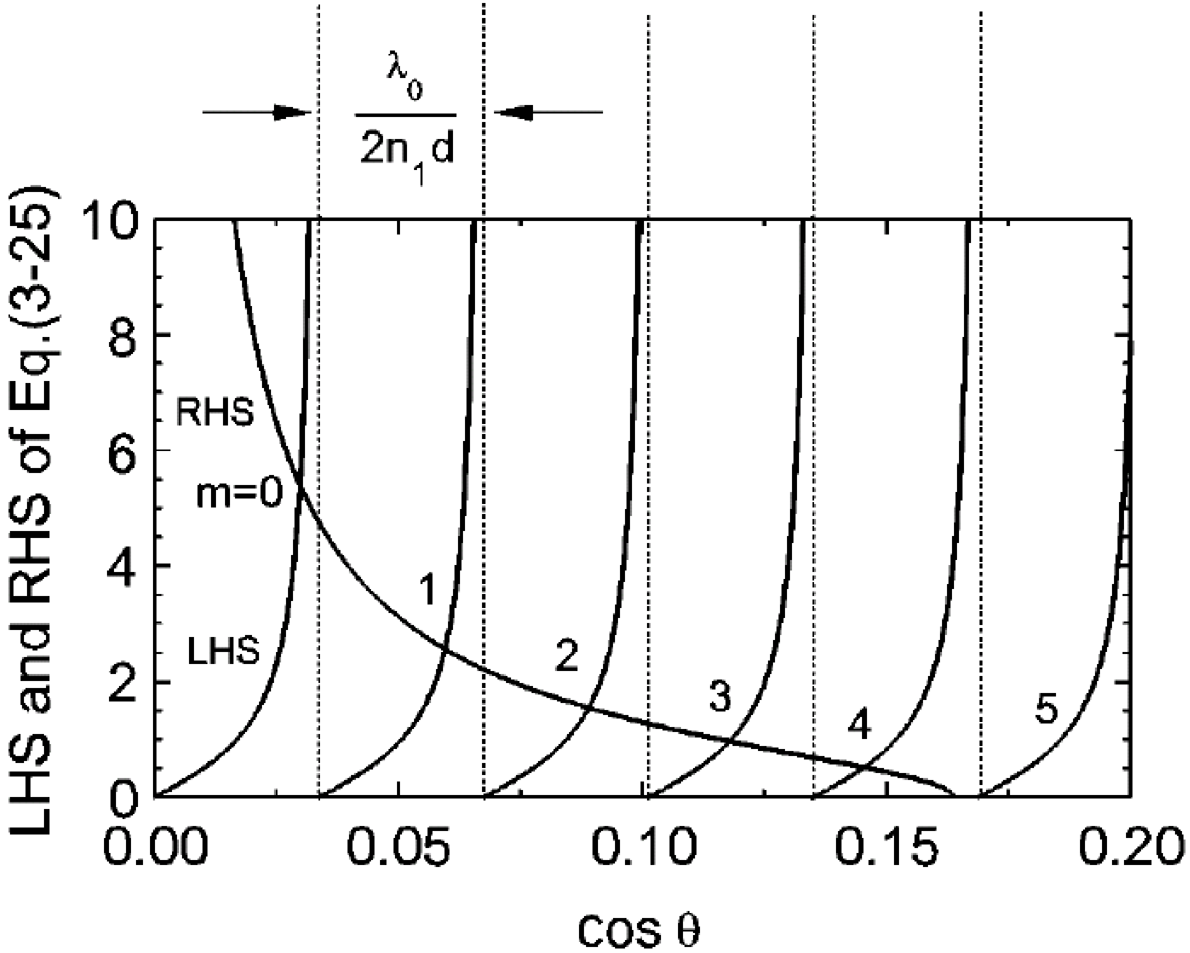
$$\boxed{\tan\left[\frac{\pi n_1 d\cos\theta}{\lambda_0} - m\frac{\pi}{2}\right] = \frac{\sqrt{1-\left(\frac{n_2}{n_1}\right)^2 - \cos^2\theta}}{\cos\theta}} \quad (25)$$

3-2. MODE CHART (2)

Figure 3-5 Graphical solution of Eq. (3-25) for modes in a **planar waveguide**, with $n_1 = 1.48$, $n_2 = 1.46$, and $d/\lambda_0 = 10$.

These parameters lead to five allowed modes, which correspond to the five line crossings.

(LHS): Left-Hand Side and (RHS): Right-Hand Side



3-2. MODE CHART (3)

- As the waveguide thickness d *is decreased, the modes become more widely separated* and fewer in number, until at some point there is only one allowed mode.

- The **condition** for such a *single-mode waveguide is:*

$$\frac{\lambda_0}{2n_1 d} > \cos\theta_c$$

$$\boxed{\sin\theta_c = \frac{n_2}{n_1}} \Rightarrow \boxed{d < \frac{\lambda_0}{2\sqrt{n_1^2 - n_2^2}}} \qquad (26)$$

- The **condition** for a single-mode waveguide can also be written in terms of the V_p *parameter* :

$$\boxed{V_p \equiv \frac{2\pi d}{\lambda_0}\sqrt{n_1^2 - n_2^2}} \Rightarrow \boxed{V_p < \pi} \qquad (27)$$

3-2. MODE CHART (4)

- If $\cos\theta$ is taken as **given**, then the **equation** can be solved **analytically** for the waveguide **thickness** d. After some manipulation, we find:

thickness for **mode number** m :

$$d_m = d_0 + \frac{m\lambda_0}{2n_1 \cos\theta} \tag{28}$$

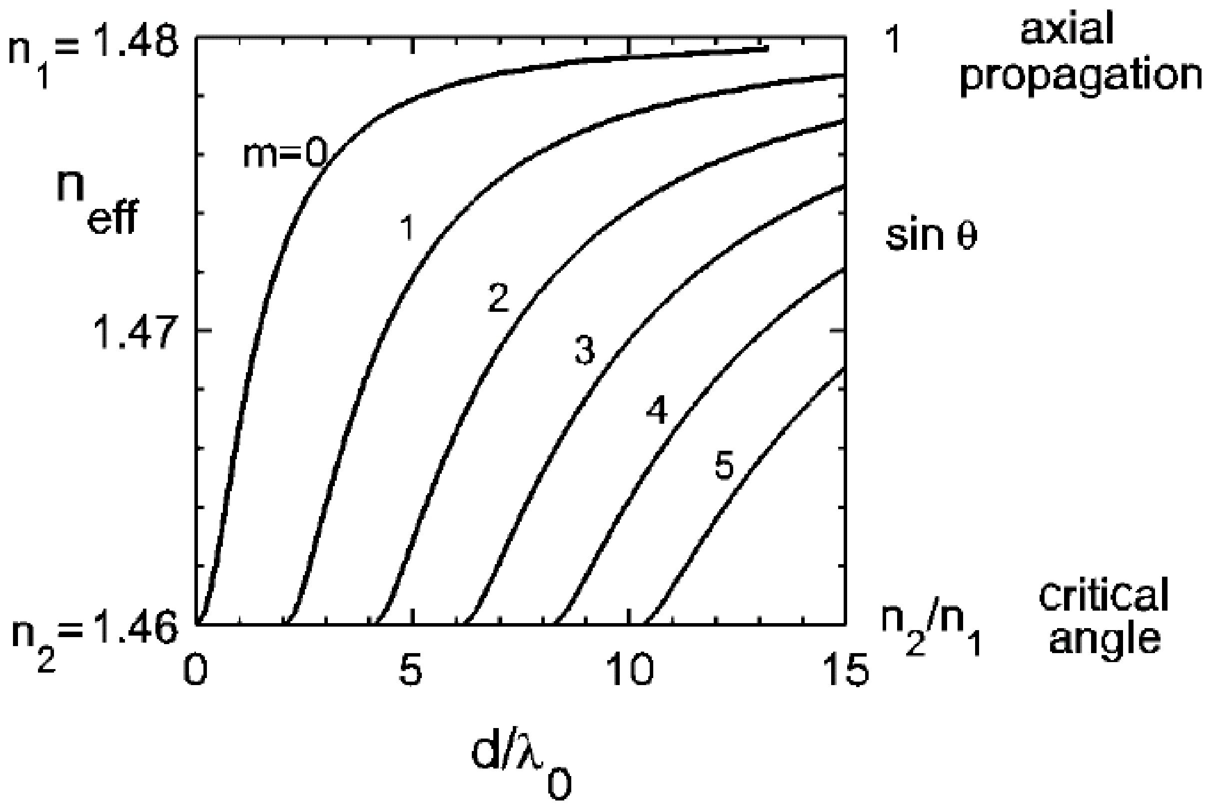
$$d_0 = \frac{\lambda_0}{\pi n_1 \cos\theta} \cos^{-1}\left(\frac{n_1 \cos\theta}{\sqrt{n_1^2 - n_2^2}}\right) \tag{29}$$

$$\boxed{n_{\text{eff}} = n_1 \sin\theta} \quad \Rightarrow \quad n_1 \cos\theta = \sqrt{n_1^2 - n_{\text{eff}}^2} \tag{30}$$

3-2. MODE CHART (5)

Figure 3-6

Mode chart calculated from Eqs. 3-28 and 3-29, using *n_1 = 1.48 and n_2 = 1.46 as in Fig.* 3-5. Values of *n_{eff} for the various modes are obtained by drawing vertical lines and looking for curve* crossings.



Field Distribution in a Mode

❖ Inside the waveguide (in the higher index n_1), *the field is oscillatory both in x and z, with the form*

$$E(x, z, t) = E_{\max} \cos(k_x x) \cos(\omega t - \beta z) \quad (31)$$

$$\beta \equiv k_z$$

$$v_p = \omega/\beta$$

➢ With mode number *m*, *there are m nodal* lines, resulting in *m + 1 lines of maximum intensity*.

➢ *The lowest-order mode, with m = 0*, has just a single intensity maximum and no nodal lines.

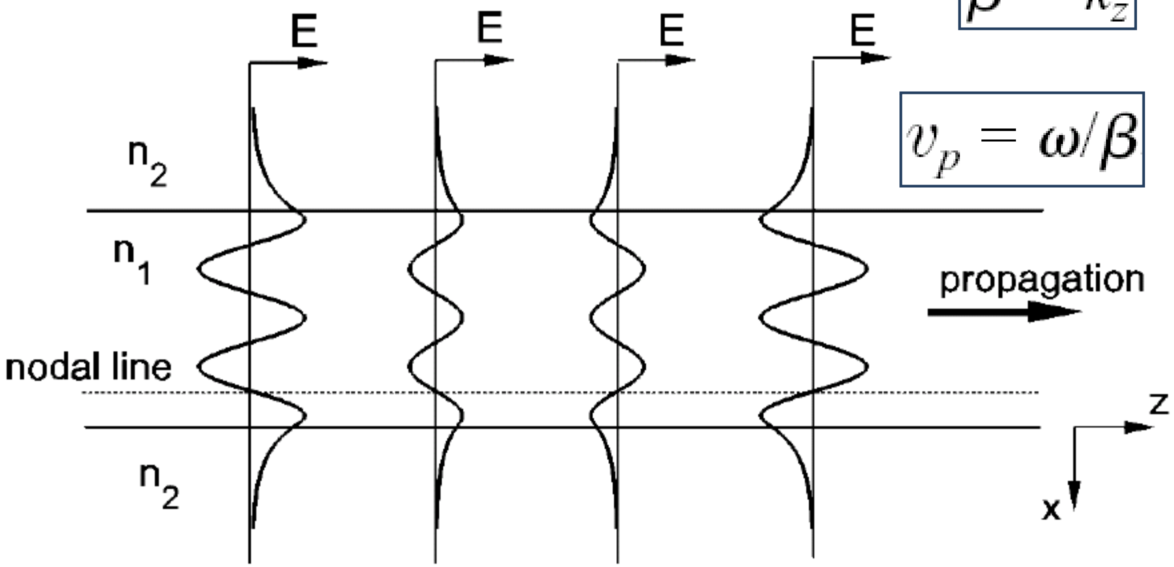


Figure 3-7 Transverse **spatial distribution** of the *E field at one instant in time for four positions* along the *z axis of a planar waveguide. Parameters are those of Fig. 3-5, with mode number m = 4.*

3-3. DISPERSION (1)

$$\boxed{v_g = v_1 \sin\theta}$$

The time it takes for a pulse of light to propagate a distance L *down the waveguide will then vary with* θ *as:*

$$\boxed{t = \frac{L}{v_g} = \frac{Ln_1}{c\sin\theta}} \quad (32)$$

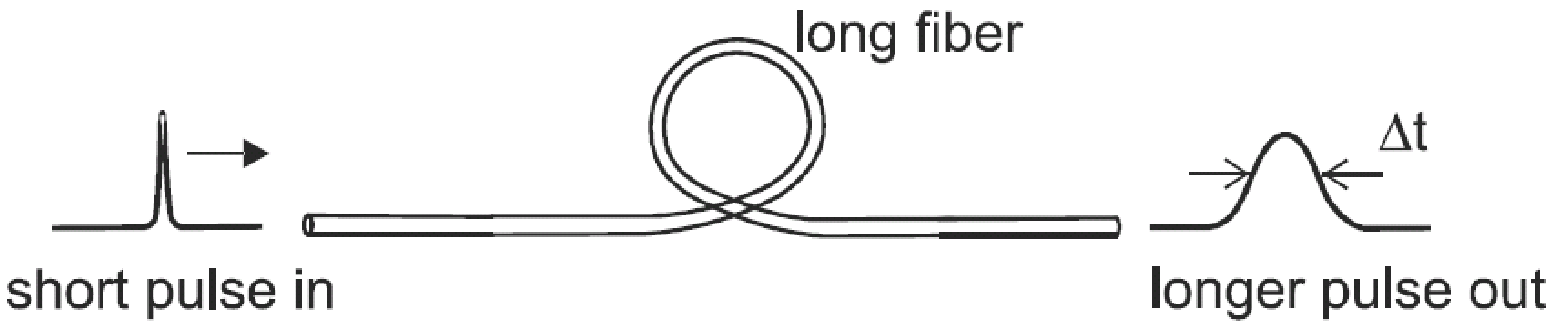
$$\Delta t = \frac{Ln_1}{c}\left(\frac{1}{\sin\theta_{\min}} - \frac{1}{\sin\theta_{\max}}\right) \quad (33)$$

$$\boxed{\theta_c < \theta < 90^\circ}$$

$$\boxed{\theta_c = n_2/n_1}$$

$$\frac{n_2}{n_1} < \sin\theta < 1 \quad (34)$$

$$\Delta t = \frac{Ln_1}{cn_2}(n_1 - n_2) \simeq \frac{L}{c}(n_1 - n_2)$$



3-3. DISPERSION (2)

$$\Delta t = \frac{Ln_1}{cn_2}(n_1 - n_2) \simeq \frac{L}{c}(n_1 - n_2)$$

where we have assumed $n_1 \approx n_2$, generally a good assumption for optical fibers. This can also be written as

intermodal dispersion

$$\Delta t \simeq \frac{Ln}{c}\Delta \qquad (35)$$

fractional index difference

$$\Delta \equiv \frac{n_1 - n_2}{n_1} \qquad (36)$$

- Note that the dispersion **does not** depend on the waveguide thickness.
- Since Δt is **proportional** to L, it is customary to specify the degree of dispersion as $\Delta t/L$, in units of ns/km.

EXAMPLE 3-2

- Determine the intermodal dispersion of an optical fiber with a core index of 1.5 and a fractional index difference of 0.01.

Solution: with $L = 1$ km, we have

$$\Delta t \simeq \frac{Ln}{c}\,\Delta$$

$$\Delta t = \frac{(10^3\ \text{m})(1.5)}{3 \times 10^8\ \text{m/s}}(10^{-2}) = 50\ \text{ns}$$

The intermodal dispersion is thus approximately 50 ns/km.

3-3. DISPERSION (3)

The criterion for distinguishable pulses is then

$$T > \frac{T}{2} + \Delta t$$

$$\boxed{\mathrm{BR} \equiv \frac{1}{T} < \frac{1}{2\Delta t}}$$

where BR is the number of pulses per second or the *bit rate.*

The effect of dispersion can be characterized by the *length × bit rate product* ($L \times BR$).

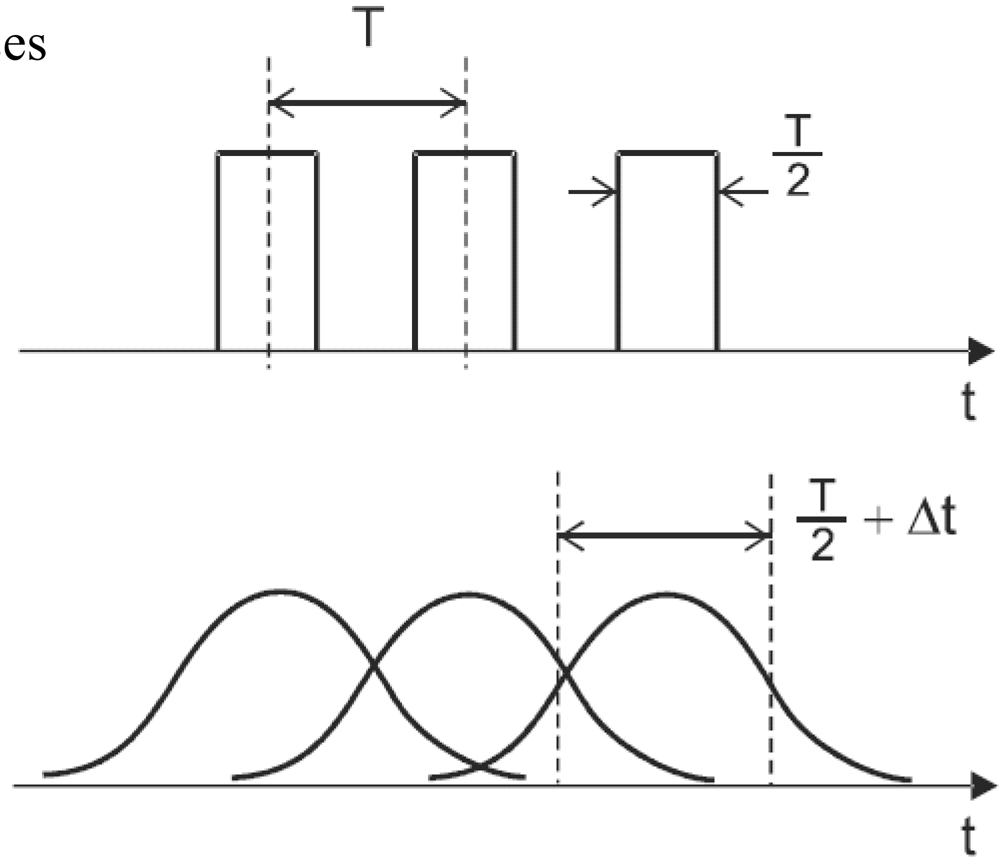


Figure 3-9 Pulses are broadened by Δt after propagating a distance L, which **limits** the rate at which data can be transmitted down a long waveguide.

Problems (chap.3): 1, 3, 4, 8, 9, 10, 12