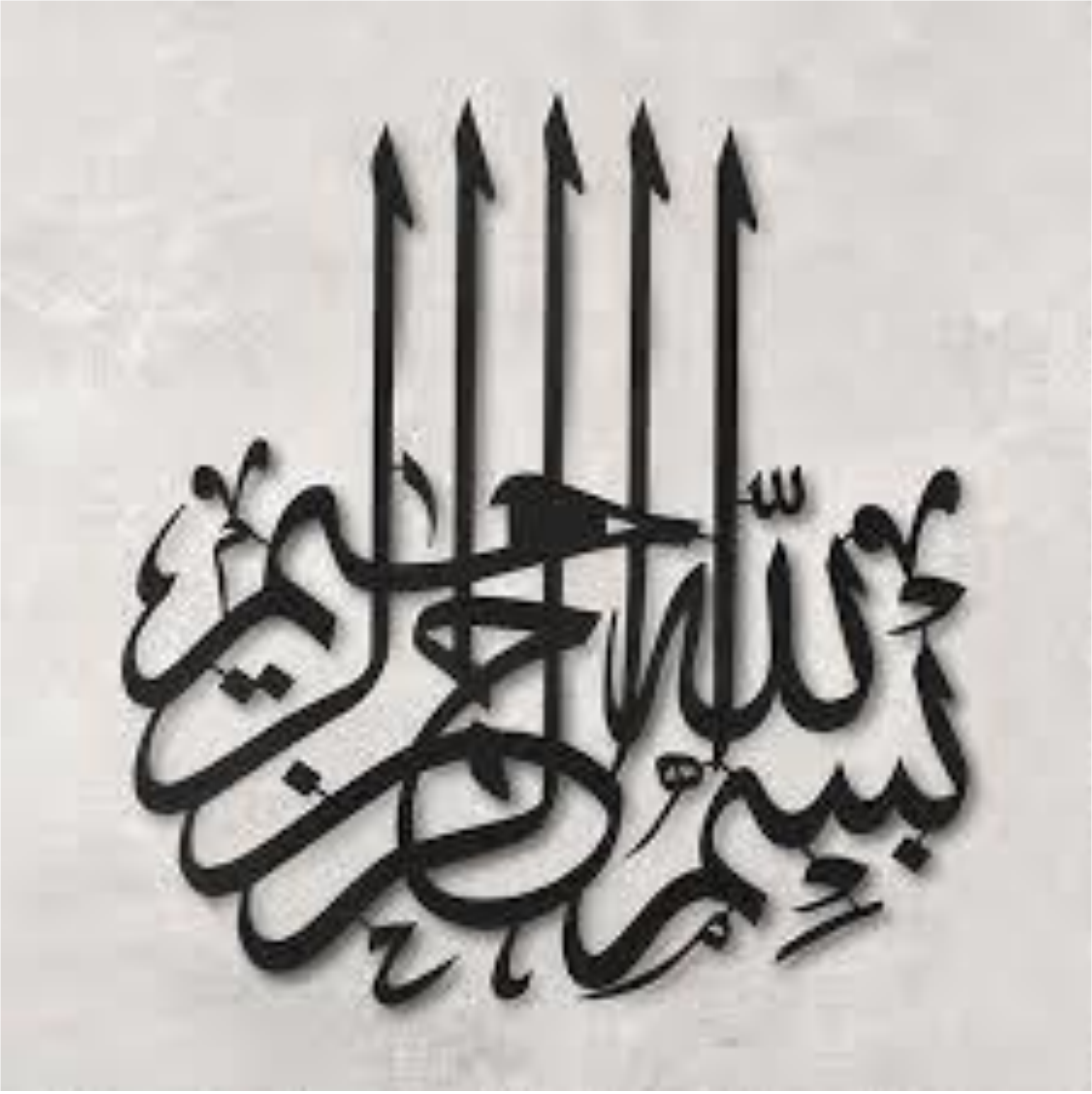
بسم الله الرحمن الرحيم

CHAPTER

3

p–n Junction

3.2 DEPLETION REGION

- To solve Poisson's equation (Eq. 13) we must know the impurity distribution.

- In this section we consider two important cases—the abrupt junction(پیوند تیز) and the linearly graded junction(پیوند شیب دار خطی).

- For either deep diffusions or high-energy ion implantations, the impurity profiles may be approximated by linearly graded junctions: that is, the impurity distribution varies linearly across the junction.

- We consider the depletion regions of both types of junction.

(**linearly graded junction:** A pn junction in which the impurity concentration does not change abruptly from donors to acceptors, but varies smoothly across the junction, and is a linear function of position.)

3.2.1 Abrupt Junction

- The space charge distribution of an abrupt junction is shown in Fig. 6*a*.

- In the depletion region, free carriers are totally depleted so that Poisson's equation (Eq. 13) simplifies to

$$\frac{d^2\psi}{dx^2} = \frac{q}{\varepsilon_s}(N_A - N_D) \qquad (13)$$

$$\frac{d^2\psi}{dx^2} = +\frac{qN_A}{\varepsilon_s} \quad \text{for} \quad -x_p \leq x < 0, \qquad (14a)$$

$$\frac{d^2\psi}{dx^2} = -\frac{qN_D}{\varepsilon_s} \quad \text{for} \quad 0 < x \leq x_n. \qquad (14b)$$

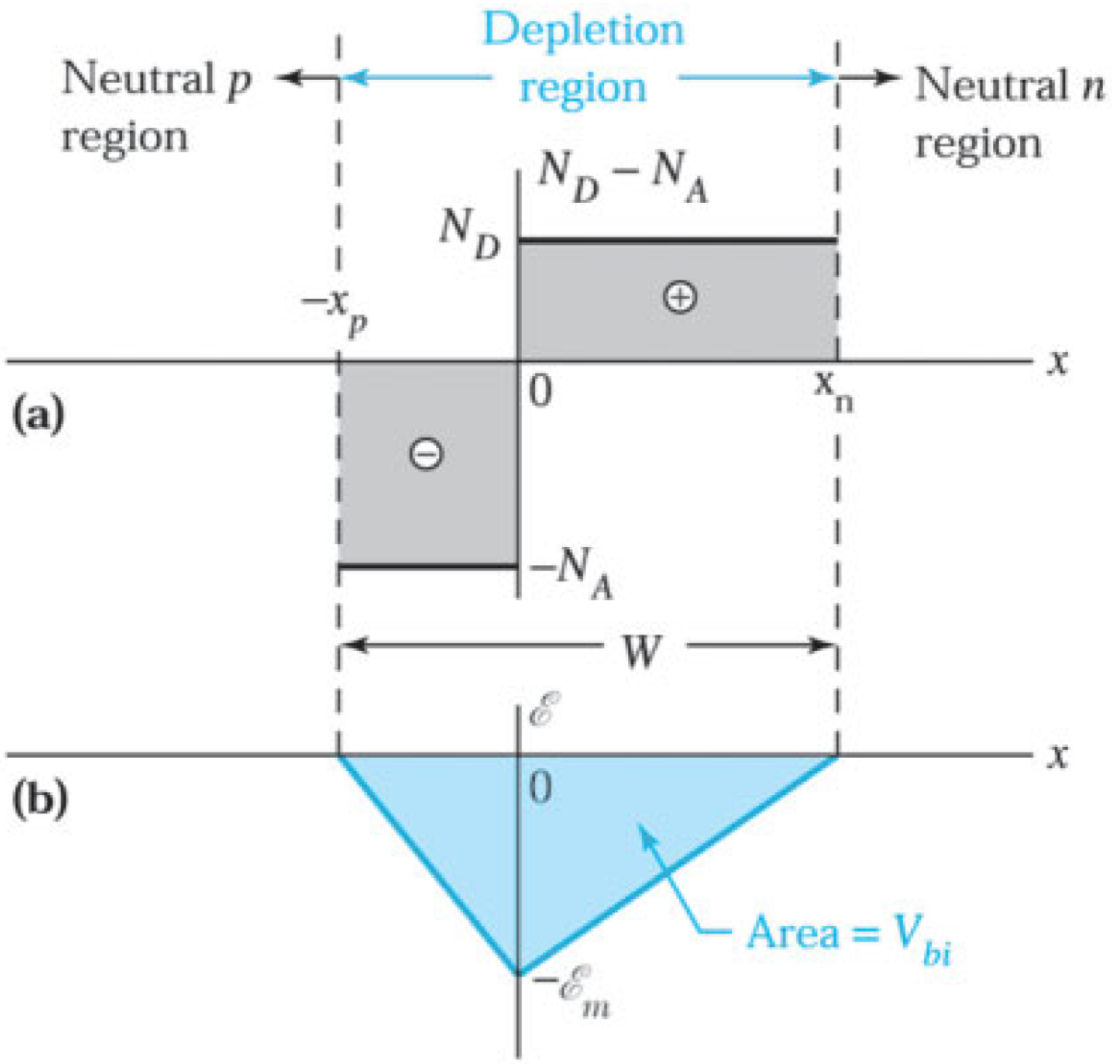

Fig. 6 (*a*) Space charge distribution in the depletion region at thermal equilibrium. (*b*) Electric-field distribution. The shaded area corresponds to the built-in potential.

➢ The overall space charge neutrality of the semiconductor requires that the total negative space charge per unit area in the p-side must precisely equal the total positive space charge per unit area in the n-side:

$$N_A x_p = N_D x_n . \tag{15}$$

➢ The total depletion layer width W is given by

$$W = x_p + x_n . \tag{16}$$

➢ The electric field shown in Fig. 6b is obtained by integrating Eqs. 14a and 14b, which gives

$$\mathcal{E} = -\frac{d\psi}{dx} \Rightarrow \frac{d\mathcal{E}}{dx} = -\frac{d^2\psi}{dx^2}$$

$$\left.\frac{d^2\psi}{dx^2} = \frac{qN_A}{\varepsilon_s} \quad \text{for} \quad -x_p \le x < 0 \quad (14a)\right\} \frac{d\mathcal{E}}{dx} = -\frac{qN_A}{\varepsilon_s} \rightarrow \int_0^{\mathcal{E}(x)} d\mathcal{E} = -\frac{qN_A}{\varepsilon_s}\int_{-x_p}^{x} dx$$

$$\Rightarrow \mathcal{E}\Big|_0^{\mathcal{E}(x)} = -\frac{qN_A}{\varepsilon_s}\, x\Big|_{-x_p}^{x}$$

$$\Rightarrow \mathcal{E}(x) = -\frac{qN_A}{\varepsilon_s}(x + x_p) \quad \text{for } -x_p \le x < 0 \quad (1)$$

$$\frac{d^2\psi}{dx^2} = -\frac{qN_D}{\varepsilon_s} \quad \text{for} \quad 0 < x \le x_n \quad (14b)$$

$$\frac{d\mathcal{E}}{dx} = -\frac{d^2\psi}{dx^2} \Rightarrow \frac{d\mathcal{E}}{dx} = \frac{qN_D}{\varepsilon_s} \rightarrow \int_{\mathcal{E}(x)}^{0} d\mathcal{E} = \frac{qN_D}{\varepsilon_s}\int_x^{x_n} dx$$

$$\Rightarrow 0 - \mathcal{E}(x) = \frac{qN_D}{\varepsilon_s}(x_n - x) \rightarrow \mathcal{E}(x) = \frac{qN_D}{\varepsilon_s}(x - x_n) \quad (2)$$

$$(1): \Rightarrow \mathcal{E}(x=0) = \mathcal{E}_m = -\frac{qN_A}{\varepsilon_s}x_p \qquad \cancel{\mathcal{E}_m = \frac{qN_A}{\varepsilon_s}}$$

$$(2): \Rightarrow \mathcal{E}(x=0) = \mathcal{E}_m = -\frac{qN_D}{\varepsilon_s}x_n \qquad \rightarrow |\mathcal{E}_m| = \frac{qN_A}{\varepsilon_s}x_p = \frac{qN_D}{\varepsilon_s}x_n \quad (18)$$

$\mathcal{E}_m$ بیشینه میدان است که در $x=0$ قرار دارد.

با توجه به رابطه ۲ و ۱ و نتایج اخیر:

$$-x_p \le x < 0 \Rightarrow \mathcal{E}(x) = -\frac{qN_A}{\varepsilon_s}x - \frac{qN_A}{\varepsilon_s}x_p = -\mathcal{E}_m - \frac{qN_A}{\varepsilon_s}x \quad (17a)$$

$$0 < x \le x_n \Rightarrow \mathcal{E}(x) = \frac{qN_D}{\varepsilon_s}x - \frac{qN_D}{\varepsilon_s}x_n = -\mathcal{E}_m + \frac{qN_D}{\varepsilon_s}x \quad (17b)$$

محاسبه پتانسیل داخلی (یا پتانسیل درون ناحیه تخلیه)

$$V_{bi} = -\int_{-x_p}^{x_n}\mathcal{E}(x)\,dx = \underbrace{-\int_{-x_p}^{0}\mathcal{E}(x)\,dx\Big|_{p\ \text{side}}}_{A} - \underbrace{\int_{0}^{x_n}\mathcal{E}(x)\,dx\Big|_{n\ \text{side}}}_{B}$$

$$A = -\int_{-x_p}^{0}\left(-\frac{qN_A}{\varepsilon_s}x - \frac{qN_A}{\varepsilon_s}x_p\right)dx = -\int_{-x_p}^{0}\left(-\mathcal{E}_m - \frac{qN_A}{\varepsilon_s}x\right)dx = +\mathcal{E}_m x\Big|_{-x_p}^{0} + \frac{qN_A}{\varepsilon_s}\frac{x^2}{2}\Big|_{-x_p}^{0}$$

$$= +\mathcal{E}_m(0 + x_p) - \frac{qN_A}{\varepsilon_s}\frac{x_p^2}{2} = +\left(+\frac{qN_A}{\varepsilon_s}x_p\right)x_p - \frac{qN_A}{\varepsilon_s}\frac{x_p^2}{2} = \frac{qN_A}{2\varepsilon_s}x_p^2$$

همچنین با محاسبات مشابه:

$$B = \frac{qN_D x_n^2}{2\varepsilon_s}$$

$$V_{bi} = A+B = \frac{q N_A}{۲\varepsilon_s} x_p \cdot x_p + \frac{q N_D}{۲\varepsilon_s} x_n \cdot x_n = \frac{۱}{۲}|\underline{\varepsilon}| \cdot x_p + |\underline{\varepsilon}| \cdot x_n \times \frac{۱}{۲}$$

$$= \underline{\frac{۱}{۲}|\underline{\varepsilon}|(x_p + x_n)} = |\underline{\varepsilon}| W \times \frac{۱}{۲}$$ (رابطه ۱۹)

برای محاسبه رابطه ۲۰

$$V_{bi} = \frac{۱}{۲}\varepsilon_m W \longrightarrow W = \frac{۲V_{bi}}{|\varepsilon_m|} \quad (*)$$

(19)

از رابطه ۱۹:

باید ε_m را از روابط قبلی محاسبه کنیم.

رابطه (۱۵):

$$N_A x_p = N_D x_n$$

$$\longrightarrow x_p = \frac{N_D}{N_A} x_n$$

رابطه (۱۶):

$$W = x_p + x_n$$

از ترکیب دو رابطه فوق:

$$W = \frac{N_D}{N_A} x_n + x_n = \left(\frac{N_D}{N_A} + ۱\right) x_n = \frac{N_A + N_D}{N_A} x_n \longrightarrow x_n = \frac{N_A}{N_A + N_D} W$$

از طرفی طبق رابطه (۱۸)

$$|\varepsilon_m| = \frac{q N_D x_n}{\varepsilon_s} = \frac{q N_D}{\varepsilon_s}\left(\frac{N_A}{N_A + N_D}\right) W$$

با جایگذاری نتیجه فوق در $(*)$:

$$W = \frac{۲V_{bi}}{\frac{q N_D}{\varepsilon_s}\frac{N_A}{(N_A + N_D)} W} \longrightarrow W^۲ = \frac{۲V_{bi} \times \varepsilon_s (N_A + N_D)}{q N_A N_D}$$

$$\longrightarrow W = \sqrt{\frac{۲\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right) V_{bi}}$$

(20)

- When the impurity concentration on one side of an abrupt junction is much higher than that on the other side, the junction is called a one-sided(يكطرفه) abrupt junction (Fig. 7a).
- Figure 7b shows the space charge distribution of a one-sided abrupt p^+–n junction, where $N_A >> N_D$.
- In this case, the depletion layer width of the p-side is much smaller than that of the n-side (i.e., $x_p << x_n$), and the expression for W can be simplified to

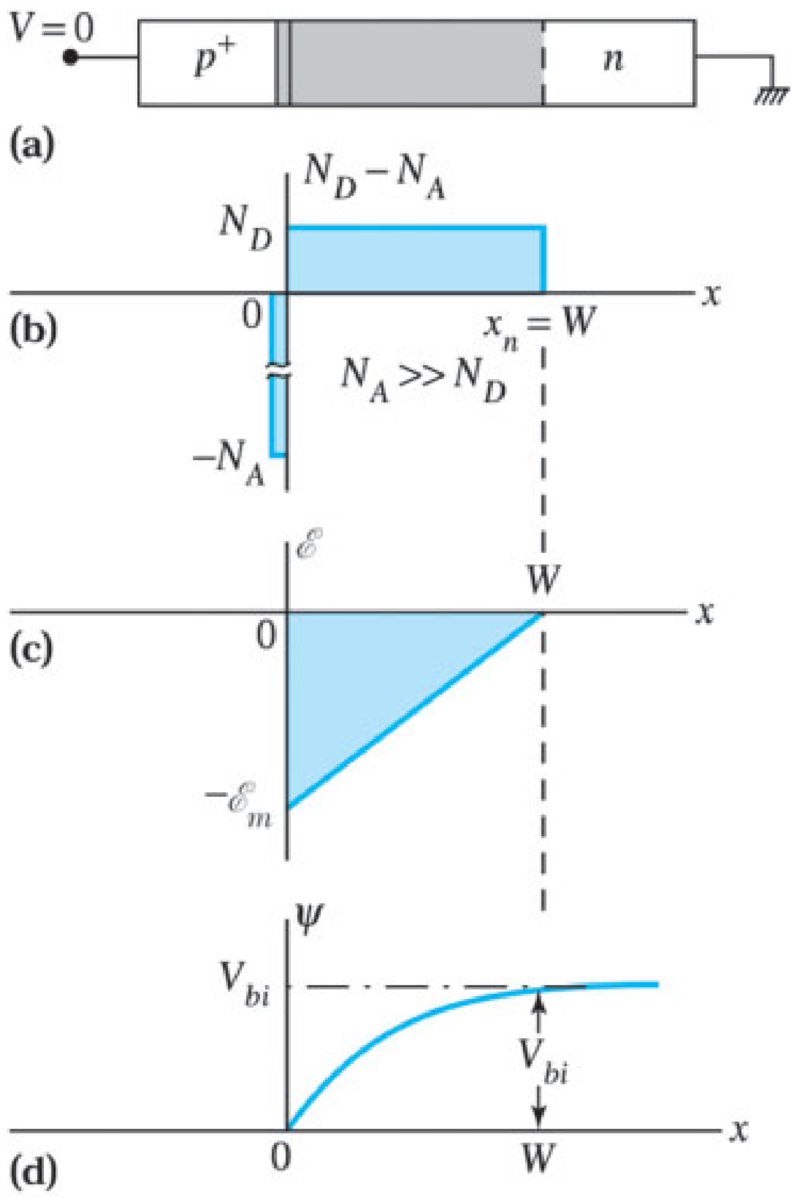

Fig. 7 (*a*) One-sided abrupt junction (with $N_A >> N_D$) in thermal equilibrium. (*b*) Space charge distribution. (*c*) Electric-field distribution. (*d*) Potential distribution with distance, where V_{bi} is the built-in potential.

$$\boxed{W \cong x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}}.} \tag{21}$$

- The expression for the electric-field distribution is the same as Eq. 17b:

$$\mathscr{E}(x) = -\mathscr{E}_m + \frac{qN_B x}{\varepsilon_s}, \tag{22}$$

where N_B is the lightly doped bulk concentration (i.e., N_D for a p^+–n junction).

- The field decreases to zero at x = W. Therefore,

$$\boxed{\mathscr{E}_m = \frac{qN_B W}{\varepsilon_s}} \tag{23}$$

and

$$\mathscr{E}(x) = \frac{qN_B}{\varepsilon_s}(-W + x) = -\mathscr{E}_m\left(1 - \frac{x}{W}\right), \tag{24}$$

as shown in Fig. 7c.

- Integrating Poisson's equation once more gives the potential distribution

$$\psi(x) = -\int_0^x \mathscr{E}dx = \mathscr{E}_m\left(x - \frac{x^2}{2W}\right) \tag{25}$$

- With zero potential in the neutral p-region as a reference, or ψ(0) = 0, and employing Eq. 19, we have

$$\boxed{\psi(x) = \frac{V_{bi}x}{W}\left(2 - \frac{x}{W}\right).} \qquad (26)$$

- The potential distribution is shown in Fig. 7d.

▶ EXAMPLE 2

For a silicon one-sided abrupt junction with $N_A = 10^{19}$ cm^{-3} and $N_D = 10^{16}$ cm^{-3}, calculate the depletion layer width and the maximum field at zero bias ($T = 300$ K).

SOLUTION: From Eqs. 12, 21, and 23, we obtain

$$\boxed{V_{bi} = \psi_n - \psi_p = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right).} \tag{12}$$

$$\boxed{W \cong x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}}.} \tag{21}$$

$$\boxed{\mathscr{E}_m = \frac{qN_B W}{\varepsilon_s}} \tag{23}$$