New models for estimating compressive strength of concrete confined with FRP sheets in circular sections

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Abstract
Models for determining the compressive strength of concrete columns confined by FRP have been presented in previous studies. In this study, a large set of experimental data regarding circular columns confined with different types of FRP has been collected. In order to increase the accuracy in the existing models, three modified models for predicting the compressive strength of circular columns confined with FRP has been proposed by using the collected data. The FRP strain efficiency factor in the proposed models is considered as: (i) a function of the strain ratio, (ii) a function of the confinement stiffness ratio, and (iii) a function of the combination of these ratios. Studying the analytical results using the proposed factors revealed that models wherein the FRP strain efficiency factor is a function of the strain ratio or the combination of the confinement stiffness ratio and strain ratio give results closer to the experimental results.

Keywords
Concrete, confinement, fiber reinforced polymers, compressive strength, circular section

Introduction
Confinement is an effective method for increasing the compressive strength of concrete columns. Concrete and steel jackets, still in common use today, were widely used in the past to reinforce concrete columns. Although these methods increase the structure’s load bearing capacity, concrete jackets increase column section dimensions significantly and steel jackets perform weakly against adverse environmental conditions. Hence, older methods are to be replaced by innovative retrofitting systems which are economical and easy to implement.1

The idea of using FRP to reinforce the existing RC columns against seismic loads was first proposed in the mid-80s.2 The 1990 (California) and 1995 (Kobe, Japan) earthquakes were important and effective factors for an extensive investigation regarding the application of FRP for the retrofitting of concrete and masonry structures in seismic zones.3

The first experimental study on concrete specimens confined with three types of FRP under axial compressive loads has been carried out by Nanni and Bradford.4 They showed that the two mechanical properties of concrete, i.e. compressive strength and ductility, would increase by confining the specimens using FRP sheets.

Experimental studies that have examined the behavior of FRP-confined columns are many and models that have used these experimental results to estimate the concrete compressive strength are numerous. Fardis and Khalili5 were the first to propose one of such models. Mander et al.6 presented a model for estimating the compressive strength and axial strain of FRP-confined concrete. The result of their research was latter used by ACI 440.2R-02 Code.7 To estimate the compressive strength of FRP-confined columns, many models have been proposed wherein the actual hoop rupture strain of the FRP wrap is not considered and their developments are based on the ultimate FRP strain reported by the manufacturer or on the Coupon...
Lam and Teng have considered a constant FRP strain efficiency factor (defined as the ratio of actual hoop rupture strain to ultimate tensile strain) for each FRP type. Sadeghian and Fam have considered the maximum confinement stress in the “confinement stiffness ratio” as well as in “strain ratio” factors to take into account the actual rupture strain and have proposed some models to estimate the compressive strength of concrete cylinders confined with FRP by analyzing 518 specimens. Moodi et al. proposed a model for estimating compressive strength of rectangular and square columns confined by FRP sheet. In their study, the effective strain coefficient of FRP was considered as function of shape section.

In this study, a large set of experimental data has been collected for circular columns confined with different types of FRP. Since modeling with larger statistical populations will lead to more reliable results, the statistical population used in this study is larger than those used in previous studies. FRP strain efficiency factor was considered as a factor of “strain ratio”, “effective stiffness ratio”, or their combination and three models were presented to estimate the compressive strength of circular, FRP-confined, concrete columns. Analyses of the results of 732 specimens show that the proposed models estimate the strength values more accurately.

Confinement mechanism

When concrete undergoes axial compression, it dilates laterally (volumetric expansion or dilation). This dilation is controlled by FRP jackets tensioned in hoop directions. Figure 1 shows the confinement effects in the FRP-confined concrete. Neglecting tangential stresses in the longitudinal direction of the column specimen and considering the equilibrium of the stresses applied on the FRP, the actual maximum confining pressure \( f_{t,a} \) can be found as follows

\[
f_{t,a} = \frac{2tf_{frp}}{D} = \frac{2E_{frp}e_{h,rup}t_j}{D} \quad (1)
\]

where \( D \) is the specimen diameter, \( E_{frp} \) is the elastic modulus of FRP material, \( t_j \) is the thickness of FRP wrap, and \( e_{h,rup} \) is the actual FRP rupture strain in hoop directions defined as follows

\[
e_{h,rup} = k_e e_{frp} \quad (2)
\]

where \( e_{frp} \) is the ultimate tensile strain of FRP materials and \( k_e \) is the strain efficiency factor. In most models presented for estimating the compressive strength of FRP-confined columns, actual FRP rupture strain has not been considered meaning that \( k_e = 1 \) in these models, but in some past studies, \( k_e \) has been considered. Lam and Tang have considered a constant value for the strain factor for each types of FRP; for AFRP, CFRP, GFRP, and HM-CFRP, this factor is 0.851, 0.586, 0.624, and 0.788, respectively. Lim and Ozbakkaloglu have presented a relationship for this factor in the form of the product of the following three factors:

1. Strain localization factor that considers the effect of non-uniform strain distribution in FRP.
2. Local (in-situ) factor that considers the effect of the difference between maximum strain measured on the column and that found from Coupon (flat tension) test.
3. FRP-to-fiber strain ratio.

They proposed equation (3) for strain efficiency factor as follows

\[
k_e = 0.9 - 2.3f'_c \times 10^{-3} - 0.75E_{frp} \times 10^{-9} \quad (3)
\]

where \( f'_c \) is the strength of unconfined concrete.

Sadeghian and Fam proposed equation (4) as follows for FRP confinement stress considering the actual FRP rupture strain

\[
f_{t,a} = \rho_c \rho_k f'_c \quad (4)
\]

where \( \rho_c \) and \( \rho_k \) are the strain and effective stiffness ratio factors, respectively, defined as follows

\[
\rho_k = 2E_{frp}f_j \left( \frac{E_{frp}}{E_{co}} \right) D \quad (5)
\]

\[
\rho_c = \frac{e_{h,rup}}{e_{co}} \quad (6)
\]
where \( e_{co} \) is the strain related to \( f_c \) in the unconfined concrete.

The FRP strain efficiency factor in this study has been considered as: (1) ratio of the ultimate FRP tensile strain to the strain related to \( f_c \) and (2) ratio of the effective stiffness which is defined as the ratio of FRP elasticity modulus to the compressive strength of the unconfined concrete. More information will be provided in Section of compressive strength of confined concrete.

### Experimental database

Tests performed on the FRP-confined concrete are numerous. In this study, a statistical population of 662 FRP-confined circular concrete specimens, extracted from various studies, has been used for modeling. The statistical population used in this study is more complete than those used in earlier researches and details of the specimens are given in Table 1. They have diameters of 47–406 mm (average 155 mm) and unconfined compressive strengths of 2.6–55.2 MPa (average 35 MPa). FRP types used in these database include CFRP, AFRP, GFRP, and HM-CFRP with moduli of elasticity ranging from 4.9 to 640 GPa (average 170 GPa) and ultimate tensile strength of 75–4810 MPa (average 2712 MPa). All FRP jackets used in these data are single direction (hoop direction). These experimental data will be used for formulation purposes.

### Genetic algorithm

Optimization of structures has always been a noticeable developing area of research in the field of engineering optimization and has made highly progress in the last decade.96,97

The natural selection and evaluation process was first observed and documented by Charles Darwin. The fittest's survival philosophy makes it easier to reach the globally optimal solution. The methodology is implemented numerically and developed for optimization problems, where the mathematical use of GAs simulates natural evaluation and adaptation to environmental variation. The process is initiated by randomly or heuristically selecting a number of candidate design variables to create an initial population, which is then encouraged to evolve over generations to produce new designs that are better or fitter. It is necessary to device a genetic coding system for representation of design variable, which can be considered to be a direct analogy of DNA structure of chromosomes.

The design variables are coded by a bit string. With this binary representation, the design variables can be
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coded only as integers. So it is usually necessary to introduce a linear scaling conversion system to appropriately obtain the required range of parameter values. For example, a design having two continuous variables $b_1$ and $b_2$ each is coded as a fixed length 10 bit string like $b_1=1001000011$, $b_2=1101010110$. Values of these parameters are selected at random. A 10 bit string is employed with regard to the precision with which the variables should be represented. Total design is represented by a 20 bit string created simply by concatenating $b_1$ with $b_2$.

By doing so, it is possible to create a founding population with members each represented by a 20 bit string. The next stage of the procedure, following the specification of the initial design population, is that of reproduction, which incorporates the concept of natural selection. The fitness of different members of the population must be evaluated before mating to produce the next generation. The fitness $F$ is computed from the objective function of the chosen problem accordingly.

The selection of mating pairs for reproduction is a crucial step in GA. There are two methods of mating pool selection: fitness proportional or roulette wheel (RW) and tournament selection (TS). The latter method is proved to provide good selective pressure by holding a tournament competition among $N=2$ individuals. The best individual (winner) from this tournament is one with highest fitness and the winner is then inserted into the mating pool. The tournament competition continues until the mating pool is filled to generate new offspring. The tournament winners’ mating pool has a single average fitness.

In biological reproduction, the child’s chromosomal pattern is derived from the two parents’ chromosomal strings and thus the child inherits both characteristics. In GA, it is the crossover process that ensures the transfer of design information from generation to generation, essentially by a simple swapping of one (single-point) or two sections (two-point) of bit string representation of two parent designs to obtain two offspring design solutions.

The positioning and extent of the crossover time is randomly selected and may differ in each generation for each mating couple. Following the crossover, the natural evolution of the mutation concept is introduced into GA by occasionally switching the bit value at a randomly selected location of the generated strings. This action is important as it protects against premature design convergence to an optimal solution. The procedure is repeated until according to the objective function the new generation ceases to improve. When this happens, the youngest generation’s most fitting individual is an optimal design solution. Figure 2 shows sketch of the GA used in this work.

In this study, GA is used for optimizing target function defined in the following sections. For all cases, initial population and iteration are considered 10,000 and 100, respectively.

**Compressive strength of confined concrete**

Compressive strength of FRP-confined concrete can be estimated by various models some of which are provided in Table 2. As shown, effects of actual FRP rupture...
strain has not been considered in models proposed by Wu and Wei,\textsuperscript{100} Pham and Hadi,\textsuperscript{101} Fahmy and Wu,\textsuperscript{102} Youssef et al.,\textsuperscript{94} and Kumutha et al.\textsuperscript{103}

In this research, the equation of the compressive strength of the FRP-confined concrete with circular section has been considered as follows

\[
f'_{cc} = f'_{cc} + z f_{l,a}
\]

(7)

wherein the actual confinement stress caused by the FRP wrap \(f_{l,a}\) has been considered based on equation (1). Considering the strength of the unconfined concrete, the value of \(z\) is found as follows

\[
z = \begin{cases} 
  x_1 & f'_{cc} \leq 35 \text{ MPa} \\
  x_2 & f'_{cc} > 35 \text{ MPa}
\end{cases}
\]

(8)

Based on the average unconfined compressive strength of the specimens in Table 1, 35 MPa has been selected. In this study, the strain efficiency factor \(k_e\) has been considered as two separate functions (or their combination) which create three separate models for estimating the compressive strength of the FRP-confined concrete. These two functions are the strain ratio \(\rho_s\) and effective stiffness ratio \(\rho_k\) defined as follows

\[
\rho_k = \frac{E_{frp}}{f'_c}
\]

(9)

\[
\rho_s = \frac{\varepsilon_{frp}}{\varepsilon_{cc}}
\]

(10)

Next, models presented for estimating the compressive strength of the FRP-confined concrete are discussed wherein the FRP strain efficiency factor is considered in three different forms.

**Model I: FRP strain efficiency factor as a function of effective stiffness**

In this case, the FRP strain efficiency factor is considered as a function of the stiffness ratio

\[
k_e = f(\rho_s) = \beta_1 + \gamma_1 \rho_s^{1/3}
\]

(11)
Values of $z_1$ and $z_2$ in equation (8) and $\beta_1$, $\gamma_1$, and $\lambda_1$ in equation (11) have been so calculated, using the optimization GA, that the difference between the compression strength estimated by the analytical model (equation (7)) and that obtained in the test will be the lowest. Accordingly, $z_1$, $z_2$, $\beta_1$, $\gamma_1$, and $\lambda_1$ can be so calculated, using the GA, that the value of $Z$, calculated by equation (12), reaches its minimum. Therefore, in the optimization algorithm, equation (12) was considered as the optimal function as follows:

$$Z = 1 - R^2 + e$$

$$e_{tot} = \frac{\sum (f'_{ec})_{exp} - (f'_{ec})_{pre}}{\sum (f'_{ec})_{exp}}$$

In equation (12), $R^2$ is the correlation coefficient and $e_{tot}$ is the total error. In equation (13), $exp$ and $pre$ suffixes represent the experimental results and those estimated through the model in equation (7), respectively.

Considering a range 0 to 6 for $z_1$ and $z_2$ and $-3$ to 3 for $\beta_1$, $\gamma_1$, and $\lambda_1$, values of $z_1$, $z_2$, $\beta_1$, $\gamma_1$, and $\lambda_1$ were estimated to be $5.2812$, $4.4537$, $0.4748$, $-1.9181$, and $-0.8035$, respectively, through optimizing Z (equation (12)) for the test results presented in Table 1. Based on the above procedure, a value of $Z = 0.5694$ was calculated for the optimal function. According to the obtained results, an increase in the effective stiffness ratio increases the strain efficiency factor, meaning that the latter is directly related to the former. Considering the strain efficiency factor as a function of the effective stiffness ratio (equation (11)), the percentage of increase in the confined compressive strength caused by FRP for a concrete specimen with an unconfined compressive strength less than 35 MPa will be about 19% higher than that the same specimen with an unconfined compressive strength greater than 35 MPa.

Model II: FRP strain efficiency factor as a function of strain ratio

As in previous case, the strain efficiency factor is considered as a function of strain ratio

$$k_c = f(\rho_k) = \beta_2 + \gamma_2\rho_k^2$$

In this study, the strain related to $f'_{ec}$ in an unconfined concrete is calculated as follows

$$\varepsilon_{ço} = \frac{f'_{ec}}{E_c}$$

Finally, the FRP strain efficiency factor is considered as a combinatory function of the strain ratio and effective stiffness ratio as follows:

$$k_c = f(\rho_k) = \beta_3 + \gamma_3\rho_k^3 + \delta_3\rho_k^3 + \theta_3\rho_k^3$$

For this model, values of $z_1$ and $z_2$ in equation (8) and $\beta_3$, $\gamma_3$, $\lambda_3$, $\delta_3$, $\phi_3$, $\theta_3$, $\eta_3$, and $\xi_3$ are calculated using the optimization GA as mentioned in previous sections. Values of $z_1$, $z_2$, $\beta_3$, $\gamma_3$, $\lambda_3$, $\delta_3$, $\phi_3$, $\theta_3$, $\eta_3$, and $\xi_3$ were estimated to be $3.499$, $3.0481$, $0.2071$, $1.4729$, $-0.0305$, $1.0841$, $0.0045$, $20$, and $-7.1945$, respectively, through optimizing test specimens in Table 1 with an initial population of 10,000 after 100 iterations and considering a range 0 to 6 for $z_1$ and $z_2$, and $-3$ to 3 for $\beta_3$, $\gamma_3$, $\lambda_3$, $\delta_3$, $\phi_3$, $\theta_3$, $\eta_3$, and $\xi_3$. The value of the optimal function calculated using equation (12) for 662 specimens used in optimization is equal to 0.4885 and results show that when the combination of the two ratios is used, the optimal function reduces by 17 and 3% compared to models I and II, respectively.
Evaluation of proposed models

To evaluate the proposed models, some additional experimental data have been used from other studies and presented in Table 3. As seen, the total number of specimens used in this table is 70.

Based on equation (12), the total error, \( e_{\text{tot}} \), for each model has been calculated and presented in Table 4. For a better comparison, the model performance is evaluated through such statistical indices as: (1) mean square error, (2) average absolute error, and (3) standard deviation determined by equations (18) to (20), respectively. These indexes, calculated for both the modeling specimens in Table 1, and the evaluating specimens in Table 3, are outlined in Table 4.

As shown, considering the statistical results, the proposed models have less error compared to other models. Models I, II, and III have averagely reduced the total error for all the specimens in Tables 1 and 3 by 21.9, 24.1 and 21.6%, respectively, compared to the models proposed by Wu and Wei,100 Pham and Hadi,101 Ozbakkaloglu and Lim,104 Fahmy and Wu,102 Teng et al.,105 Youssef et al.,94 Kumutha et al.,103 Guralnick and Gunawan,106 and Lam and Teng.107

Figure 3(a) to (m) shows the performance of the models proposed by Wu and Wei,100 Pham and Hadi,101 Ozbakkaloglu and Lim,104 Fahmy and Wu,102 Teng et al.,105 Youssef et al.,94 Kumutha et al.,103 Guralnick and Gunawan,106 and Lam and Teng,107 and proposed models I, II, and III for all the specimens in Tables 1 and 3 (732 specimens). As shown, the proposed models estimate the compressive strength of the FRP-confined, circular section concrete specimens better.

For more comparisons, the values of optimal functions, that involve the effects of the total error and correlation coefficient, are provided in Table 5 for all the models studied in this research (732 specimens). And to check the effects of the proposed models, percent reductions created by all three models compared to other mentioned models are given in Table 5.

In Table 5, a negative/positive sign indicates a decrease/increase in the value of the optimal function compared to other mentioned models. Results in Table 5 show that models II and III perform better, but

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Table 3. Details of the FRP-confined circular concrete specimens for evaluating procedure.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Total number of database</th>
<th>Diameter (mm)</th>
<th>Unconfined concrete strength range (MPa)</th>
<th>FRP type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abdollahi et al.109</td>
<td>5</td>
<td>150</td>
<td>14.8–41.7</td>
<td>GFRP</td>
</tr>
<tr>
<td>Almusallam110</td>
<td>4</td>
<td>150</td>
<td>47.7–50.8</td>
<td>GFRP</td>
</tr>
<tr>
<td>Howie and Karbhari111</td>
<td>12</td>
<td>152</td>
<td>38.6</td>
<td>GFRP</td>
</tr>
<tr>
<td>Ilki et al.112</td>
<td>12</td>
<td>150</td>
<td>6.2</td>
<td>CFPR</td>
</tr>
<tr>
<td>Issa and Karam113</td>
<td>9</td>
<td>150</td>
<td>30.5</td>
<td>CFPR</td>
</tr>
<tr>
<td>Lin and Chen114</td>
<td>10</td>
<td>120</td>
<td>32.7</td>
<td>GFRP, HM-CFRP</td>
</tr>
<tr>
<td>Lin and Liao115</td>
<td>6</td>
<td>100</td>
<td>23.9</td>
<td>CFPR</td>
</tr>
<tr>
<td>Miyachi et al.116</td>
<td>6</td>
<td>100–150</td>
<td>23.6–26.3</td>
<td>CFPR</td>
</tr>
<tr>
<td>Vincent and Ozbakkaloglu117</td>
<td>6</td>
<td>152</td>
<td>49.4</td>
<td>AFRP</td>
</tr>
</tbody>
</table>

Table 4. Statistical indicators for FRP-confined circular concrete specimens.

<table>
<thead>
<tr>
<th>Theoretical models</th>
<th>Specimens of Table 1</th>
<th>Specimens of Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wu and Wei100</td>
<td>MSE 8.06, AAE 17.15, SD 28.26</td>
<td>MSE 1.66, AAE 10.17, SD 12.99</td>
</tr>
<tr>
<td>Pham and Hadi101</td>
<td>MSE 5.58, AAE 16.9, SD 23.67</td>
<td>MSE 2.29, AAE 11.96, SD 15.16</td>
</tr>
<tr>
<td>Ozbakkaloglu and Lim104</td>
<td>MSE 6.52, AAE 17.43, SD 25.66</td>
<td>MSE 2.21, AAE 11.62, SD 15.03</td>
</tr>
<tr>
<td>Fahmy and Wu102</td>
<td>MSE 4.61, AAE 17.27, SD 21.08</td>
<td>MSE 3.17, AAE 15.45, SD 15.76</td>
</tr>
<tr>
<td>Teng et al.105</td>
<td>MSE 6.98, AAE 18.92, SD 24.34</td>
<td>MSE 2.47, AAE 13.85, SD 10.79</td>
</tr>
<tr>
<td>Youssef et al.94</td>
<td>MSE 12.94, AAE 19.47, SD 35.81</td>
<td>MSE 5.76, AAE 18.79, SD 24.15</td>
</tr>
<tr>
<td>Kumutha et al.103</td>
<td>MSE 10.17, AAE 28.27, SD 18.05</td>
<td>MSE 12.65, AAE 32.59, SD 15.12</td>
</tr>
<tr>
<td>Guralnick and Gunawan106</td>
<td>MSE 6, AAE 17.7, SD 23.75</td>
<td>MSE 2.79, AAE 12.26, SD 16.89</td>
</tr>
<tr>
<td>Lam and Teng107</td>
<td>MSE 6.31, AAE 17.35, SD 25.18</td>
<td>MSE 1.83, AAE 11.61, SD 12.26</td>
</tr>
<tr>
<td>Oliveira et al.108</td>
<td>MSE 8.71, AAE 17.79, SD 28.72</td>
<td>MSE 2.05, AAE 11.02, SD 14.2</td>
</tr>
<tr>
<td>Model I</td>
<td>MSE 7.89, AAE 16.01, SD 28.14</td>
<td>MSE 1.79, AAE 11.27, SD 13.52</td>
</tr>
<tr>
<td>Model II</td>
<td>MSE 6.36, AAE 15.48, SD 25.27</td>
<td>MSE 2.47, AAE 12.5, SD 14.66</td>
</tr>
<tr>
<td>Model III</td>
<td>MSE 5.8, AAE 15.5, SD 24.15</td>
<td>MSE 5.19, AAE 16.73, SD 21.28</td>
</tr>
</tbody>
</table>
Figure 3. Performance of the selected models in comparison with experimental data: a) Ref. [100], b) Ref. [101], c) Ref. [104], d) Ref. [102], e) Ref. [105], f) Ref. [94], g) Ref. [103], h) Ref. [106], i) Ref. [107], j) Ref. [108], k) the proposed model I, l) the proposed model II and m) the proposed model III.
Table 5. Comparison of optimal function for models.

<table>
<thead>
<tr>
<th>Theoretical model</th>
<th>Z</th>
<th>Percent increase/ decrease model I</th>
<th>Percent increase/ decrease model II</th>
<th>Percent increase/ decrease model III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wu and Wei&lt;sup&gt;100&lt;/sup&gt;</td>
<td>0.51</td>
<td>6.21</td>
<td>-5.24</td>
<td>-5.43</td>
</tr>
<tr>
<td>Pham and Hadi&lt;sup&gt;101&lt;/sup&gt;</td>
<td>0.54</td>
<td>1.93</td>
<td>-10.03</td>
<td>-10.24</td>
</tr>
<tr>
<td>Ozbakkaloglu and Lim&lt;sup&gt;104&lt;/sup&gt;</td>
<td>0.56</td>
<td>-2.01</td>
<td>-14.46</td>
<td>-14.67</td>
</tr>
<tr>
<td>Fahmy and Wu&lt;sup&gt;102&lt;/sup&gt;</td>
<td>0.55</td>
<td>-0.63</td>
<td>-12.92</td>
<td>-13.13</td>
</tr>
<tr>
<td>Teng et al.&lt;sup&gt;103&lt;/sup&gt;</td>
<td>0.55</td>
<td>-0.74</td>
<td>-13.04</td>
<td>-13.25</td>
</tr>
<tr>
<td>Youssef et al.&lt;sup&gt;94&lt;/sup&gt;</td>
<td>0.68</td>
<td>-23.94</td>
<td>-39.07</td>
<td>-39.33</td>
</tr>
<tr>
<td>Kumutha et al.&lt;sup&gt;103&lt;/sup&gt;</td>
<td>0.73</td>
<td>-32.52</td>
<td>-48.70</td>
<td>-48.98</td>
</tr>
<tr>
<td>Guralnick and Gunawan&lt;sup&gt;106&lt;/sup&gt;</td>
<td>0.50</td>
<td>8.80</td>
<td>-2.33</td>
<td>-2.52</td>
</tr>
<tr>
<td>Lam and Teng&lt;sup&gt;107&lt;/sup&gt;</td>
<td>0.54</td>
<td>0.50</td>
<td>-11.64</td>
<td>-11.84</td>
</tr>
<tr>
<td>Oliveira et al.&lt;sup&gt;108&lt;/sup&gt;</td>
<td>0.50</td>
<td>9.76</td>
<td>-1.26</td>
<td>-1.44</td>
</tr>
</tbody>
</table>

Figure 3. Continued
it should be noted that their accuracies do not differ much. According to this table, models I, II, and III averagely reduce the value of the optimum function by 3.2, 15.9, and 16.1%, respectively, compared to other models mentioned in this study.

Conclusions

In this study, three models have been proposed for estimating the compressive strength of FRP-confined, circular-section columns. The strain efficiency factor of FRP in these models has been considered as: (i) a function of the strain ratio, (ii) a function of the effective stiffness ratio, and (iii) a function of the combination of these ratios. Results from this research are shown as follows:

1. Compared to the effective stiffness ratio, the effect of the strain ratio for estimating the compressive strength of confined circular columns is greater.
2. Models proposed in this research estimate the compressive strength of circular columns confined with different types of FRP better; the three proposed models averagely reduce the total error by 21.9, 24.1, and 21.6% and values of the optimal function by 3.2, 15.9, and 16.1% compared to the models mentioned in this study.
3. There is not much difference between the value of the optimal function (which is a combination of total error and correlation coefficient) of the model wherein the FRP strain efficiency factor is a function of the strain ratio and that of the one wherein FRP strain efficiency factor is a function of the combination of the strain ratio and the effective stiffness ratio. Therefore, considering the convenience of the model wherein the FRP strain efficiency factor is a function of the strain ratio, it can be selected as the optimal model.

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