Solutions Manual

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CONTENTS

CHAPTER 1 PROBLEMS

1.1 The demand estimation is the starting point for planning the future electric power supply. The consistency of demand growth over the years has led to numerous attempts to fit mathematical curves to this trend. One of the simplest curves is

$$P = P_0 e^{a(t-t_0)}$$

where a is the average per unit growth rate, P is the demand in year t, and P_0 is the given demand at year t_0.

Assume the peak power demand in the United States in 1984 is 480 GW with an average growth rate of 3.4 percent. Using *MATLAB*, plot the predicated peak demand in GW from 1984 to 1999. Estimate the peak power demand for the year 1999.

We use the following commands to plot the demand growth

```
t0 = 84; P0 = 480;
a =.034;
t =(84:1:99)';
P =P0*exp(a*(t-t0));
disp('Predicted Peak Demand - GW')
disp([t, P])
plot(t, P), grid
xlabel('Year'), ylabel('Peak power demand GW')
P99 =P0*exp(a*(99 - t0))
```

The result is

```
Predicted Peak Demand - GW
   84.0000  480.0000
   85.0000  496.6006
   86.0000  513.7753
   87.0000  531.5441
   88.0000  549.9273
   89.0000  568.9463
   90.0000  588.6231
   91.0000  608.9804
   92.0000  630.0418
   93.0000  651.8315
   94.0000  674.3740
   95.0000  697.6978
   96.0000  721.8274
   97.0000  746.7916
   98.0000  772.6190
   99.0000  799.3398

P99 =

      799.3398
```

The plot of the predicated demand is shown n Figure 1.

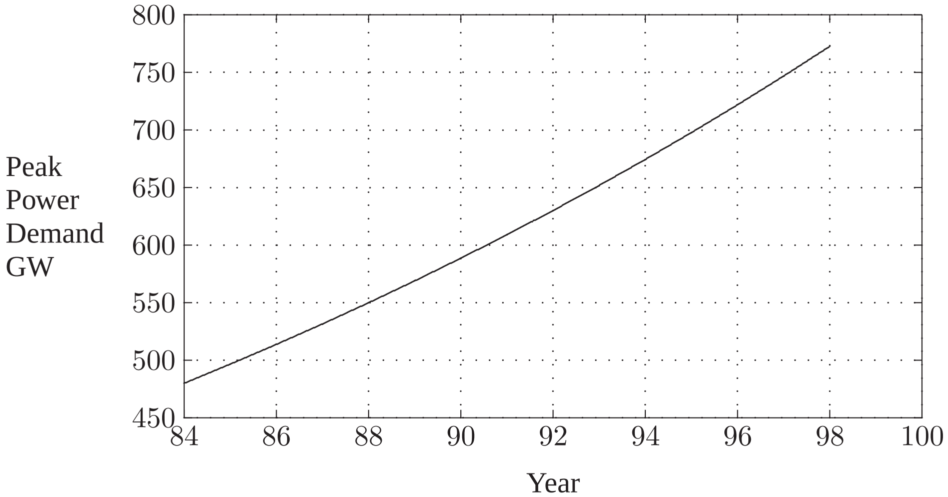


FIGURE 1
Peak Power Demand for Problem 1.1

1.2 In a certain country, the energy consumption is expected to double in 10 years.

Assuming a simple exponential growth given by

$$P = P_0 e^{at}$$

calculate the growth rate a.

$$\begin{aligned} 2P_0 &= P_0 e^{10a} \\ \ln 2 &= 10a \end{aligned}$$

Solving for a, we have

$$a = \frac{0.693}{10} = 0.0693 = 6.93\%$$

1.3. The annual load of a substation is given in the following table. During each month, the power is assumed constant at an average value. Using *MATLAB* and the **barcycle** function, obtain a plot of the annual load curve. Write the necessary statements to find the average load and the annual load factor.

Annual System Load	
Interval – Month	Load – MW
January	8
February	6
March	4
April	2
May	6
June	12
July	16
August	14
September	10
October	4
November	6
December	8

The following commands

```
data = [ 0   1    8
         1   2    6
         2   3    4
         3   4    2
         4   5    6
         5   6   12
```

```
             6   7   16
             7   8   14
             8   9   10
             9  10    4
            10  11    6
            11  12    8];
P = data(:,3);                              % Column array of load
Dt = data(:, 2) - data(:,1); % Column array of demand interval
W = P'*Dt;                    % Total energy, area under the curve
Pavg = W/sum(Dt)                                    % Average load
Peak = max(P)                                          % Peak load
LF = Pavg/Peak*100                           % Percent load factor
barcycle(data)                              % Plots the load cycle
xlabel('time, month'), ylabel('P, MW'), grid
```

result in

```
    Pavg =
             8
    Peak =
            16
    LF =
            50
```

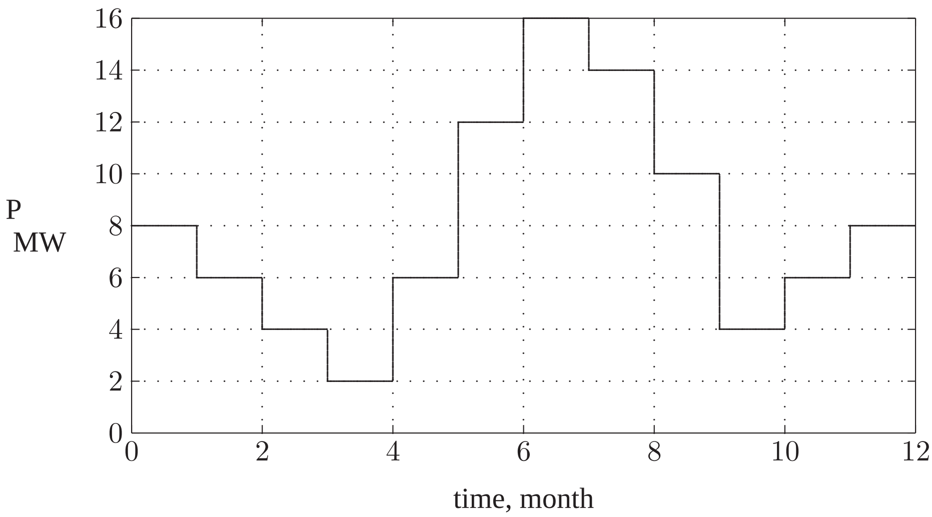


FIGURE 2
Monthly load cycle for Problem 1.3

CHAPTER 2 PROBLEMS

2.1. Modify the program in Example 2.1 such that the following quantities can be entered by the user:
The peak amplitude V_m, and the phase angle θ_v of the sinusoidal supply $v(t) = V_m \cos(\omega t + \theta_v)$. The impedance magnitude Z, and its phase angle γ of the load.
The program should produce plots for $i(t)$, $v(t)$, $p(t)$, $p_r(t)$ and $p_x(t)$, similar to Example 2.1. Run the program for $V_m = 100$ V, $\theta_v = 0$ and the following loads:

An inductive load, $Z = 1.25\angle 60^\circ\ \Omega$
A capacitive load, $Z = 2.0\angle -30^\circ\ \Omega$
A resistive load, $Z = 2.5\angle 0^\circ\ \Omega$

(a) From $p_r(t)$ and $p_x(t)$ plots, estimate the real and reactive power for each load. Draw a conclusion regarding the sign of reactive power for inductive and capacitive loads.
(b) Using phasor values of current and voltage, calculate the real and reactive power for each load and compare with the results obtained from the curves.
(c) If the above loads are all connected across the same power supply, determine the total real and reactive power taken from the supply.

The following statements are used to plot the instantaneous voltage, current, and the instantaneous terms given by(2-6) and (2-8).

```
Vm = input('Enter voltage peak amplitude Vm = ');
thetav =input('Enter voltage phase angle in degree thetav = ');
Vm = 100; thetav = 0;      % Voltage amplitude and phase angle
Z = input('Enter magnitude of the load impedance  Z = ');
gama = input('Enter load phase angle in degree gama = ');
thetai = thetav - gama;        % Current phase angle in degree
```

```
theta = (thetav - thetai)*pi/180;          % Degree to radian
Im = Vm/Z;                                 % Current amplitude
wt=0:.05:2*pi;                             % wt from 0 to 2*pi
v=Vm*cos(wt);                          % Instantaneous voltage
i=Im*cos(wt + thetai*pi/180);          % Instantaneous current
p=v.*i;                                  % Instantaneous power
V=Vm/sqrt(2); I=Im/sqrt(2);          % RMS voltage and current
pr = V*I*cos(theta)*(1 + cos(2*wt));               % Eq. (2.6)
px = V*I*sin(theta)*sin(2*wt);                     % Eq. (2.8)
disp('(a) Estimate from the plots')
P = max(pr)/2, Q = V*I*sin(theta)*sin(2*pi/4)
P = P*ones(1, length(wt));            % Average power for plot
xline = zeros(1, length(wt));        % generates a zero vector
wt=180/pi*wt;                    % converting radian to degree
subplot(221), plot(wt, v, wt, i,wt, xline), grid
title(['v(t)=Vm coswt, i(t)=Im cos(wt +',num2str(thetai),')'])
xlabel('wt, degrees')
subplot(222), plot(wt, p, wt, xline), grid
title('p(t)=v(t) i(t)'), xlabel('wt, degrees')
subplot(223), plot(wt, pr, wt, P, wt,xline), grid
title('pr(t)   Eq. 2.6'), xlabel('wt, degrees')
subplot(224), plot(wt, px, wt, xline), grid
title('px(t)  Eq. 2.8'), xlabel('wt, degrees')
subplot(111)
disp('(b) From P and Q formulas using phasor values ')
P=V*I*cos(theta)                               % Average power
Q = V*I*sin(theta)                            % Reactive power
```

The result for the inductive load $Z = 1.25\angle 60°\Omega$ is

```
    Enter voltage peak amplitude Vm = 100
    Enter voltage phase angle in degree thatav = 0
    Enter magnitude of the load impedance  Z = 1.25
    Enter load phase angle in degree gama = 60

(a) Estimate from the plots

    P =
         2000
    Q =
         3464
```

(b) For the inductive load $Z = 1.25\angle 60°\Omega$, the rms values of voltage and current are

$$V = \frac{100\angle 0°}{1.414} = 70.71\angle 0° \text{ V}$$

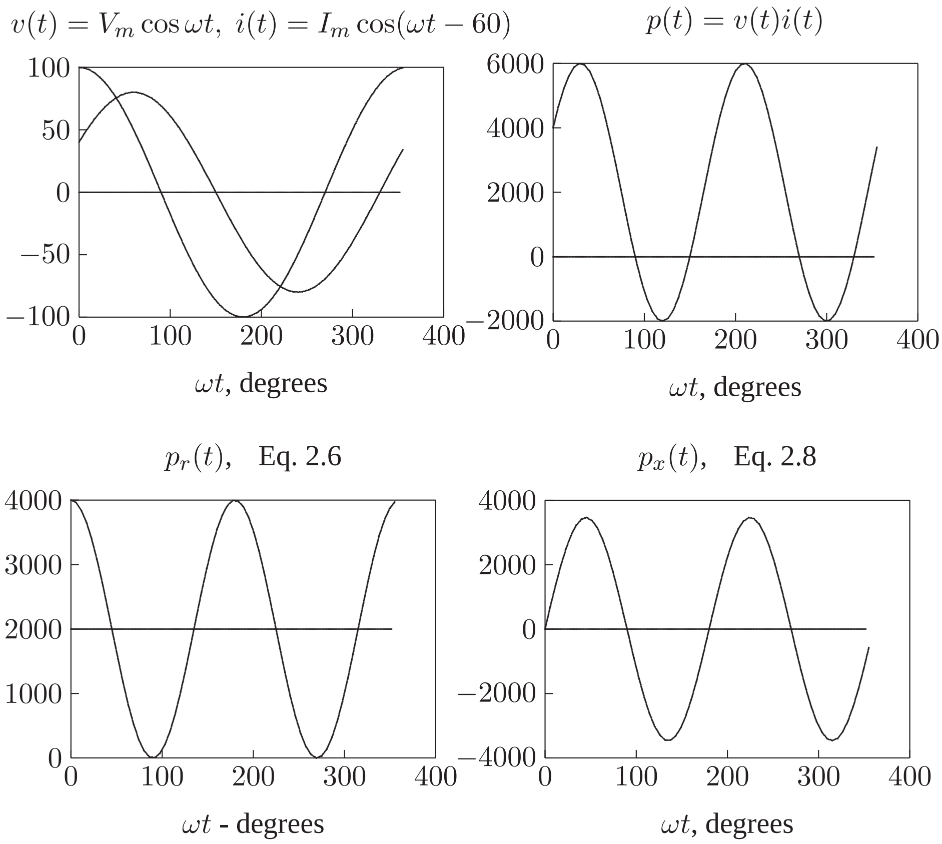


FIGURE 3
Instantaneous current, voltage, power, Eqs. 2.6 and 2.8.

$$I = \frac{70.71\angle 0^\circ}{1.25\angle 60^\circ} = 56.57\angle{-60^\circ} \text{ A}$$

Using (2.7) and (2.9), we have

$$P = (70.71)(56.57)\cos(60) = 2000 \text{ W}$$
$$Q = (70.71)(56.57)\sin(60) = 3464 \text{ Var}$$

Running the above program for the capacitive load $Z = 2.0\angle{-30^\circ}\ \Omega$ will result in

```
(a) Estimate from the plots

P =
     2165
Q =
    -1250
```

Similarly, for $Z = 2.5\angle 0^\circ\ \Omega$, we get

```
    P =
         2000
Q =
         0
```

(c) With the above three loads connected in parallel across the supply, the total real and reactive powers are

$$P = 2000 + 2165 + 2000 = 6165 \text{ W}$$
$$Q = 3464 - 1250 + 0 = 2214 \text{ Var}$$

2.2. A single-phase load is supplied with a sinusoidal voltage

$$v(t) = 200\cos(377t)$$

The resulting instantaneous power is

$$p(t) = 800 + 1000\cos(754t - 36.87^\circ)$$

(a) Find the complex power supplied to the load.
(b) Find the instantaneous current $i(t)$ and the rms value of the current supplied to the load.
(c) Find the load impedance.
(d) Use *MATLAB* to plot $v(t)$, $p(t)$, and $i(t) = p(t)/v(t)$ over a range of 0 to 16.67 ms in steps of 0.1 ms. From the current plot, estimate the peak amplitude, phase angle and the angular frequency of the current, and verify the results obtained in part (b). Note in *MATLAB* the command for array or element-by-element division is `./`.

$$\begin{aligned} p(t) &= 800 + 1000\cos(754t - 36.87^\circ) \\ &= 800 + 1000\cos 36.87^\circ \cos 754t + \sin 36.87^\circ \sin 754t \\ &= 800 + 800\cos 754t + 600\sin 754t \\ &= 800[1 + \cos 2(377)t] + 600\sin 2(377)t \end{aligned}$$

$p(t)$ is in the same form as (2.5), thus $P = 600$ W, and $Q = 600$, Var, or

$$S = 800 + j600 = 1000\angle 36.87^\circ \text{ VA}$$

(b) Using $S = \frac{1}{2}V_m I_m{}^*$, we have

$$1000\angle 36.87^\circ = \frac{1}{2}200\angle 0^\circ I_m$$

or

$$I_m = 10\angle{-36.87^\circ} \;\; \text{A}$$

Therefore, the instantaneous current is

$$i(t) = 10cos(377t - 36.87^\circ) \;\; \text{A}$$

(c)

$$Z_L = \frac{V}{I} = \frac{200\angle 0^\circ}{10\angle{-36.87^\circ}} = 20\angle 36.87^\circ \;\; \Omega$$

(d) We use the following command

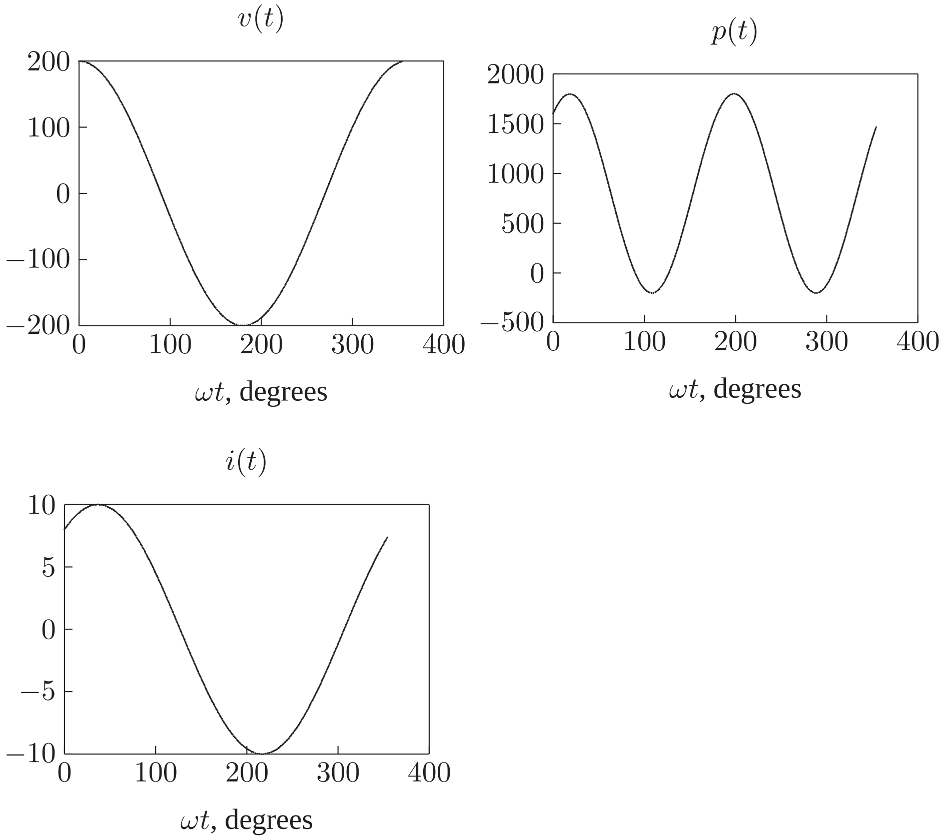


FIGURE 4
Instantaneous voltage, power, and current for Problem 2.2.

```
Vm = 200;
t=0:.0001:0.01667;                               % wt from 0 to 2*pi
v=Vm*cos(377*t);                             % Instantaneous voltage
p = 800 + 1000*cos(754*t - 36.87*pi/180);% Instantaneous power
i=p./v;                                      % Instantaneous current
wt=180/pi*377*t;                       % converting radian to degree
xline = zeros(1, length(wt));          % generates a zero vector
subplot(221), plot(wt, v, wt, xline), grid
xlabel('wt,  degrees'), title('v(t)')
subplot(222), plot(wt, p, wt, xline), grid
xlabel('wt,  degrees'), title('p(t)')
subplot(223), plot(wt, i, wt, xline), grid
xlabel('wt, degrees'), title('i(t)'), subplot(111)
```

The result is shown in Figure 4. The inspection of current plot shows that the peak amplitude of the current is 10 A, lagging voltage by $36.87°$, with an angular frequency of 377 Rad/sec.

2.3. An inductive load consisting of R and X in series feeding from a 2400-V rms supply absorbs 288 kW at a lagging power factor of 0.8. Determine R and X.

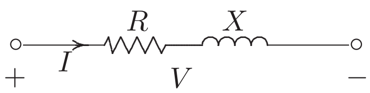


FIGURE 5
An inductive load, with R and X in series.

$$\theta = \cos^{-1} 0.8 = 36.87°$$

The complex power is

$$S = \frac{288}{0.8}\angle 36.87° = 360\angle 36.87° \ \text{kVA}$$

The current given from $S = VI^*$, is

$$I = \frac{360 \times 10^3\angle -36.87°}{2400\angle 0°} = 150\angle -36.87 \ \text{A}$$

Therefore, the series impedance is

$$Z = R + jX = \frac{V}{I} = \frac{2400\angle 0°}{150\angle -36.87°} = 12.8 + j9.6 \ \Omega$$

Therefore, $R = 12.8\ \Omega$ and $X = 9.6\ \Omega$.

2.4. An inductive load consisting of R and X in parallel feeding from a 2400-V rms supply absorbs 288 kW at a lagging power factor of 0.8. Determine R and X.

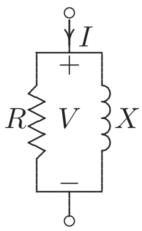


FIGURE 6
An inductive load, with R and X in parallel.

The complex power is

$$S = \frac{288}{0.8}\angle 36.87^\circ = 360\angle 36.87^\circ \text{ kVA}$$
$$= 288 \text{ kW} + j216 \text{ kvar}$$

$$R = \frac{|V|^2}{P} = \frac{(2400)^2}{288 \times 10^3} = 20 \ \Omega$$
$$X = \frac{|V|^2}{Q} = \frac{(2400)^2}{216 \times 10^3} = 26.667 \ \Omega$$

2.5. Two loads connected in parallel are supplied from a single-phase 240-V rms source. The two loads draw a total real power of 400 kW at a power factor of 0.8 lagging. One of the loads draws 120 kW at a power factor of 0.96 leading. Find the complex power of the other load.

$$\theta = \cos^{-1} 0.8 = 36.87^\circ$$

The total complex load is

$$S = \frac{400}{0.8}\angle 36.87^\circ = 500\angle 36.87^\circ \text{ kVA}$$
$$= 400 \text{ kW} + j300 \text{ kvar}$$

The 120 kW load complex power is

$$S = \frac{120}{0.96}\angle -16.26^\circ = 125\angle -16.26^\circ \text{ kVA}$$
$$= 120 \text{ kW} - j35 \text{ kvar}$$

Therefore, the second load complex power is

$$S_2 = 400 + j300 - (120 - j35) = 280 \text{ kW} + j335 \text{ kvar}$$

2.6. The load shown in Figure 7 consists of a resistance R in parallel with a capacitor of reactance X. The load is fed from a single-phase supply through a line of impedance $8.4 + j11.2\ \Omega$. The rms voltage at the load terminal is $1200\angle 0°$ V rms, and the load is taking 30 kVA at 0.8 power factor leading.
(a) Find the values of R and X.
(b) Determine the supply voltage V.

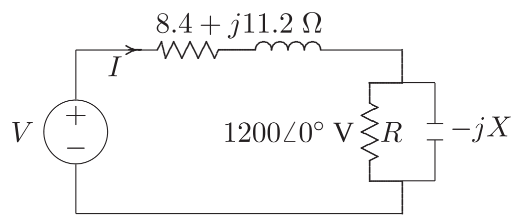


FIGURE 7
Circuit for Problem 2.6.

$$\theta = \cos^{-1} 0.8 = 36.87°$$

The complex power is

$$S = 30\angle{-36.87°} = 24 \text{ kW} - j18 \text{ kvar}$$

(a)

$$R = \frac{|V|^2}{P} = \frac{(1200)^2}{24000} = 60\ \ \Omega$$
$$X = \frac{|V|^2}{Q} = \frac{(1200)^2}{18000} = 80\ \ \Omega$$

From $S = VI^*$, the current is

$$I = \frac{30000\angle 36.87°}{1200\angle 0°} = 25\angle 36.87\ \ \text{A}$$

Thus, the supply voltage is

$$\begin{aligned} V &= 1200\angle 0° + 25\angle 36.87°(8.4 + j11.2) \\ &= 1200 + j350 = 1250\angle 16.26°\ \ \text{V} \end{aligned}$$

2.7. Two impedances, $Z_1 = 0.8 + j5.6\ \Omega$ and $Z_2 = 8 - j16\ \Omega$, and a single-phase motor are connected in parallel across a 200-V rms, 60-Hz supply as shown in Figure 8. The motor draws 5 kVA at 0.8 power factor lagging.

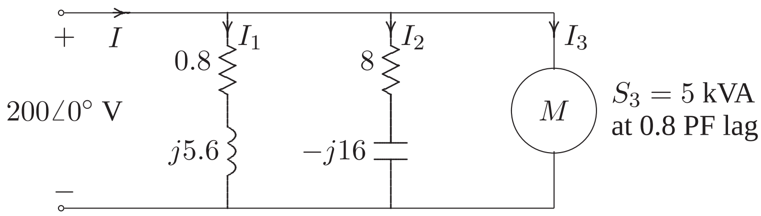


FIGURE 8
Circuit for Problem 2.7.

(a) Find the complex powers S_1, S_2 for the two impedances, and S_3 for the motor.
(b) Determine the total power taken from the supply, the supply current, and the overall power factor.
(c) A capacitor is connected in parallel with the loads. Find the kvar and the capacitance in μF to improve the overall power factor to unity. What is the new line current?

(a) The load complex power are

$$S_1 = \frac{|V|^2}{Z_1^*} = \frac{(200)^2}{0.8 - j5.6} = 1000 + j7000 \text{ VA}$$
$$S_2 = \frac{|V|^2}{Z_2^*} = \frac{(200)^2}{8 + j16} = 1000 - j2000 \text{ VA}$$
$$S_3 = 5000\angle 36.87^\circ = 4000 + j3000 \text{ VA}$$

Therefore, the total complex power is

$$S_t = 6 + j8 = 10\angle 53.13^\circ \text{ kVA}$$

(b) From $S = VI^*$, the current is

$$I = \frac{10000\angle -53.13^\circ}{200\angle 0^\circ} = 50\angle -53.13 \text{ A}$$

and the power factor is $\cos 53.13^\circ = 0.6$ lagging.

(c) For overall unity power factor, $Q_C = 8000$ Var, and the capacitive impedance is

$$Z_C = \frac{|V|^2}{S_C{}^*} = \frac{(200)^2}{j8000} = -j5 \ \ \Omega$$

and the capacitance is

$$C = \frac{10^6}{(2\pi)(60)(5)} = 530.5 \ \ \mu\text{F}$$

The new current is

$$I = \frac{6000\angle 0^\circ}{200\angle 0^\circ} = 30\angle 0 \ \ \text{A}$$

2.8. Two single-phase ideal voltage sources are connected by a line of impedance of $0.7 + j2.4\ \Omega$ as shown in Figure 9. $V_1 = 500\angle 16.26^\circ$ V and $V_2 = 585\angle 0^\circ$ V. Find the complex power for each machine and determine whether they are delivering or receiving real and reactive power. Also, find the real and the reactive power loss in the line.

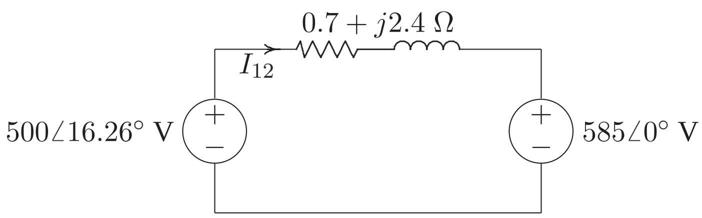


FIGURE 9
Circuit for Problem 2.8.

$$I_{12} = \frac{500\angle 16.26^\circ - 585\angle 0^\circ}{0.7 + j2.4} = 42 + j56 = 70\angle 53.13^\circ \ \ \text{A}$$

$$\begin{aligned} S_{12} = V_1 I_{12}^* = (500\angle 16.26^\circ)(70\angle{-53.13^\circ}) &= 35000\angle{-36.87^\circ} \\ &= 28000 - j21000 \ \ \text{VA} \\ S_{21} = V_2 I_{21}^* = (585\angle 0^\circ)(-70\angle{-53.13^\circ}) &= 40950\angle{-53.13^\circ} \\ &= -24570 + j32760 \ \ \text{VA} \end{aligned}$$

From the above results, since P_1 is positive and P_2 is negative, source 1 generates 28 kW, and source 2 receives 24.57 kW, and the real power loss is 3.43 kW. Similarly, since Q_1 is negative, source 1 receives 21 kvar and source 2 delivers 32.76 kvar. The reactive power loss in the line is 11.76 kvar.

2.9. Write a *MATLAB* program for the system of Example 2.5 such that the voltage magnitude of source 1 is changed from 75 percent to 100 percent of the given value in steps of 1 volt. The voltage magnitude of source 2 and the phase angles of the two sources is to be kept constant. Compute the complex power for each source and the line loss. Tabulate the reactive powers and plot Q_1, Q_2, and Q_L versus voltage magnitude $|V_1|$. From the results, show that the flow of reactive power along the interconnection is determined by the magnitude difference of the terminal voltages.

We use the following commands

```
E1 = input('Source # 1 Voltage Mag. = ');
a1 = input('Source # 1 Phase Angle  = ');
E2 = input('Source # 2 Voltage Mag. = ');
a2 = input('Source # 2 Phase Angle  = ');
R = input('Line Resistance = ');
X = input('Line Reactance = ');
Z = R + j*X;                                   % Line impedance
E1 = (0.75*E1:1:E1)';      % Change E1 form 75% to 100% E1
a1r = a1*pi/180;                     % Convert degree to radian
k = length(E1);
E2 = ones(k,1)*E2;%create col. Array of same length for E2
a2r = a2*pi/180;                     % Convert degree to radian
V1=E1.*cos(a1r) + j*E1.*sin(a1r);
V2=E2.*cos(a2r) + j*E2.*sin(a2r);
I12 = (V1 - V2)./Z;  I21=-I12;
S1= V1.*conj(I12);  P1 = real(S1);  Q1 = imag(S1);
S2= V2.*conj(I21);  P2 = real(S2);  Q2 = imag(S2);
SL= S1+S2;        PL = real(SL);  QL = imag(SL);
Result1=[E1, Q1,  Q2, QL];
disp('     E1       Q-1       Q-2       Q-L ')
disp(Result1)
plot(E1, Q1,  E1, Q2,  E1, QL), grid
xlabel(' Source #1 Voltage Magnitude')
ylabel(' Q, var')
text(112.5, -180, 'Q2')
text(112.5,  5,'QL'), text(112.5, 197, 'Q1')
```

The result is

```
Source # 1 Voltage Mag. = 120
Source # 1 Phase Angle  = -5
Source # 2 Voltage Mag. = 100
Source # 2 Phase Angle  = 0
Line Resistance = 1
Line Reactance = 7

E1        Q-1       Q-2       Q-L

 90.0000 -105.5173  129.1066   23.5894
 91.0000  -93.9497  114.9856   21.0359
 92.0000  -82.1021  100.8646   18.7625
 93.0000  -69.9745   86.7435   16.7690
 94.0000  -57.5669   72.6225   15.0556
 95.0000  -44.8794   58.5015   13.6221
 96.0000  -31.9118   44.3804   12.4687
 97.0000  -18.6642   30.2594   11.5952
 98.0000   -5.1366   16.1383   11.0017
 99.0000    8.6710    2.0173   10.6883
100.0000   22.7586  -12.1037   10.6548
101.0000   37.1262  -26.2248   10.9014
102.0000   51.7737  -40.3458   11.4279
103.0000   66.7013  -54.4668   12.2345
104.0000   81.9089  -68.5879   13.3210
105.0000   97.3965  -82.7089   14.6876
106.0000  113.1641  -96.8299   16.3341
107.0000  129.2117 -110.9510   18.2607
108.0000  145.5393 -125.0720   20.4672
109.0000  162.1468 -139.1931   22.9538
110.0000  179.0344 -153.3141   25.7203
111.0000  196.2020 -167.4351   28.7669
112.0000  213.6496 -181.5562   32.0934
113.0000  231.3772 -195.6772   35.7000
114.0000  249.3848 -209.7982   39.5865
115.0000  267.6724 -223.9193   43.7531
116.0000  286.2399 -238.0403   48.1996
117.0000  305.0875 -252.1614   52.9262
118.0000  324.2151 -266.2824   57.9327
119.0000  343.6227 -280.4034   63.2193
120.0000  363.3103 -294.5245   68.7858
```

Examination of Figure 10 shows that the flow of reactive power along the interconnection is determined by the voltage magnitude difference of terminal voltages.

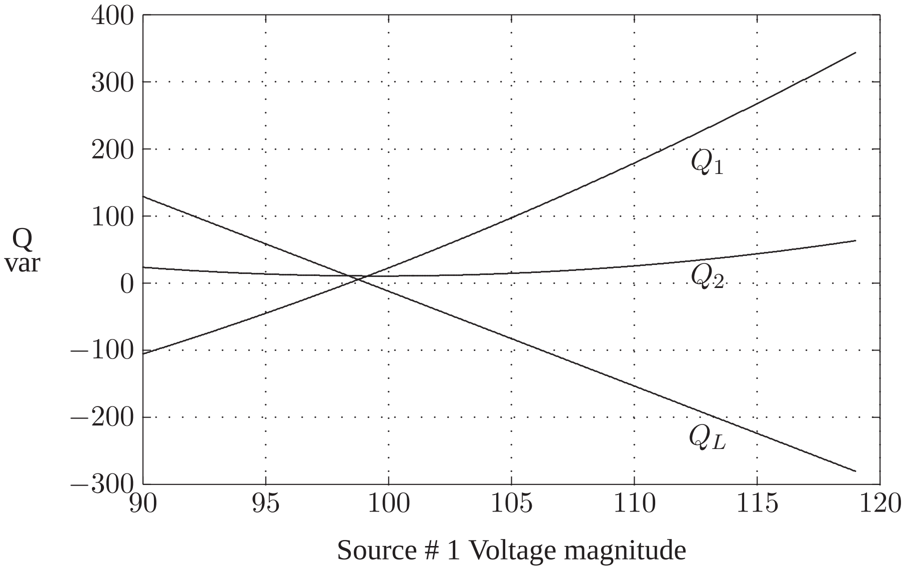


FIGURE 10
Reactive power versus voltage magnitude.

2.10. A balanced three-phase source with the following instantaneous phase voltages

$$v_{an} = 2500\cos(\omega t)$$
$$v_{bn} = 2500\cos(\omega t - 120°)$$
$$v_{cn} = 2500\cos(\omega t - 240°)$$

supplies a balanced Y-connected load of impedance $Z = 250\angle 36.87°\ \Omega$ per phase.
(a) Using *MATLAB*, plot the instantaneous powers p_a, p_b, p_c and their sum versus ωt over a range of $0 : 0.05 : 2\pi$ on the same graph. Comment on the nature of the instantaneous power in each phase and the total three-phase real power.
(b) Use (2.44) to verify the total power obtained in part (a).

We use the following commands

```
wt=0:.02:2*pi;
pa=25000*cos(wt).*cos(wt-36.87*pi/180);
pb=25000*cos(wt-120*pi/180).*cos(wt-120*pi/180-36.87*pi/180);
pc=25000*cos(wt-240*pi/180).*cos(wt-240*pi/180-36.87*pi/180);
p = pa+pb+pc;
```

```
plot(wt, pa, wt, pb, wt, pc, wt, p), grid
xlabel('Radian')
disp('(b)')
V = 2500/sqrt(2);
gama = acos(0.8);
Z = 250*(cos(gama)+j*sin(gama));
I = V/Z;
P = 3*V*abs(I)*0.8
```

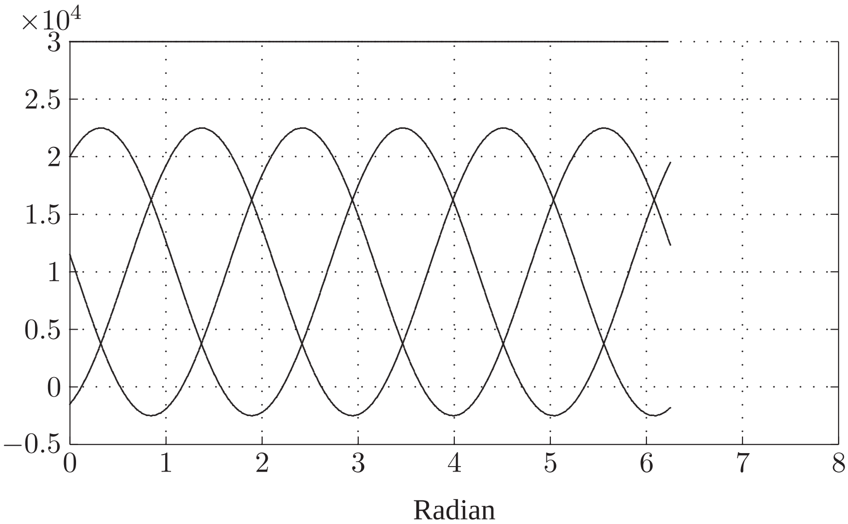


FIGURE 11
Instantaneous powers and their sum for Problem 2.10.

(b)

$$V = \frac{2500}{\sqrt{2}} = 1767.77\angle 0^\circ \ \ \text{V}$$
$$I = \frac{1767.77\angle 0^\circ}{250\angle 36.87^\circ} = 7.071\angle{-36.87^\circ} \ \ \text{A}$$
$$P = (3)(1767.77)(7.071)(0.8) = 30000 \ \ \text{W}$$

2.11. A 4157-V rms three-phase supply is applied to a balanced Y-connected three-phase load consisting of three identical impedances of $48\angle 36.87^\circ \ \Omega$. Taking the phase to neutral voltage V_{an} as reference, calculate
(a) The phasor currents in each line.
(b) The total active and reactive power supplied to the load.

$$V_{an} = \frac{4157}{\sqrt{3}} = 2400 \ \ \text{V}$$

With V_{an} as reference, the phase voltages are:

$$V_{an} = 2400\angle 0^\circ \text{ V} \qquad V_{bn} = 2400\angle{-120^\circ} \text{ V} \qquad V_{an} = 2400\angle{-240^\circ} \text{ V}$$

(a) The phasor currents are:

$$\begin{aligned} I_a &= \frac{V_{an}}{Z} = \frac{2400\angle 0^\circ}{48\angle 36.87^\circ} = 50\angle{-36.87^\circ} \ \text{ A} \\ I_b &= \frac{V_{bn}}{Z} = \frac{2400\angle{-120^\circ}}{48\angle 36.87^\circ} = 50\angle{-156.87^\circ} \ \text{ A} \\ I_c &= \frac{V_{cn}}{Z} = \frac{2400\angle{-240^\circ}}{48\angle 36.87^\circ} = 50\angle{-276.87^\circ} \ \text{ A} \end{aligned}$$

(b) The total complex power is

$$\begin{aligned} S = 3V_{an}I_a^* = (3)(2400\angle 0^\circ)(50\angle 36.87^\circ) &= 360\angle 36.87^\circ \text{ kVA} \\ &= 288 \text{ kW} + j216 \text{ KVAR} \end{aligned}$$

2.12. Repeat Problem 2.11 with the same three-phase impedances arranged in a Δ connection. Take V_{ab} as reference.

$$V_{an} = \frac{4157}{\sqrt{3}} = 2400 \ \text{ V}$$

With V_{ab} as reference, the phase voltages are:

$$I_{ab} = \frac{V_{ab}}{Z} = \frac{4157\angle 0^\circ}{48\angle 36.87^\circ} = 86.6\angle{-36.87^\circ} \ \text{ A}$$

$$I_a = \sqrt{3}\angle{-30^\circ} I_{ab} = (\sqrt{3}\angle{-30^\circ})(86.6\angle{-36.87^\circ} = 150\angle{-66.87^\circ} \ \text{ A}$$

For positive phase sequence, current in other lines are

$$I_b = 150\angle{-186.87^\circ} \text{ A}, \text{ and } I_c = 150\angle 53.13^\circ \text{ A}$$

(b) The total complex power is

$$\begin{aligned} S = 3V_{ab}I_{ab}^* = (3)(4157\angle 0^\circ)(86.6\angle 36.87^\circ) &= 1080\angle 36.87^\circ \text{ kVA} \\ &= 864 \text{ kW} + j648 \text{ kvar} \end{aligned}$$

2.13. A balanced delta connected load of $15 + j18\ \Omega$ per phase is connected at the end of a three-phase line as shown in Figure 12. The line impedance is $1 + j2\ \Omega$ per phase. The line is supplied from a three-phase source with a line-to-line voltage of 207.85 V rms. Taking V_{an} as reference, determine the following:

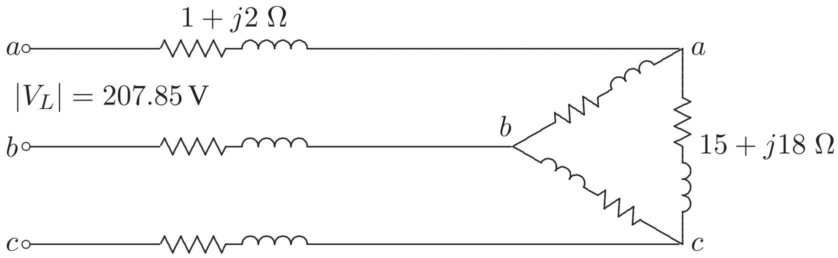


FIGURE 12
Circuit for Problem 2.13.

(a) Current in phase a.
(b) Total complex power supplied from the source.
(c) Magnitude of the line-to-line voltage at the load terminal.

$$V_{an} = \frac{207.85}{\sqrt{3}} = 120 \ \text{ V}$$

Transforming the delta connected load to an equivalent Y-connected load, result in the phase 'a' equivalent circuit, shown in Figure 13.

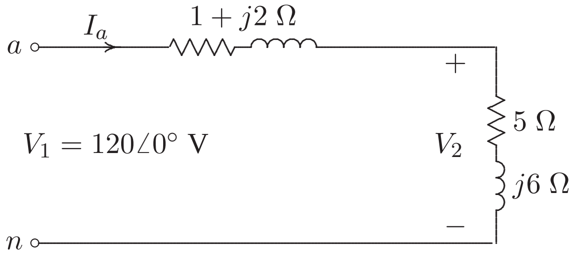


FIGURE 13
The per phase equivalent circuit for Problem 2.13.

(a)

$$I_a = \frac{120\angle 0^\circ}{6 + j8} = 12\angle{-53.13^\circ} \ \text{ A}$$

(b) The total complex power is

$$\begin{aligned} S = 3V_{an}I_a^* = (3)(120\angle 0^\circ)(12\angle 53.13^\circ &= 4320\angle 53.13^\circ \text{ VA} \\ &= 2592 \text{ W} + j3456 \text{ Var} \end{aligned}$$

(c)

$$V_2 = 120\angle 0^\circ - (1 + j2)(12\angle{-53.13^\circ} = 93.72\angle{-2.93^\circ} \ \text{ A}$$

Thus, the magnitude of the line-to-line voltage at the load terminal is $V_L = \sqrt{3}(93.72) = 162.3$ V.

2.14. Three parallel three-phase loads are supplied from a 207.85-V rms, 60-Hz three-phase supply. The loads are as follows:

Load 1: A 15 HP motor operating at full-load, 93.25 percent efficiency, and 0.6 lagging power factor.
Load 2: A balanced resistive load that draws a total of 6 kW.
Load 3: A Y-connected capacitor bank with a total rating of 16 kvar.

(a) What is the total system kW, kvar, power factor, and the supply current per phase?
(b) What is the system power factor and the supply current per phase when the resistive load and induction motor are operating but the capacitor bank is switched off?

The real power input to the motor is

$$P_1 = \frac{(15)(746)}{0.9325} = 12 \text{ kW}$$
$$S_1 = \frac{12}{0.6}\angle 53.13^\circ \text{ kVA} = 12 \text{ kW} + j16 \text{ kvar}$$
$$S_2 = 6 \text{ kW} + j0 \text{ kvar}$$
$$S_3 = 0 \text{ kW} - j16 \text{ kvar}$$

(a) The total complex power is

$$S = 18\angle 0^\circ \text{ kVA} = 18 \text{ kW} + j0 \text{ kvar}$$

The supply current is

$$I = \frac{18000}{(3)(120)} = 50\angle 0^\circ \text{ A}, \qquad \text{at unity power factor}$$

(b) With the capacitor switched off, the total power is

$$S = 18 + j16 = 24.08\angle 41.63^\circ \text{ kVA}$$

$$I = \frac{24083\angle -41.63}{(3)(120\angle 0^\circ)} = 66.89\angle -41.63^\circ \text{ A}$$

The power factor is $\cos 41.63^\circ = 0.747$ lagging.

2.15. Three loads are connected in parallel across a 12.47 kV three-phase supply.

Load 1: Inductive load, 60 kW and 660 kvar.
Load 2: Capacitive load, 240 kW at 0.8 power factor.
Load 3: Resistive load of 60 kW.

(a) Find the total complex power, power factor, and the supply current.
(b) A Y-connected capacitor bank is connected in parallel with the loads. Find the total kvar and the capacitance per phase in μF to improve the overall power factor to 0.8 lagging. What is the new line current?

$$
\begin{aligned}
S_1 &= \ \ 60 \text{ kW} + j660 \text{ kvar} \\
S_2 &= 240 \text{ kW} - j180 \text{ kvar} \\
S_3 &= \ \ 60 \text{ kW} + \quad j0 \text{ kvar}
\end{aligned}
$$

(a) The total complex power is

$$S = 360 \text{ kW} + j480 \text{ kvar} = 600\angle 53.13^\circ \text{ kVA}$$

The phase voltage is

$$V = \frac{12.47}{\sqrt{3}} = 7.2\angle 0^\circ \ \ \text{kV}$$

The supply current is

$$I = \frac{600\angle -53.13^\circ}{(3)(7.2)} = 27.77\angle -53.13^\circ \ \ \text{A}$$

The power factor is $\cos 53.13^\circ = 0.6$ lagging.

(b) The net reactive power for 0.8 power factor lagging is

$$Q' = 360 \tan 36.87^\circ = 270 \ \ \text{kvar}$$

Therefore, the capacitor kvar is $Q_c = 480 - 270 = 210$ kvar, or $S_c = -j210$ kVA.

$$X_c = \frac{|V_L|^2}{S_c^*} = \frac{(12.47 \times 1000)^2}{j210000} = -j740.48 \ \Omega$$

$$C = \frac{10^6}{(2\pi)(60)(740.48)} = 3.58\mu\text{F}$$

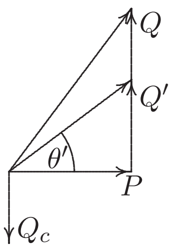


FIGURE 14
The power diagram for Problem 2.15.

$$I = \frac{S^*}{V^*} = \frac{360 - j270}{(3)(7.2)} = 20.835\angle{-36.87^\circ} \text{ A}$$

2.16. A balanced Δ-connected load consisting of a pure resistances of 18 Ω per phase is in parallel with a purely resistive balanced Y-connected load of 12 Ω per phase as shown in Figure 15. The combination is connected to a three-phase balanced supply of 346.41-V rms (line-to-line) via a three-phase line having an inductive reactance of $j3$ Ω per phase. Taking the phase voltage V_{an} as reference, determine
(a) The current, real power, and reactive power drawn from the supply.
(b) The line-to-neutral and the line-to-line voltage of phase a at the combined load terminals.

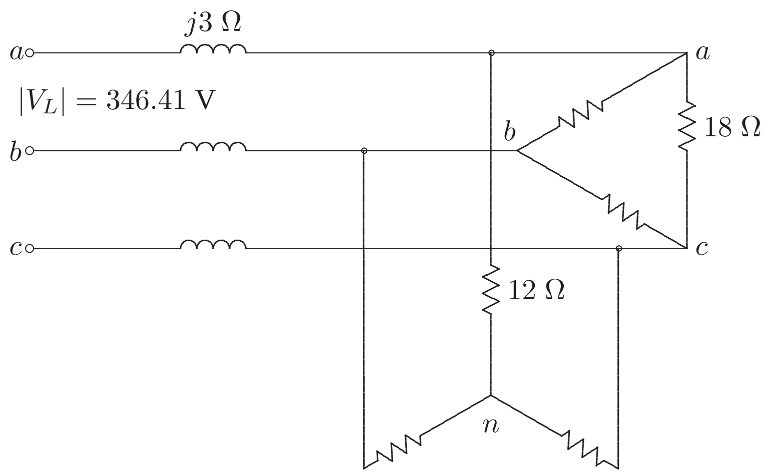


FIGURE 15
Circuit for Problem 2.16.

Transforming the delta connected load to an equivalent Y-connected load, result in the phase 'a' equivalent circuit, shown in Figure 16.

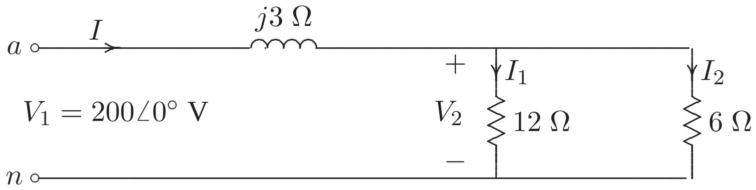


FIGURE 16
The per phase equivalent circuit for Problem 2.16.

(a)

$$V_{an} = \frac{346.41}{\sqrt{3}} = 200 \ \text{V}$$

The input impedance is

$$Z = \frac{(12)(6)}{12+6} + j3 = 4 + j3 \ \ \Omega$$

$$I_a = \frac{200\angle 0^\circ}{4+j3} = 40\angle{-36.87^\circ} \ \ \text{A}$$

The total complex power is

$$\begin{aligned} S = 3V_{an}I_a^* = (3)(200\angle 0^\circ)(40\angle 36.87^\circ) &= 24000\angle 36.87^\circ \ \text{VA} \\ &= 19200 \ \text{W} + j14400 \ \text{Var} \end{aligned}$$

(b)

$$V_2 = 200\angle 0^\circ - (j3)(40\angle{-36.87^\circ}) = 160\angle{-36.87^\circ} \ \ \text{A}$$

Thus, the magnitude of the line-to-line voltage at the load terminal is $V_L = \sqrt{3}(160) = 277.1$ V.

CHAPTER 3 PROBLEMS

3.1. A three-phase, 318.75-kVA, 2300-V alternator has an armature resistance of 0.35 Ω/phase and a synchronous reactance of 1.2 Ω/phase. Determine the no-load line-to-line generated voltage and the voltage regulation at
(a) Full-load kVA, 0.8 power factor lagging, and rated voltage.
(b) Full-load kVA, 0.6 power factor leading, and rated voltage.

$$V_\phi = \frac{2300}{\sqrt{3}} = 1327.9 \text{ V}$$

(a) For 318.75 kVA, 0.8 power factor lagging, $S = 318750\angle 36.87°$ VA.

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{318750\angle{-36.87°}}{(3)(1327.9)} = 80\angle{-36.87°} \text{ A}$$

$$E_\phi = 1327.9 + (0.35 + j1.2)(80\angle{-36.87°}) = 1409.2\angle 2.44° \text{ V}$$

The magnitude of the no-load generated voltage is
$E_{LL} = \sqrt{3}\, 1409.2 = 2440.8$ V, and the voltage regulation is

$$V.R. = \frac{2440.8 - 2300}{2300} \times 100 = 6.12\%$$

(b) For 318.75 kVA, 0.6 power factor leading, $S = 318750\angle{-53.13°}$ VA.

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{318750\angle 53.13°}{(3)(1327.9} = 80\angle 53.13° \text{ A}$$

$$E_\phi = 1327.9 + (0.35 + j1.2)(80\angle 53.13^\circ) = 1270.4\angle 3.61^\circ \ \text{ V}$$

The magnitude of the no-load generated voltage is
$E_{LL} = \sqrt{3}\ 1270.4 = 2220.4$ V, and the voltage regulation is

$$V.R. = \frac{2200.4 - 2300}{2300} \times 100 = -4.33\%$$

3.2. A 60-MVA, 69.3-kV, three-phase synchronous generator has a synchronous reactance of 15 Ω/phase and negligible armature resistance.

(a) The generator is delivering rated power at 0.8 power factor lagging at the rated terminal voltage to an infinite bus bar. Determine the magnitude of the generated emf per phase and the power angle δ.

(b) If the generated emf is 36 kV per phase, what is the maximum three-phase power that the generator can deliver before losing its synchronism?

(c) The generator is delivering 48 MW to the bus bar at the rated voltage with its field current adjusted for a generated emf of 46 kV per phase. Determine the armature current and the power factor. State whether power factor is lagging or leading?

$$V_\phi = \frac{69.3}{\sqrt{3}} = 40 \ \text{ kV}$$

(a) For 60 kVA, 0.8 power factor lagging, $S = 60000\angle 36.87^\circ$ kVA.

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{60000\angle{-36.87^\circ}}{(3)(40)} = 500\angle{-36.87^\circ} \ \text{ A}$$

$$E_\phi = 40 + (j15)(500\angle{-36.87^\circ}) \times 10^{-3} = 44.9\angle 7.675^\circ \ \text{ kV}$$

(b)

$$P_{max} = \frac{3|E||V|}{X_s} = \frac{(3)(36)(40)}{15} = 288 \ \text{ MW}$$

(c) For $P = 48$ MW, and $E = 46$ KV/phase, the power angle is given by

$$48 = \frac{(3)(46)(40)}{15} \sin\delta$$

or

$$\delta = 7.4947^\circ$$

and solving for the armature current from $E = V + jX_sI_a$, we have

$$I_a = \frac{46000\angle 7.4947^\circ - 40000\angle 0^\circ}{j15} = 547.47\angle -43.06^\circ \ \ \text{A}$$

The power factor is $\cos^{-1} 43.06 = 0.7306$ lagging.

3.3. A 24,000-kVA, 17.32-kV, Y-connected synchronous generator has a synchronous reactance of 5 Ω/phase and negligible armature resistance.

(a) At a certain excitation, the generator delivers rated load, 0.8 power factor lagging to an infinite bus bar at a line-to-line voltage of 17.32 kV. Determine the excitation voltage per phase.

(b) The excitation voltage is maintained at 13.4 KV/phase and the terminal voltage at 10 KV/phase. What is the maximum three-phase real power that the generator can develop before pulling out of synchronism?

(c) Determine the armature current for the condition of part (b).

$$V_\phi = \frac{17.32}{\sqrt{3}} = 10 \ \ \text{kV}$$

(a) For 24000 kVA, 0.8 power factor lagging, $S = 24000\angle 36.87^\circ$ kVA.

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{24000\angle -36.87^\circ}{(3)(10)} = 800\angle -36.87^\circ \ \ \text{A}$$

$$E_\phi = 10 + (j5)(800\angle -36.87^\circ) \times 10^{-3} = 12.806\angle 14.47^\circ \ \ \text{kV}$$

(b)

$$P_{max} = \frac{3|E||V|}{X_s} = \frac{(3)(13.4)(10)}{5} = 80.4 \ \ \text{MW}$$

(c) At maximum power transfer $\delta = 90^\circ$, and solving for the armature current from $E = V + jX_sI_a$, we have

$$I_a = \frac{13400\angle 90^\circ - 10000\angle 0^\circ}{j5} = 3344\angle 36.73^\circ \ \ \text{A}$$

The power factor is $\cos^{-1} 36.73 = 0.7306$ leading.

3.4. A 34.64-kV, 60-MVA, three-phase salient-pole synchronous generator has a direct axis reactance of 13.5 Ω and a quadrature-axis reactance of 9.333 Ω. The armature resistance is negligible.

(a) Referring to the phasor diagram of a salient-pole generator shown in Figure 17, show that the power angle δ is given by

$$\delta = \tan^{-1}\left(\frac{X_q|I_a|\cos\theta}{V + X_q|I_a|\sin\theta}\right)$$

(b) Compute the load angle δ and the per phase excitation voltage E when the generator delivers rated MVA, 0.8 power factor lagging to an infinite bus bar of 34.64-kV line-to-line voltage.

(c) The generator excitation voltage is kept constant at the value found in part (b). Use *MATLAB* to obtain a plot of the power angle curve, i.e., equation (3.26) over a range of $d = 0 : .05 : 180°$. Use the command $[P_{max}, k]$ = max(P); $d_{max} = d(k)$, to obtain the steady-state maximum power P_{max} and the corresponding power angle d_{max}.

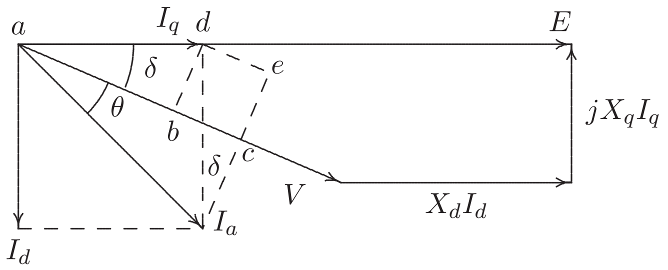


FIGURE 17
Phasor diagram of a salient-pole generator for Problem 3.4.

(a) Form the phasor diagram shown in Figure 17, we have

$$\begin{aligned}|V|\sin\delta &= X_qI_q \\ &= X_q|I_a|\cos(\theta+\delta) \\ &= X_q|I_a|(\cos\theta\cos\delta - \sin\theta\sin\delta)\end{aligned}$$

or

$$\frac{\sin\delta}{\cos\delta} = \frac{X_q|I_a|\cos\theta}{V + X_q|I_a|\sin\theta}$$

or

$$\delta = \tan^{-1} \frac{X_q|I_a|\cos\theta}{V + X_q|I_a|\sin\theta}$$

(b)

$$V_\phi = \frac{34.64}{\sqrt{3}} = 20 \text{ kV}$$

(a) For 60 MVA, 0.8 power factor lagging, $S = 60\angle 36.87^\circ$ MVA

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{60000\angle{-36.87^\circ}}{(3)(20)} = 1000\angle{-36.87^\circ} \text{ A}$$

$$\delta = \tan^{-1} \frac{(9.333)(1000)(0.8)}{20000 + (9.333)(1000)(0.6)} = 16.26^\circ$$

The magnitude of the no-load generated emf per phase is given by

$$\begin{aligned} |E| &= |V|\cos\delta + X_d|I_a|\sin(\theta + \delta) \\ &= 20\cos 16.26^\circ + (13.5)(1000)(10^{-3})\sin 53.13^\circ = 30 \text{ kV} \end{aligned}$$

(c) We use the following commands

```
V = 20000; Xd = 13.5; Xq = 9.333;
theta=acos(0.8);
Ia = 20E06/20000;
delta = atan(Xq*Ia*cos(theta)/(V + Xq*Ia*sin(theta)));
deltadg=delta*180/pi;
E = V*cos(delta)+Xd*Ia*sin(theta+delta);
E_KV = E/1000;  % Excitaiton voltage in kV
fprintf('Power angle = %g Degree \n', deltadg)
fprintf('E = %g kV \n\n', E_KV)
deltadg = (0:.25:180)';
delta=deltadg*pi/180;
P=3*E*V/Xd*sin(delta)+3*V^2*(Xd-Xq)/(2*Xd*Xq)*sin(2*delta);
P = P/1000000;      % Power in MW
plot(deltadg, P), grid
xlabel('Delta - Degree'), ylabel('P - MW')
[Pmax, k]=max(P); delmax=deltadg(k);
fprintf('Max power = %g MW',Pmax)
fprintf(' at power angle %g degree \n', delmax)
```

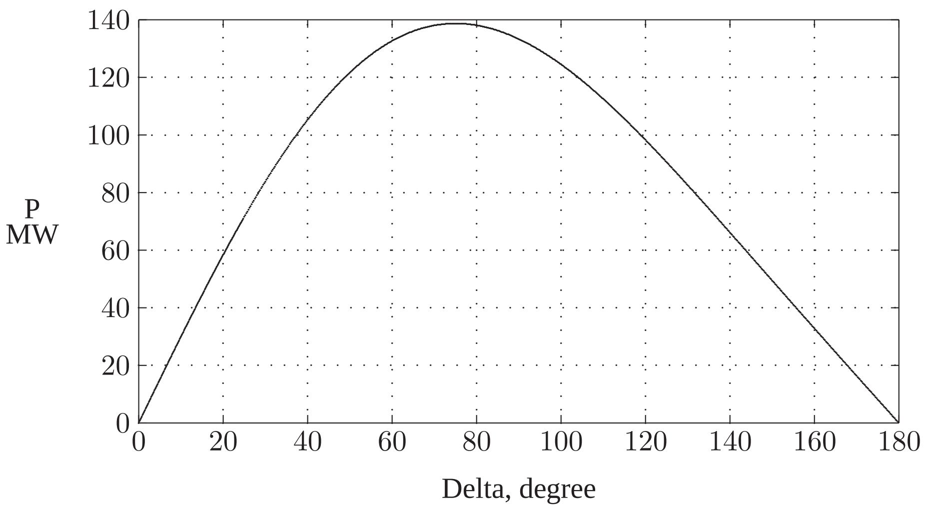


FIGURE 18
Power angle curve for Problem 3.4.

The result is

```
Power angle = 16.2598 Degree
E = 30 kV
Max power = 138.712 MW at power angle 75 degree
```

3.5. A 150-kVA, 2400/240-V single-phase transformer has the parameters as shown in Figure 19.

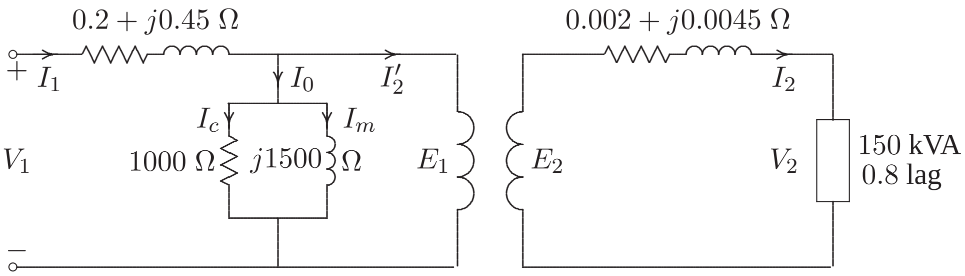


FIGURE 19
Transformer circuit for Problem 3.5.

(a) Determine the equivalent circuit referred to the high-voltage side.
(b) Find the primary voltage and voltage regulation when transformer is operating at full load 0.8 power factor lagging and 240 V.

(c) Find the primary voltage and voltage regulation when the transformer is operating at full-load 0.8 power factor leading.
(d) Verify your answers by running the **trans** program in *MATLAB* and obtain the transformer efficiency curve.

(a) Referring the secondary impedance to the primary side, the transformer equivalent impedance referred to the high voltage-side is

$$Z_{e1} = 0.2 + j0.45 + \left(\frac{2400}{240}\right)^2 (0.002 + j0.0045) = 0.4 + j0.9 \ \ \Omega$$

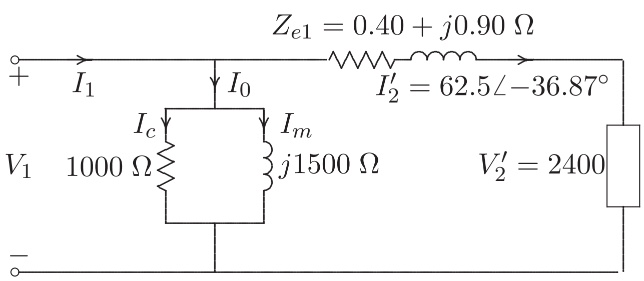


FIGURE 20
Equivalent circuit referred to the high-voltage side.

(b) At full-load 0.8 power factor lagging $S = 150\angle 36.87°$ kVA, and the referred primary current is

$$I_2{}' = \frac{S^*}{V^*} = \frac{150000\angle -36.87°}{2400} = 62.5\angle -36.87° \ \ \text{A}$$

$$V_1 = 2400 + (0.4 + j0.9)(62.5\angle -36.87°) = 2453.933\angle 0.7° \ \ \text{V}$$

The voltage regulation is

$$V.R. = \frac{2453.933 - 2400}{2400} \times 100 = 2.247\%$$

(c) At full-load 0.8 power factor leading $S = 150\angle -36.87°$ kVA, and the referred primary current is

$$I_2{}' = \frac{S^*}{V^*} = \frac{150000\angle +36.87°}{2400} = 62.5\angle +36.87° \ \ \text{A}$$

$$V_1 = 2400 + (0.4 + j0.9)(62.5\angle +36.87°) = 2387.004\angle 1.44° \ \ \text{V}$$

The voltage regulation is

$$V.R. = \frac{2453.933 - 2400}{2400} \times 100 = -0.541\%$$

(d) Entering **trans** at the *MATLAB* prompt, result in

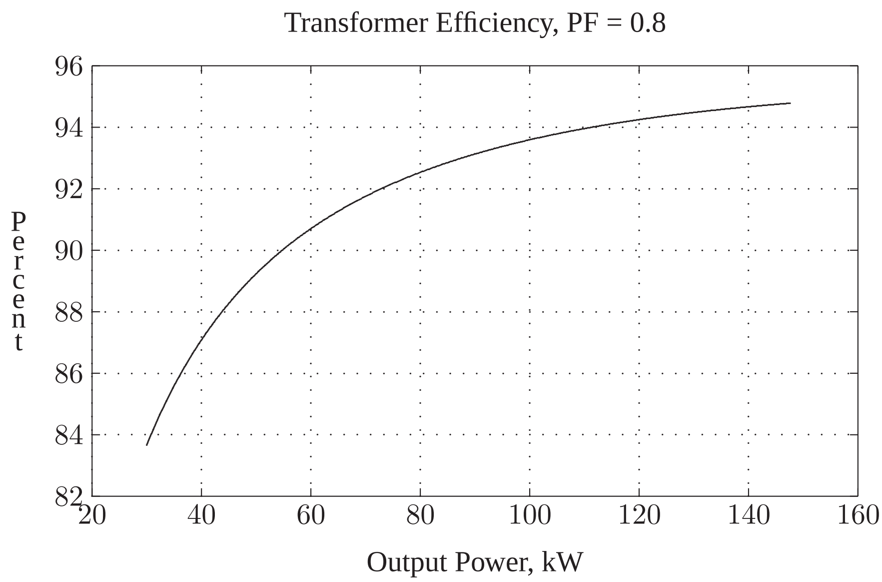


FIGURE 21
Efficiency curve of Problem 3.5.

```
                    TRANSFORMER ANALYSIS

Type of parameters for input                Select
----------------------------                ------
To obtain equivalent circuit from tests        1
To input individual winding impedances         2
To input transformer equivalent impedance      3
To quit                                        0
Select number of menu --> 2

Enter Transformer rated power in kVA, S = 150
Enter rated low voltage in volts  = 240
Enter rated high voltage in volts = 2400
```

```
Enter LV winding series impedance R1+j*X1 in ohm=0.002+j*.0045
Enter HV winding series impedance R2+j*X2 in ohm=0.2+j*0.45
Enter 'lv' within quotes for low side shunt branch or
enter 'hv' within quotes for high side shunt branch -> 'hv'
Shunt resistance in ohm (if neglected enter inf) = 1000
Shunt reactance in ohm (if neglected enter inf) = 1500

Shunt branch ref. to LV side      Shunt branch ref. to HV side
    Rc =       10.000 ohm             Rc =     1000.000 ohm
    Xm =       15.000 ohm             Xm =     1500.000 ohm

Series branch ref. to LV side     Series branch ref.to HV side
    Ze =  0.0040 + j  0.0090 ohm    Ze =  0.4000 + j  0.90 ohm

Hit return to continue
    Enter load kVA, S2 = 150
    Enter load power factor, pf = 0.8
    Enter 'lg' within quotes for lagging pf
    or 'ld' within quotes for leading pf -> 'lg'
    Enter load terminal voltage in volt, V2 = 240
    Secondary load voltage  =    240.000 V
    Secondary load current  =    625.000 A  at -36.87 degrees
    Current ref. to primary =     62.500 A  at -36.87 degrees
    Primary no-load current =      2.949 A  at -33.69 degrees
    Primary input current   =     65.445 A  at -36.73 degrees
    Primary input voltage   =   2453.933 V  at  0.70  degrees
    Voltage regulation      =      2.247 percent
    Transformer efficiency  =     94.249 percent
    Maximum efficiency is 95.238 %occurs at 288 kVA with 0.8pf
```

The efficiency curve is shown in Figure 21. The analysis is repeated for the full-load kVA, 0.8 power factor leading.

3.6. A 60-kVA, 4800/2400-V single-phase transformer gave the following test results:

1. Rated voltage is applied to the low voltage winding and the high voltage winding is open-circuited. Under this condition, the current into the low voltage winding is 2.4 A and the power taken from the 2400 V source is 3456 W.

2. A reduced voltage of 1250 V is applied to the high voltage winding and the low voltage winding is short-circuited. Under this condition, the current flowing into the high voltage winding is 12.5 A and the power taken from the 1250 V source is 4375 W.

(a) Determine parameters of the equivalent circuit referred to the high voltage side.

(b) Determine voltage regulation and efficiency when transformer is operating at full-load, 0.8 power factor lagging, and a terminal voltage of 2400 V.

(c) What is the load kVA for maximum efficiency and the maximum efficiency at 0.8 power factor?

(d) Determine the efficiency when transformer is operating at 3/4 full-load, 0.8 power factor lagging, and a terminal voltage of 2400 V.

(e) Verify your answers by running the **trans** program in *MATLAB* and obtain the transformer efficiency curve.

(a) From the no-load test data

$$R_{cLV} = \frac{|V_2|^2}{P_0} = \frac{(2400)^2}{3456} = 1666.667\ \Omega$$

$$I_c = \frac{|V_2|}{R_{cLV}} = \frac{2400}{1666.67} = 1.44 \ \ \text{A}$$

$$I_m = \sqrt{I_0^2 - I_c^2} = \sqrt{(2.4)^2 - (1.44)^2} = 1.92 \ \ \text{A}$$

$$X_{mLV} = \frac{|V_2|}{I_m} = \frac{2400}{1.92} = 1250 \ \ \Omega$$

The shunt parameters referred to the high-voltage side are

$$R_{CHV} = \left(\frac{4800}{2400}\right)^2 1666.667 = 6666.667\ \Omega$$

$$X_{mHV} = \left(\frac{4800}{2400}\right)^2 1250 = 5000\ \Omega$$

From the short-circuit test data

$$Z_{e1} = \frac{V_{sc}}{I_{sc}} = \frac{1250}{12.5} = 100\ \Omega$$

$$R_{e1} = \frac{P_{sc}}{(I_{sc})^2} = \frac{4375}{(12.5)^2} = 28\ \Omega$$

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = \sqrt{(100)^2 - (28)^2} = 96 \;\; \Omega$$

(b) At full-load 0.8 power factor lagging $S = 60\angle 36.87^\circ$ kVA, and the referred primary current is

$$I_2{}' = \frac{S^*}{V^*} = \frac{60000\angle -36.87^\circ}{4800} = 12.5\angle -36.87^\circ \;\; \text{A}$$

$$V_1 = 4800 + (28 + j96)(12.5\angle -36.87^\circ) = 5848.29\angle 7.368^\circ \;\; \text{V}$$

The voltage regulation is

$$V.R. = \frac{5848.29 - 4800}{4800} \times 100 = 21.839\%$$

Efficiency at full-load, 0.8 power factor is

$$\eta = \frac{(60)(0.8)}{(60)(0.8) + 3.456 + 4.375} \times 100 = 85.974 \;\%$$

(c) At Maximum efficiency $P_c = P_{cu}$, and we have

$$\begin{aligned} P_{cu} &\propto S_{fl}^2 \\ P_c &\propto S_{max\eta}^2 \end{aligned}$$

Therefore, the load kVA at maximum efficiency is

$$S_{max\eta} = \sqrt{\frac{P_c}{P_{cu}}} S_{fl} = \sqrt{\frac{3456}{4375}}\, 60 = 53.327 \;\; \text{kVA}$$

and the maximum efficiency at 0.8 power factor is

$$\eta = \frac{(53.327)(0.8)}{(53.327)(0.8) + 3.456 + 3.456} \times 100 = 86.057 \;\%$$

(d) At $3/4$ full-load, the copper loss is

$$P_{cu3/4} = (3/4)^2(4375) = 2460.9375 \;\; \text{W}$$

Therefore, the efficiency at $3/4$ full-load, 0.8 power factor is

$$\eta = \frac{(45)(0.8)}{(45)(0.8) + 3.456 + 2.4609} \times 100 = 85.884 \;\%$$

(e) The commands **trans** display the following menu

```
                    TRANSFORMER ANALYSIS

Type of parameters for input                Select
----------------------------                ------
To obtain equivalent circuit from tests        1
To input individual winding impedances         2
To input transformer equivalent impedance      3
To quit                                        0
Select number of menu --> 1

Enter Transformer rated power in kVA, S = 60
Enter rated low voltage in volts  = 2400
Enter rated high voltage in volts = 4800

Open circuit test data
----------------------
Enter 'lv' within quotes for data referred to low side or
enter 'hv' within quotes for data referred to high side ->'lv'
Enter input voltage in volts, Vo = 2400
Enter no-load current in Amp, Io = 2.4
Enter no-load input power in Watt, Po = 3456

Short circuit test data
-----------------------
Enter 'lv' within quotes for data referred to low side or
enter 'hv' within quotes for data referred to high side ->'hv'
Enter reduced input voltage in volts, Vsc = 1250
Enter input current in Amp, Isc = 12.5
Enter input power in Watt, Psc = 4375

Shunt branch ref. to LV side      Shunt branch ref. to HV side
    Rc =     1666.667 ohm             Rc =     6666.667 ohm
    Xm =     1250.000 ohm             Xm =     5000.000 ohm

Series branch ref. to LV side     Series branch ref.to HV side
    Ze = 7.0000 + j  24.0000 ohm    Ze = 28.000 + j 96.000 ohm
Hit return to continue
    Enter load KVA, S2 = 60
    Enter load power factor, pf = 0.8
    Enter 'lg' within quotes for lagging pf
    or 'ld' within quotes for leading pf -> 'lg'
    Enter load terminal voltage in volt, V2 = 2400
    Secondary load voltage  =   2400.000 V
    Secondary load current  =     25.000 A  at -36.87 degrees
```

```
Current ref. to primary =     12.500 A  at -36.87 degrees
Primary no-load current =      1.462 A  at -53.13 degrees
Primary input current   =     13.910 A  at -38.56 degrees
Primary input voltage   =   5848.290 V  at  7.37  degrees
Voltage regulation      =     21.839 percent
Transformer efficiency  =     85.974 percent
Maximum efficiency is 86.057%occurs at 53.33kVA with 0.8pf
```

The efficiency curve is shown in Figure 22. The analysis is repeated for the $3/4$ full-load kVA, 0.8 power factor lagging.

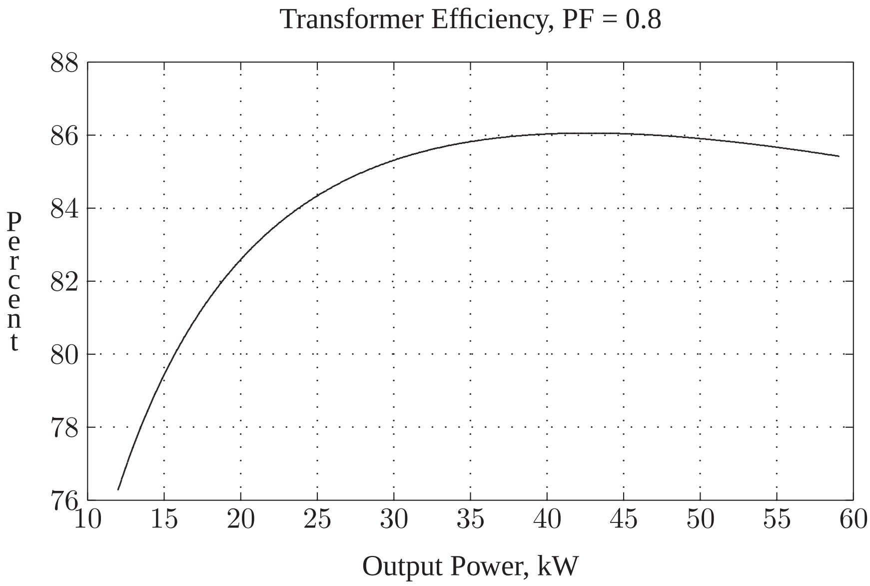


FIGURE 22
Efficiency curve of Problem 3.6.

3.7. A two-winding transformer rated at 9-kVA, 120/90-V, 60-HZ has a core loss of 200 W and a full-load copper loss of 500 W.

(a) The above transformer is to be connected as an auto transformer to supply a load at 120 V from 210 V source. What kVA load can be supplied without exceeding the current rating of the windings? (For this part assume an ideal transformer.)

(b) Find the efficiency with the kVA loading of part (a) and 0.8 power factor.

The two-winding transformer rated currents are:

$$I_{LV} = \frac{9000}{90} = 100 \text{ A}$$

$$I_{HV} = \frac{9000}{120} = 75 \text{ A}$$

The auto transformer connection is as shown in Figure 23.

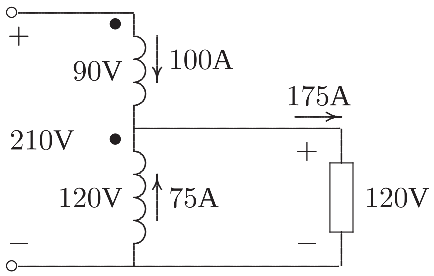


FIGURE 23
Auto transformer connection for Problem 3.7.

(a) With windings carrying rated currents, the auto transformer rating is

$$S = (120)(175)(10^{-3}) = 21 \text{ kVA}$$

(b) Since the windings are subjected to the same rated voltages and currents as the two-winding transformer, the auto transformer copper loss and the core loss at the rated values are the same as the two-winding transformer. Therefore, the auto transformer efficiency at 0.8 power factor is

$$\eta = \frac{(21000)(0.8)}{(21000)(0.8) + 200 + 500} \times 100 = 96 \text{ \%}$$

3.8. Three identical 9-MVA, 7.2-kV/4.16-kV, single-phase transformers are connected in Y on the high-voltage side and Δ on the low voltage side. The equivalent series impedance of each transformer referred to the high-voltage side is $0.12 + j0.82\ \Omega$ per phase. The transformer supplies a balanced three-phase load of 18 MVA, 0.8 power factor lagging at 4.16 kV. Determine the line-to-line voltage at the high-voltage terminals of the transformer.

The per phase equivalent circuit is shown in Figure 24. The load complex power is

$$S = 18000\angle 36.87^\circ \text{ kVA}$$

$$I_1 = \frac{18000\angle -36.87^\circ}{(3)(7.2)} = 833.333\angle -36.87^\circ \text{ A}$$

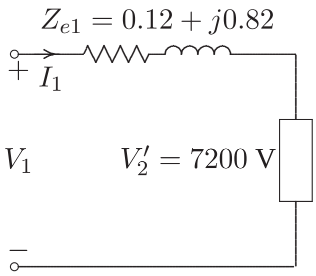


FIGURE 24
The per phase equivalent circuit of Problem 3.8.

$$V_1 = 7.2\angle 0^\circ + (0.12 + j0.82)(833.333\angle{-36.87^\circ})(10^{-3}) = 7.7054\angle 3.62^\circ \text{ kV}$$

Therefore, the magnitude of the primary line-to-line supply voltage is $V_{1LL} = \sqrt{3}(7.7054) = 13.346$ kV.

3.9. A 400-MVA, 240-kV/24-KV, three-phase Y-Δ transformer has an equivalent series impedance of $1.2 + j6\ \Omega$ per phase referred to the high-voltage side. The transformer is supplying a three-phase load of 400-MVA, 0.8 power factor lagging at a terminal voltage of 24 kV (line to line) on its low-voltage side. The primary is supplied from a feeder with an impedance of $0.6 + j1.2\ \Omega$ per phase. Determine the line-to-line voltage at the high-voltage terminals of the transformer and the sending-end of the feeder.

The per phase equivalent circuit referred to the primary is is shown in Figure 25. The load complex power is

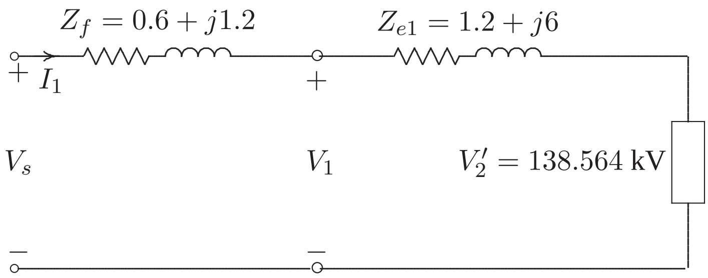


FIGURE 25
The per phase equivalent circuit for Problem 3.9.

$$S = 400000\angle 36.87^\circ \text{ kVA}$$

and the referred secondary voltage per phase is

$$V_2^{'} = \frac{240}{\sqrt{3}} = 138.564 \ \text{kV}$$

$$I_1 = \frac{400000\angle{-36.87^\circ}}{(3)(138.564)} = 962.25\angle{-36.87^\circ} \ \text{A}$$

$$V_1 = 138.564\angle 0^\circ + (1.2 + j6)(962.25\angle{-36.87^\circ})(10^{-3}) = 143\angle 1.57^\circ \ \text{kV}$$

Therefore, the line-to-line voltage magnitude at the high voltage terminal of the transformer is $V_{1LL} = \sqrt{3}(143) = 247.69$ kV.

$$V_s = 138.564\angle 0^\circ + (1.8 + j7.2)(962.25\angle{-36.87^\circ})(10^{-3}) = 144.177\angle 1.79^\circ \ \text{kV}$$

Therefore, the line-to-line voltage magnitude at the sending end of the feeder is $V_{1LL} = \sqrt{3}(143) = 249.72$ kV.

3.10. In Problem 3.9, with transformer rated values as base quantities, express all impedances in per unit. Working with per-unit values, determine the line-to-line voltage at the high-voltage terminals of the transformer and the sending-end of the feeder.

With the transformer rated MVA as base, the HV-side base impedance is

$$Z_B = \frac{(240)^2}{400} = 144 \ \Omega$$

The transformer and the feeder impedances in per unit are

$$\begin{aligned} Z_e &= \frac{(1.2 + j6)}{144} = 0.008333 + j0.04166 \text{ pu} \\ Z_f &= \frac{(0.6 + j1.2)}{144} = 0.004166 + j0.00833 \text{ pu} \end{aligned}$$

$$I = \frac{1\angle{-36.87^\circ}}{1\angle 0^\circ} = 1\angle{-36.87^\circ} \text{ pu}$$

$$V_1 = 1.0\angle 0^\circ + (0.008333 + j0.04166)(1\angle{-36.87^\circ} = 1.03205\angle 1.57^\circ \text{ pu}$$

Therefore, the line-to-line voltage magnitude at the high-voltage terminal of the transformer is $(1.03205)(240) = 247.69$ kV

$$V_s = 1.0\angle 0^\circ + (0.0125 + j0.05)(1\angle{-36.87^\circ} = 1.0405\angle 1.79^\circ \text{ pu}$$

Therefore, the line-to-line voltage magnitude at the high voltage terminal of the transformer is $(1.0405)(240) = 249.72$ kV

3.11. A three-phase, Y-connected, 75-MVA, 27-kV synchronous generator has a synchronous reactance of $9.0\ \Omega$ per phase. Using rated MVA and voltage as base values, determine the per unit reactance. Then refer this per unit value to a 100-MVA, 30-kV base.

The base impedance is

$$Z_B = \frac{(KV_B)^2}{MVA_B} = \frac{(27)^2}{75} = 9.72\ \Omega$$

$$X_{pu} = \frac{9}{9.72} = 0.926 \text{ pu}$$

The generator reactance on a 100-MVA, 30-kV base is

$$X_{pu_{new}} = 0.926\left(\frac{100}{75}\right)\left(\frac{27}{30}\right)^2 = 1.0 \text{ pu}$$

3.12. A 40-MVA, 20-kV/400-kV, single-phase transformer has the following series impedances:

$Z_1 = 0.9 + j1.8\ \Omega$ and $Z_2 = 128 + j288\ \Omega$

Using the transformer rating as base, determine the per unit impedance of the transformer from the ohmic value referred to the low-voltage side. Compute the per unit impedance using the ohmic value referred to the high-voltage side.

The transformer equivalent impedance referred to the low-voltage side is

$$Z_{e1} = 0.9 + j1.8 + \left(\frac{20}{400}\right)^2 (128 + j288) = 1.22 + j2.52\ \Omega$$

The low-voltage base impedance is

$$Z_{B1} = \frac{(20)^2}{40} = 10\ \Omega$$

$$Z_{pu_1} = \frac{1.22 + j2.52}{10} = 0.122 + j0.252 \text{ pu}$$

The transformer equivalent impedance referred to the high-voltage side is

$$Z_{e2} = \left(\frac{400}{20}\right)^2 (0.9 + j1.8) + (128 + j288) = 488 + j1008\ \Omega$$

The high-voltage base impedance is

$$Z_{B2} = \frac{(400)^2}{40} = 4000\ \Omega$$

$$Z_{pu_2} = \frac{488 + j1008}{4000} = 0.122 + j0.252\ \text{ pu}$$

We note that the transformer per unit impedance has the same value regardless of whether it is referred to the primary or the secondary side.

3.13. Draw an impedance diagram for the electric power system shown in Figure 26 showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

G_1 :	90 MVA	20 kV	$X = 9\%$
T_1 :	80 MVA	20/200 kV	$X = 16\%$
T_2 :	80 MVA	200/20 kV	$X = 20\%$
G_2 :	90 MVA	18 kV	$X = 9\%$
Line:		200 kV	$X = 120\ \Omega$
Load:		200 kV	$S = 48$ MW $+j64$ Mvar

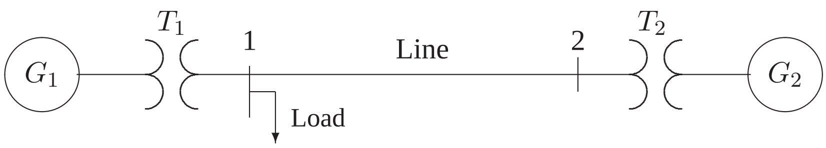


FIGURE 26
One-line diagram for Problem 3.13

The base voltage V_{BG1}on the LV side of T_1 is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20\left(\frac{200}{20}\right) = 200 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{BG2} = 200\left(\frac{20}{200}\right) = 20 \quad \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69)and (3.70) are

$$G\text{:} \qquad X = 0.09\left(\frac{100}{90}\right) = 0.10 \ \text{ pu}$$

$$T_1\text{:} \qquad X = 0.16\left(\frac{100}{80}\right) = 0.20 \ \text{ pu}$$

$$T_2\text{:} \qquad X = 0.20\left(\frac{100}{80}\right) = 0.25 \ \text{ pu}$$

$$G_2\text{:} \qquad X = 0.09\left(\frac{100}{90}\right)\left(\frac{18}{20}\right)^2 = 0.081 \ \text{ pu}$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \ \Omega$$

The per unit line reactance is

$$Line\text{:} \qquad X = \left(\frac{120}{400}\right) = 0.30 \ \text{ pu}$$

The load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(200)^2}{48 - j64} = 300 + j400 \ \ \Omega$$

The load impedance in per unit is

$$Z_{L(pu)} = \frac{300 + j400}{400} = 0.75 + j1.0 \ \text{ pu}$$

The per unit equivalent circuit is shown in Figure 27.

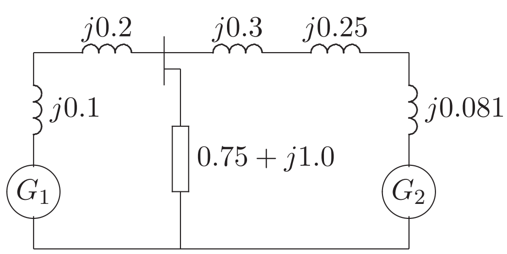


FIGURE 27
Per unit impedance diagram for Problem 3.11.

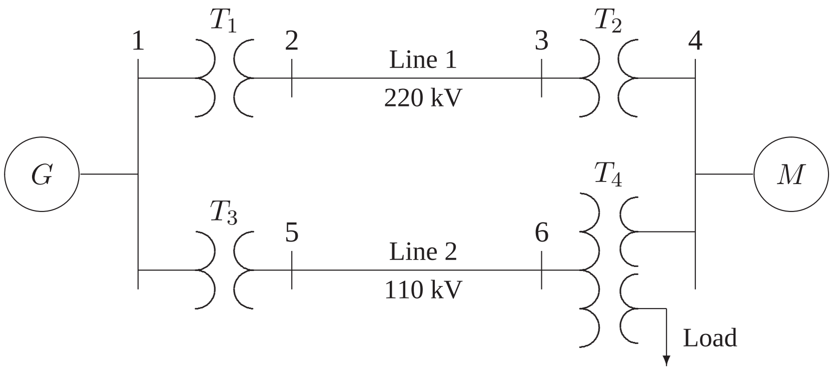


FIGURE 28
One-line diagram for Problem 3.14

3.14. The one-line diagram of a power system is shown in Figure 28.

The three-phase power and line-line ratings are given below.

G:	80 MVA	22 kV	$X = 24\%$
T_1:	50 MVA	22/220 kV	$X = 10\%$
T_2:	40 MVA	220/22 kV	$X = 6.0\%$
T_3:	40 MVA	22/110 kV	$X = 6.4\%$
Line 1:		220 kV	$X = 121\ \Omega$
Line 2:		110 kV	$X = 42.35\ \Omega$
M:	68.85 MVA	20 kV	$X = 22.5\%$
Load:	10 Mvar	4 kV	Δ-connected capacitors

The three-phase ratings of the three-phase transformer are

Primary:	Y-connected	40MVA, 110 kV
Secondary:	Y-connected	40 MVA, 22 kV
Tertiary:	Δ-connected	15 MVA, 4 kV

The per phase measured reactances at the terminal of a winding with the second one short-circuited and the third open-circuited are

$Z_{PS} = 9.6\%$	40 MVA, 110 kV / 22 kV
$Z_{PT} = 7.2\%$	40 MVA, 110 kV / 4 kV
$Z_{ST} = 12\%$	40 MVA, 22 kV / 4 kV

Obtain the T-circuit equivalent impedances of the three-winding transformer to the common MVA base. Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 22 kV as the voltage base for generator.

The base voltage V_{B1} on the LV side of T_1 is 22 kV. Hence the base on its HV side is

$$V_{B2} = 22\left(\frac{220}{22}\right) = 220 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B3} = 220$ kV, and on its LV side at

$$V_{B4} = 220\left(\frac{22}{220}\right) = 22 \quad \text{kV}$$

Similarly, the voltage base at buses 5 and 6 are

$$V_{B5} = V_{B6} = 22\left(\frac{110}{22}\right) = 110 \quad \text{kV}$$

Voltage base for the tertiary side of T_4 is

$$V_{BT} = 110\left(\frac{4}{110}\right) = 4 \quad \text{kV}$$

The per unit impedances on a 100 MVA base are:

$$\begin{aligned}
G: &\qquad X = 0.24\left(\frac{100}{80}\right) = 0.30 \ \text{pu} \\
T_1: &\qquad X = 0.10\left(\frac{100}{50}\right) = 0.20 \ \text{pu} \\
T_2: &\qquad X = 0.06\left(\frac{100}{40}\right) = 0.15 \ \text{pu} \\
T_3: &\qquad X = 0.064\left(\frac{100}{40}\right) = 0.16 \quad \text{pu}
\end{aligned}$$

The motor reactance is expressed on its nameplate rating of 68.85 MVA, and 20 kV. However, the base voltage at bus 4 for the motor is 22 kV, therefore

$$M: \qquad X = 0.225\left(\frac{100}{68.85}\right)\left(\frac{20}{22}\right)^2 = 0.27 \ \text{pu}$$

Impedance bases for lines 1 and 2 are

$$\begin{aligned}
Z_{B2} &= \frac{(220)^2}{100} = 484 \ \Omega \\
Z_{B5} &= \frac{(110)^2}{100} = 121 \ \Omega
\end{aligned}$$

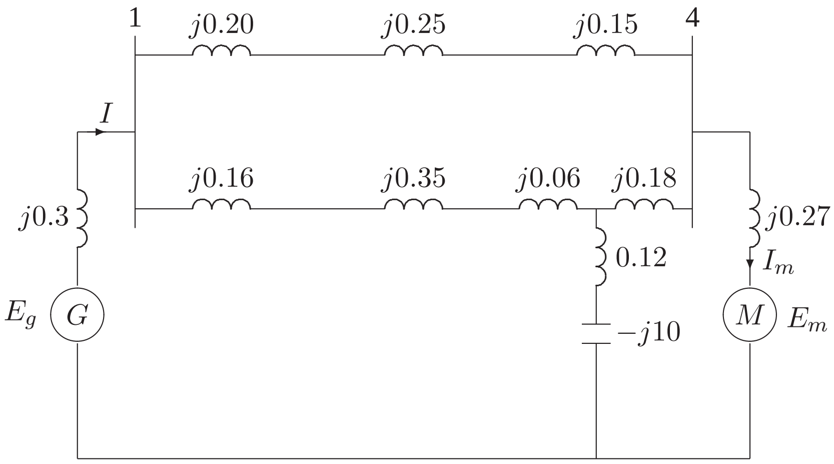


FIGURE 29
Per unit impedance diagram for Problem 3.14.

Line 1 and 2 per unit reactances are

$$Line_1: \quad X = \left(\frac{121}{484}\right) = 0.25 \text{ pu}$$
$$Line_2: \quad X = \left(\frac{42.35}{121}\right) = 0.35 \text{ pu}$$

The load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(4)^2}{j10} = -j1.6 \ \Omega$$

The base impedance for the load is

$$Z_{BT} = \frac{(4)^2}{100} = 0.16 \ \Omega$$

Therefore, the load impedance in per unit is

$$Z_{L(pu)} = \frac{-j1.6}{0.16} = -j10 \text{ pu}$$

The three-winding impedances on a 100 MVA base are

$$Z_{PS} = 0.096\left(\frac{100}{40}\right) = 0.24 \text{ pu}$$
$$Z_{PT} = 0.072\left(\frac{100}{40}\right) = 0.18 \text{ pu}$$
$$Z_{ST} = 0.120\left(\frac{100}{40}\right) = 0.30 \text{ pu}$$

The equivalent T circuit impedances are

$$Z_P = \frac{1}{2}(j0.24 + j0.18 - j0.30) = j0.06 \text{ pu}$$
$$Z_S = \frac{1}{2}(j0.24 + j0.30 - j0.18) = j0.18 \text{ pu}$$
$$Z_T = \frac{1}{2}(j0.18 + j0.30 - j0.24) = j0.12 \text{ pu}$$

The per unit equivalent circuit is shown in Figure 29.

3.15. The three-phase power and line-line ratings of the electric power system shown in Figure 30 are given below.

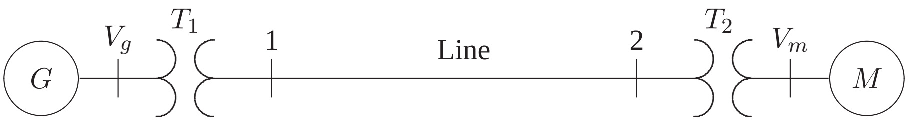


FIGURE 30
One-line diagram for Problem 3.15

G_1 :	60 MVA	20 kV	$X = 9\%$
T_1 :	50 MVA	20/200 kV	$X = 10\%$
T_2 :	50 MVA	200/20 kV	$X = 10\%$
M :	43.2 MVA	18 kV	$X = 8\%$
Line:		200 kV	$Z = 120 + j200\ \Omega$

(a) Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

(b) The motor is drawing 45 MVA, 0.80 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

The base voltage V_{BG1}on the LV side of T_1 is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20\left(\frac{200}{20}\right) = 200 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{Bm} = 200\left(\frac{20}{200}\right) = 20 \quad \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69)and (3.70) are

$$
\begin{aligned}
G: &\qquad X = 0.09\left(\frac{100}{60}\right) = 0.15 \ \text{ pu}\\
T_1: &\qquad X = 0.10\left(\frac{100}{50}\right) = 0.20 \ \text{ pu}\\
T_2: &\qquad X = 0.10\left(\frac{100}{50}\right) = 0.20 \ \text{ pu}\\
M: &\qquad X = 0.08\left(\frac{100}{43.2}\right)\left(\frac{18}{20}\right)^2 = 0.15 \ \text{ pu}
\end{aligned}
$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \ \Omega$$

The per unit line impedance is

$$Line: \qquad Z_{line} = (\frac{120 + j200}{400}) = 0.30 + j0.5 \ \text{ pu}$$

The per unit equivalent circuit is shown in Figure 31.

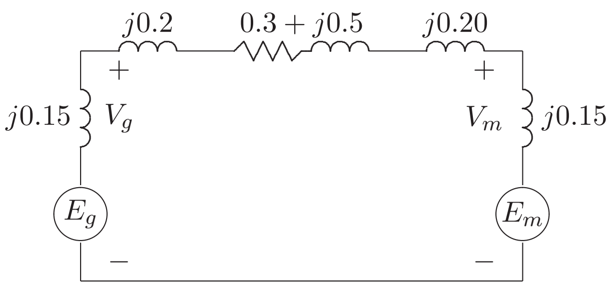


FIGURE 31
Per unit impedance diagram for Problem 3.15.

(b) The motor complex power in per unit is

$$S_m = \frac{45\angle 36.87^\circ}{100} = 0.45\angle 36.87^\circ \ \text{ pu}$$

and the motor terminal voltage is

$$V_m = \frac{18\angle 0^\circ}{20} = 0.90\angle 0^\circ \ \text{ pu}$$

$$I = \frac{0.45\angle{-36.87^\circ}}{0.90\angle 0^\circ} = 0.5\angle{-36.87^\circ} \text{ pu}$$

$$V_g = 0.90\angle 0^\circ + (0.3 + j0.9)(0.5\angle{-36.87^\circ} = 1.31795\angle 11.82^\circ \text{ pu}$$

Thus, the generator line-to-line terminal voltage is

$$V_g = (1.31795)(20) = 26.359 \text{ kV}$$

$$E_g = 0.90\angle 0^\circ + (0.3 + j1.05)(0.5\angle{-36.87^\circ} = 1.375\angle 13.88^\circ \text{ pu}$$

Thus, the generator line-to-line internal emf is

$$E_g = (1.375)(20) = 27.5 \text{ kV}$$

3.16. The one-line diagram of a three-phase power system is as shown in Figure 32. Impedances are marked in per unit on a 100-MVA, 400-kV base. The load at bus 2 is $S_2 = 15.93$ MW $-j33.4$ Mvar, and at bus 3 is $S_3 = 77$ MW $+j14$ Mvar. It is required to hold the voltage at bus 3 at $400\angle 0^\circ$ kV. Working in per unit, determine the voltage at buses 2 and 1.

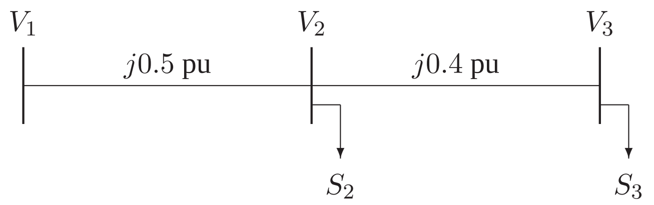


FIGURE 32
One-line diagram for Problem 3.16

$$S_2 = 15.93 \text{ MW} - j33.4 \text{ Mvar} = 0.1593 - j0.334 \text{ pu}$$
$$S_3 = 77.00 \text{ MW} + j14.0 \text{ Mvar} = 0.7700 + j0.140 \text{ pu}$$
$$V_3 = \frac{400\angle 0^\circ}{400} = 1.0\angle 0^\circ \text{ pu}$$
$$I_3 = \frac{S_3^*}{V_3^*} = \frac{0.77 - j0.14}{1.0\angle 0^\circ} = 0.77 - j0.14 \text{ pu}$$
$$V_2 = 1.0\angle 0^\circ + (j0.4)(0.77 - j0.14) = 1.1\angle 16.26^\circ \text{ pu}$$

Therefore, the line-to-line voltage at bus 2 is

$$
\begin{aligned}
V_2 &= (400)(1.1) = 440 \ \text{kV} \\
I_2 &= \frac{S_2^*}{V_2^*} = \frac{0.1593 + j0.334}{1.1\angle{-16.26^\circ}} = 0.054 + j0.332 \ \text{pu} \\
I_{12} &= (0.77 - j0.14) + (0.054 + j0.332) = 0.824 + j0.192 \ \text{pu} \\
V_1 &= 1.1\angle 16.26^\circ + (j0.5)(0.824 + j0.192) = 1.2\angle 36.87^\circ \ \text{pu}
\end{aligned}
$$

Therefore, the line-to-line voltage at bus 1 is

$$V_1 = (400)(1.2) = 480 \ \text{kV}$$

3.17. The one-line diagram of a three-phase power system is as shown in Figure 33. The transformer reactance is 20 percent on a base of 100-MVA, 23/115-kV and the line impedance is $Z = j66.125\Omega$. The load at bus 2 is $S_2 = 184.8$ MW $+j6.6$ Mar, and at bus 3 is $S_3 = 0$ MW $+j20$ Mar. It is required to hold the voltage at bus 3 at $115\angle 0^\circ$ kV. Working in per unit, determine the voltage at buses 2 and 1.

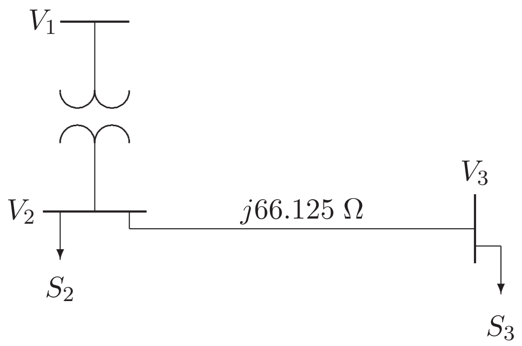


FIGURE 33
One-line diagram for Problem 3.17

$$
\begin{aligned}
S_2 &= 184.8 \text{ MW} + j6.6 \text{ Mvar} = 1.848 + j0.066 \ \text{pu} \\
S_3 &= \quad 0 \text{ MW} + j20.0 \text{ Mvar} = \quad 0 + j0.20 \ \text{pu}
\end{aligned}
$$

$$
\begin{aligned}
V_3 &= \frac{115\angle 0^\circ}{115} = 1.0\angle 0^\circ \ \text{pu} \\
I_3 &= \frac{S_3^*}{V_3^*} = \frac{-j0.2}{1.0\angle 0^\circ} = -j0.2 \ \text{pu} \\
V_2 &= 1.0\angle 0^\circ + (j0.5)(-j0.2) = 1.1\angle 0^\circ \ \text{pu}
\end{aligned}
$$

Therefore, the line-to-line voltage at bus 2 is

$$V_2 = (115)(1.1) = 126.5 \text{ kV}$$

$$I_2 = \frac{S_2^*}{V_2^*} = \frac{1.848 - j0.066}{1.1\angle 0^\circ} = 1.68 - j0.06 \text{ pu}$$
$$I_{12} = (1.68 - j0.06) + (-j0.2) = 1.68 - j0.26 \text{ pu}$$
$$V_1 = 1.1\angle 0^\circ + (j0.2)(1.68 - j0.26) = 1.2\angle 16.26^\circ \text{ pu}$$

Therefore, the line-to-line voltage at bus 1 is

$$V_1 = (23)(1.2) = 27.6 \text{ kV}$$

CHAPTER 4 PROBLEMS

4.1. A solid cylindrical aluminum conductor 25 km long has an area of 336,400 circular mils. Obtain the conductor resistance at (a) $20°$C and (b) $50°$C. The resistivity of aluminum at $20°$C is 2.8×10^{-8} Ω-m.

(a)

$$d = \sqrt{336400} = 580 \text{ mil} = (580)(10^{-3})(2.54) = 1.4732 \ \text{ cm}$$

$$A = \frac{\pi d^2}{4} = 1.704564 \ \text{ cm}^2$$

$$R_1 = \rho \, \frac{\ell}{A} = 2.8 \times 10^{-8} \, \frac{25 \times 10^3}{1.704564 \times 10^{-4}} = 4.106 \ \Omega$$

(b)

$$R_2 = R_1 \left(\frac{228 + 50}{228 + 20} \right) = 4.106 \left(\frac{278}{248} \right) = 4.6 \ \Omega$$

4.2. A transmission-line cable consists of 12 identical strands of aluminum, each 3 mm in diameter. The resistivity of aluminum strand at $20°$C is 2.8×10^{-8} Ω-m. Find the $50°$C AC resistance per Km of the cable. Assume a skin-effect correction factor of 1.02 at 60 Hz.

$$A = 12\,\frac{\pi d^2}{4} = 12\,\frac{(\pi)(3^2)}{4} = 84.823 \;\; \text{mm}^2$$

$$R_{20} = \rho\,\frac{\ell}{A} = \frac{2.8\times 10^{-8}\times 10^3}{84.823\times 10^{-6}} = 0.33\;\Omega$$

$$R_{50} = R_{20}\left(\frac{228+50}{228+20}\right) = 0.33\left(\frac{278}{248}\right) = 0.37\;\Omega$$

$$R_{AC} = (1.02)(0.37) = 0.3774\;\Omega$$

4.3. A three-phase transmission line is designed to deliver 190.5-MVA at 220-kV over a distance of 63 Km. The total transmission line loss is not to exceed 2.5 percent of the rated line MVA. If the resistivity of the conductor material is 2.84×10^{-8} Ω-m, determine the required conductor diameter and the conductor size in circular mils.

The total transmission line loss is

$$P_L = \frac{2.5}{100}(190.5) = 4.7625 \;\; \text{MW}$$

$$|I| = \frac{S}{\sqrt{3}\,V_L} = \frac{(190.5)10^3}{\sqrt{3}(220)} = 500 \;\; \text{A}$$

From $P_L = 3R|I|^2$, the line resistance per phase is

$$R = \frac{4.7625\times 10^6}{3(500)^2} = 6.35\;\Omega$$

The conductor cross sectional area is

$$A = \frac{(2.84\times 10^{-8})(63\times 10^3)}{6.35} = 2.81764\times 10^{-4} \;\; \text{m}^2$$

Therefore

$$d = 1.894 \text{ cm} = 0.7456 \text{ in} = 556000 \;\; \text{cmil}$$

4.4. A single-phase transmission line 35 Km long consists of two solid round conductors, each having a diameter of 0.9 cm. The conductor spacing is 2.5 m. Calculate the equivalent diameter of a fictitious hollow, thin-walled conductor having the same equivalent inductance as the original line. What is the value of the inductance per conductor?

$$r' = e^{-\frac{1}{4}} r = e^{-\frac{1}{4}} \left(\frac{0.9}{2}\right) = 0.35 \ \ \text{cm}$$

or $\qquad d = 0.7$ cm.

$$L = 0.2 \ln \frac{2.5}{0.35 \times 10^{-2}} = 1.314 \ \ \text{mH/Km}$$

The inductance per conductor is $\ell L = (35)(1.314) = 46$ mH.

4.5. Find the geometric mean radius of a conductor in terms of the radius r of an individual strand for
(a) Three equal strands as shown in Figure 34(a)
(b) Four equal strands as shown in Figure 34(b)

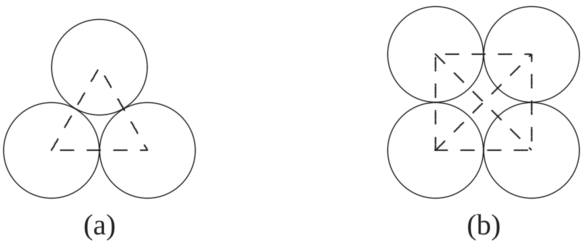


FIGURE 34
Cross section of the stranded conductor for Problem 4.5.

(a)

$$\begin{aligned} GMR &= \sqrt[9]{(r'2r \times 2r)^3} \\ &= \sqrt[3]{e^{-\frac{1}{4}} r \times 2r \times 2r} = 1.46r \end{aligned}$$

(b)

$$\begin{aligned} GMR &= \sqrt[16]{(r' \times 2r \times 2r \times \sqrt{2}\, 2r)^4} \\ &= \sqrt[4]{e^{-\frac{1}{4}} 8\sqrt{2}\, r^4} = 1.723r \end{aligned}$$

4.6. One circuit of a single-phase transmission line is composed of three solid 0.5-cm radius wires. The return circuit is composed of two solid 2.5-cm radius wires. The arrangement of conductors is as shown in Figure 35. Applying the concept of the GMD and GMR, find the inductance of the complete line in millihenry per kilometer.

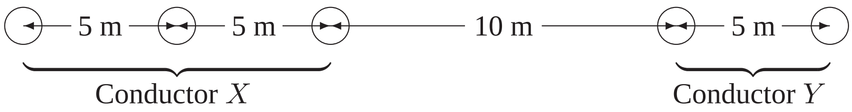


FIGURE 35
Conductor layout for Problem 4.6.

$$D_m = \sqrt[6]{(20)(25)(15)(20)(10)(15)} = 16.802 \ \text{ m}$$

$$D_{SX} = \sqrt[9]{(e^{-\frac{1}{4}} \times 0.005 \times 5 \times 10)(e^{-\frac{1}{4}} \times 0.005 \times 5 \times 5)(e^{-\frac{1}{4}} \times 0.005 \times 5 \times 10)}$$
$$= 0.5366 \ \text{ m}$$

$$D_{SY} = \sqrt[4]{\left(e^{-\frac{1}{4}} \times 0.025 \times 5\right)^2} = 0.312 \ \text{ m}$$

Therefore

$$L_X = 0.2 \ln \frac{16.802}{0.5366} = 0.6888 \ \text{ mH/Km}$$

and

$$L_Y = 0.2 \ln \frac{16.802}{0.312} = 0.79725 \ \text{ mH/Km}$$

The loop inductance is

$$L = L_X + L_Y = 0.6888 + 0.79725 = 1.48605 \ \text{ mH/Km}$$

4.7. A three-phase, 60-Hz transposed transmission line has a flat horizontal configuration as shown in Figure 36. The line reactance is 0.486 Ω per kilometer. The conductor geometric mean radius is 2.0 cm. Determine the phase spacing D in meter.

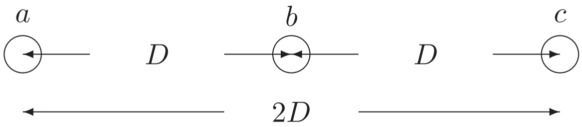


FIGURE 36
Conductor layout for Problem 4.7.

$$L = \frac{(0.486)10^3}{(2\pi)(60)} = 1.2892 \text{ mH/Km}$$

Therefore, we have

$$1.2892 = 0.2 \ln \frac{GMD}{0.02} \quad \text{or} \quad GMD = 12.6 \text{ m}$$

$$12.6 = \sqrt[3]{(D)(D)(2D)} \quad \text{or} \quad D = 10 \text{ m}$$

4.8. A three-phase transposed line is composed of one ACSR 159,000 cmil, 54/19 Lapwing conductor per phase with flat horizontal spacing of 8 meters as shown in Figure 37. The GMR of each conductor is 1.515 cm.
(a) Determine the inductance per phase per kilometer of the line.
(b) This line is to be replaced by a two-conductor bundle with 8-m spacing measured from the center of the bundles as shown in Figure 38. The spacing between the conductors in the bundle is 40 cm. If the line inductance per phase is to be 77 percent of the inductance in part (a), what would be the GMR of each new conductor in the bundle?

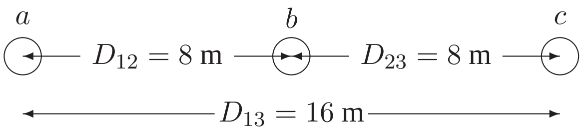


FIGURE 37
Conductor layout for Problem 4.8 (a).

(a)

$$GMD = \sqrt[3]{(8)(8)(16)} = 10.0794 \text{ m}$$

$$L = 0.2 \frac{10.0794}{1.515 \times 10^{-2}} = 1.3 \text{ mH/Km}$$

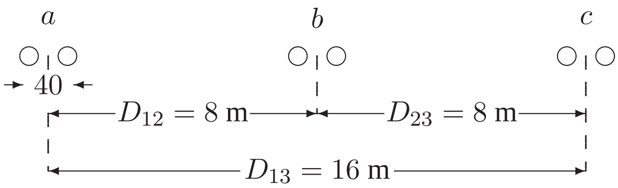


FIGURE 38
Conductor layout for Problem 4.8 (b).

(b)

$$L_2 = (1.3)(0.77) = 1.0 \text{ mH/Km}$$

$$1.0 = 0.2 \ln \frac{10.0794}{D_S^b} \quad \text{or} \quad D_S^b = 0.0679 \text{ m}$$

Now

$$S_S^b = \sqrt{D_S\, d} \quad \text{therefore} \quad D_S = 1.15 \text{ cm}$$

4.9. A three-phase transposed line is composed of one ACSR 1,431,000 cmil, 47/7 Bobolink conductor per phase with flat horizontal spacing of 11 meters as shown in Figure 39. The conductors have a diameter of 3.625 cm and a GMR of 1.439 cm. The line is to be replaced by a three-conductor bundle of ACSR 477,000 cmil, 26/7 Hawk conductors having the same cross-sectional area of aluminum as the single-conductor line. The conductors have a diameter of 2.1793 cm and a GMR of 0.8839 cm. The new line will also have a flat horizontal configuration, but it is to be operated at a higher voltage and therefore the phase spacing is increased to 14 m as measured from the center of the bundles as shown in Figure 40. The spacing between the conductors in the bundle is 45 cm. Determine
(a) The percentage change in the inductance.
(b) The percentage change in the capacitance.

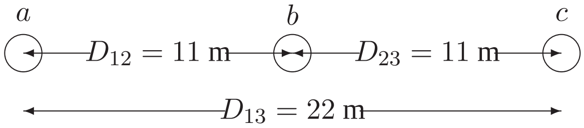


FIGURE 39
Conductor layout for Problem 4.9 (a).

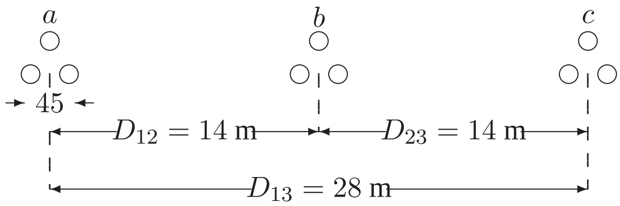


FIGURE 40
Conductor layout for Problem 4.9 (b).

For the one conductor per phase configuration , we have

$$GMD = \sqrt[3]{(11)(11)(22)} = 13.859 \ \text{ m}$$

$$L = 0.2 \frac{13.859}{1.439 \times 10^{-2}} = 1.374 \ \text{ mH/Km}$$

$$C = \frac{0.0556}{\ln \frac{13.859}{1.8125 \times 10^{-2}}} = 0.008374 \ \ \mu\text{F/Km}$$

For the three-conductor bundle per phase configuration , we have

$$GMD = \sqrt[3]{(14)(14)(28)} = 17.6389 \ \text{ m}$$

$$GMR_L = \sqrt[3]{(45)^2(0.8839)} = 12.1416 \ \text{ cm}$$

$$GMR_C = \sqrt[3]{(45)^2(2.1793/2)} = 13.01879 \ \text{ cm}$$

$$L = 0.2 \frac{17.6389}{12.1416 \times 10^{-2}} = 0.9957 \ \text{ mH/Km}$$

$$C = \frac{0.0556}{\ln \frac{17.6389}{13.01879 \times 10^{-2}}} = 0.011326 \ \ \mu\text{F/Km}$$

(a) The percentage reduction in the inductance is

$$\frac{1.374 - 0.9957}{1.374}(100) = 27.53\%$$

(b) The percentage increase in the capacitance is

$$\frac{0.001326 - 0.008374}{0.008374}(100) = 35.25\%$$

4.10. A single-circuit three-phase transmission transposed line is composed of four ACSR 1,272,000 cmil conductor per phase with horizontal configuration as shown in Figure 41. The bundle spacing is 45 cm. The conductor code name is *pheasant*. In *MATLAB*, use command **acsr** to find the conductor diameter and its GMR. Determine the inductance and capacitance per phase per kilometer of the line. Use function **[GMD, GMRL, GMRC] =gmd**, (4.58) and (4.92) in *MATLAB* to verify your results.

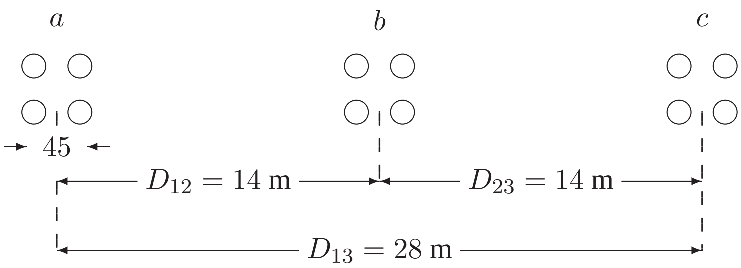


FIGURE 41
Conductor layout for Problem 4.10.

Using the command **acsr**, result in

```
Enter ACSR code name within single quotes -> 'pheasant'
Al Area  Strand  Diameter GMR    Resistance Ohm/Km Ampacity
CMILS    Al/St   cm        cm     60Hz,25C 60HZ,50C Ampere
1272000  54/19  3.5103   1.4173  0.04586  0.05290  1200
```

From (4.79)

$$GMD = \sqrt[3]{(14)(14)(28)} = 17.63889 \text{ m}$$

and From (4.53) and (4.90), we have

$$GMR_L = 1.09\sqrt[4]{(1.4173)(45)^3} = 20.66 \text{ cm}$$

$$GMR_C = 1.09\sqrt[4]{(3.5103/2)(45)^3} = 21.8 \text{ cm}$$

and from (4.58) and (4.92), we get

$$L = 0.2\ln\frac{GMD}{GMRL} = 0.2\ln\frac{17.63889}{0.2066} = 0.889 \text{ mH/Km}$$

and

$$C = \frac{0.0556}{\ln \frac{GMD}{GMRC}} = \frac{0.0556}{\ln \frac{17.63889}{0.218}} = 0.0127 \ \ \mu\text{F/Km}$$

The function **gmd** is used to verify the results. The following commands

```
[GMD, GMRL, GMRC] = gmd
L = 0.2*log(GMD/GMRL);       %mH/Km
C = 0.0556/log(GMD/GMRC);    %microF/Km
```

result in

```
GMD  = 17.63889 m
GMRL =  0.20673 m   GMRC =  0.21808 m
L =
        0.8893
C =
        0.0127
```

4.11. A double circuit three-phase transposed transmission line is composed of two ACSR 2,167,000 cmil, 72/7 Kiwi conductor per phase with vertical configuration as shown in Figure 42. The conductors have a diameter of 4.4069 cm and a GMR of 1.7374 cm. The bundle spacing is 45 cm. The circuit arrangement is $a_1b_1c_1$, $c_2b_2a_2$. Find the inductance and capacitance per phase per kilometer of the line. Find these values when the circuit arrangement is $a_1b_1c_1$, $a_2b_2c_2$. Use function **[GMD, GMRL, GMRC] =gmd**, (4.58) and (4.92) in *MATLAB* to verify your results. For the $a_1b_1c_1$,$c_2b_2a_2$ configuration, the following distances are computed in

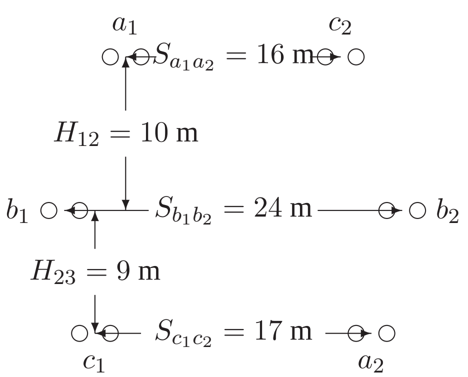


FIGURE 42
Conductor layout for Problem 4.11.

meter.

$$\begin{array}{llll} D_{a_1b_1} = 10.7703 & D_{a_1b_2} = 22.3607 & D_{a_2b_1} = 22.3886 & D_{a_2b_2} = 9.6566 \\ D_{b_1c_1} = 9.6566 & D_{b_1c_2} = 22.3607 & D_{b_2c_1} = 22.3886 & D_{b_2c_2} = 10.7703 \\ D_{a_1c_1} = 19.0065 & D_{a_1c_2} = 16.0000 & D_{a_2c_1} = 17.0000 & D_{a_2c_2} = 19.0065 \\ D_{a_1a_2} = 25.1644 & D_{b_1b_2} = 24.0000 & D_{c_1c_2} = 25.1644 & \end{array}$$

From (4.54), we have

$$D_{AB} = \sqrt[4]{(10.7703)(22.3607)(22.3886)(9.6566)} = 15.1057$$

$$D_{BC} = \sqrt[4]{(9.6566)(22.3607)(22.3886)(10.7703)} = 15.1057$$

$$D_{CA} = \sqrt[4]{(19.0065)(16.0000)(17.0000)(19.0065)} = 17.0749$$

The equivalent geometric mean distance is

$$GMD = \sqrt[3]{(15.1057)(15.1057)(17.7049)} = 15.9267$$

From (4.51) and (4.66), we have

$$D_s^b = \sqrt{D_s \times d} = \sqrt{(0.017374)(0.45)} = 0.08842$$

$$r^b = \sqrt{r \times d} = \sqrt{(0.044069/2)(0.45)} = 0.09957$$

From (4.56) and (4.93), we have

$$\begin{array}{ll} D_{SA} = \sqrt{(0.08842)(25.1644)} = 1.4916 & D_{SB} = \sqrt{(0.08842)(24.0000)} = 1.4567 \\ D_{SC} = \sqrt{(0.08842)(25.1644)} = 1.4916 & \end{array}$$

and

$$\begin{array}{ll} r_A = \sqrt{(0.09957)(25.1644)} = 1.5829 & r_B = \sqrt{(0.09957)(24.0000)} = 1.5458 \\ r_C = \sqrt{(0.09957)(25.1644)} = 1.5829 & \end{array}$$

$$GMR_L = \sqrt[3]{(1.4916)(1.4567)(1.4916)} = 1.4799$$

$$GMR_C = \sqrt[3]{(1.5829)(1.5458)(1.5829)} = 1.5705$$

Therefore, the inductance and capacitance per phase are

$$L = 0.2 \ln \frac{GMD}{GMRL} = 0.2 \ln \frac{15.9267}{1.4799} = 0.4752 \text{ mH/Km}$$

and

$$C = \frac{0.0556}{\ln \frac{GMD}{GMRC}} = \frac{0.0556}{\ln \frac{15.9267}{1.5705}} = 0.0240 \ \mu\text{F/Km}$$

The function **gmd** is used to verify the results. The following commands

```
[GMD, GMRL, GMRC] = gmd
L = 0.2*log(GMD/GMRL);        %mH/Km
C = 0.0556/log(GMD/GMRC);   %microF/Km
```

result in

```
Circuit Arrangements
--------------------
(1)  abc-c‘b‘a‘
(2)  abc-(a‘)(b‘)(c‘)
Enter (1 or 2)  -> 1
Enter spacing unit within quotes ’m’ or ’ft’-> ’m’
Enter row vector [S11, S22, S33] = [16 24 17]
Enter row vector [H12, H23] = [10 9]
Cond. size, bundle spacing unit: Enter ’cm’ or ’in’-> ’cm’
Conductor diameter in cm = 4.4069
Geometric Mean Radius in cm = 1.7374
No. of bundled cond. (enter 1 for single cond.) = 2
Bundle spacing in cm = 45

GMD  = 15.92670 m
GMRL =  1.47993 m   GMRC =  1.57052 m
L =
        0.4752
C =
        0.0240
```

For the $a_1b_1c_1$,$a_2b_2c_2$ configuration, the following distances are computed in meter.

$$
\begin{array}{llll}
D_{a_1b_1} = 10.7703 & D_{a_1b_2} = 22.3607 & D_{a_2b_1} = 22.3607 & D_{a_2b_2} = 10.7703 \\
D_{b_1c_1} = 9.6566 & D_{b_1c_2} = 22.3886 & D_{b_2c_1} = 22.3886 & D_{b_2c_2} = 9.6566 \\
D_{a_1c_1} = 19.0065 & D_{a_1c_2} = 25.1644 & D_{a_2c_1} = 25.1644 & D_{a_2c_2} = 19.0065 \\
D_{a_1a_2} = 16.0000 & D_{b_1b_2} = 24.0000 & D_{c_1c_2} = 17.0000 &
\end{array}
$$

From (4.54), we have

$$
\begin{aligned}
D_{AB} &= \sqrt[4]{(10.7703)(22.3607)(22.3607)(10.7703)} = 15.5187 \\
D_{BC} &= \sqrt[4]{(9.6566)(22.3886)(22.3886)(9.6566)} = 14.7036 \\
D_{CA} &= \sqrt[4]{(19.0065)(25.1644)(25.1644)(19.0065)} = 21.8698
\end{aligned}
$$

The equivalent geometric mean distance is

$$
GMD = \sqrt[3]{(15.5187)(14.7036)(21.8698)} = 17.0887
$$

From (4.51) and (4.66), we have

$$D_s^b = \sqrt{D_s \times d} = \sqrt{(0.017374)(0.45)} = 0.08842$$
$$r^b = \sqrt{r \times d} = \sqrt{(0.044069/2)(0.45)} = 0.09957$$

From (4.56) and (4.93), we have

$$D_{SA} = \sqrt{(0.08842)(16)} = 1.1894 \; D_{SB} = \sqrt{(0.08842)(24.0000)} = 1.4567$$
$$D_{SC} = \sqrt{(0.08842)(17)} = 1.2260$$

and

$$r_A = \sqrt{(0.09957)(16)} = 1.2622 \; r_B = \sqrt{(0.09957)(24.0000)} = 1.5458$$
$$r_C = \sqrt{(0.09957)(17)} = 1.3010$$

$$GMR_L = \sqrt[3]{(1.1894)(1.4567)(1.2260)} = 1.2855$$
$$GMR_C = \sqrt[3]{(1.2622)(1.5458)(1.3010} = 1.3641$$

Therefore, the inductance and capacitance per phase are

$$L = 0.2 \ln \frac{GMD}{GMRL} = 0.2 \ln \frac{17.0887}{1.2855} = 0.5174 \;\; \text{mH/Km}$$

and

$$C = \frac{0.0556}{\ln \frac{GMD}{GMRC}} = \frac{0.0556}{\ln \frac{17.0887}{1.3641}} = 0.02199 \;\; \mu\text{F/Km}$$

The function **gmd** is used in the same way as before to verify the results.

4.12. The conductors of a double-circuit three-phase transmission line are placed on the corner of a hexagon as shown in Figure 43. The two circuits are in parallel and are sharing the balanced load equally. The conductors of the circuits are identical, each having a radius r. Assume that the line is symmetrically transposed. Using the method of GMR, determine an expression for the capacitance per phase per meter of the line.

$$D_{a_1c_1} = D_{a_1b_2} = D_{b_1a_2} = D_{b_1c_2} = D_{c_1b_2} = D_{c_2a_2} = \sqrt{3}D$$

$$D_{AB} = D_{BC} = D_{CA} = \sqrt[4]{D_{a_1b_1}D_{a_1b_2}D_{a_2b_1}D_{a_2b_2}}$$
$$= \sqrt{D_{a_1b_1}D_{a_1b_2}} = 3^{\frac{1}{4}}D$$

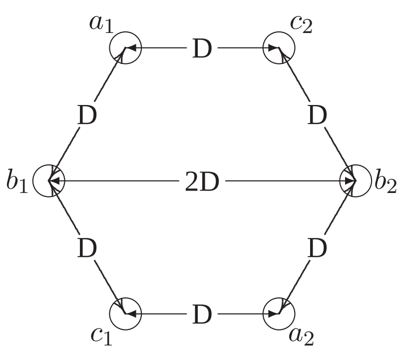


FIGURE 43
Conductor layout for Problem 4.12.

Therefore

$$GMD = 3^{\frac{1}{4}} D$$

$$D_{SA} = D_{SB} = D_{SC} = (r \times 2D)^{\frac{1}{2}}$$

and

$$C = \frac{2\pi\epsilon}{\ln(\frac{3^{\frac{1}{2}} D^2}{2rD})^{\frac{1}{2}}} = \frac{4\pi\epsilon}{\ln \frac{0.866D}{r}} \quad \text{F/m}$$

4.13. A 60-Hz, single-phase power line and a telephone line are parallel to each other as shown in Figure 44. The telephone line is symmetrically positioned directly below phase b. The power line carries an rms current of 226 A. Assume zero current flows in the ungrounded telephone wires. Find the magnitude of the voltage per Km induced in the telephone line.

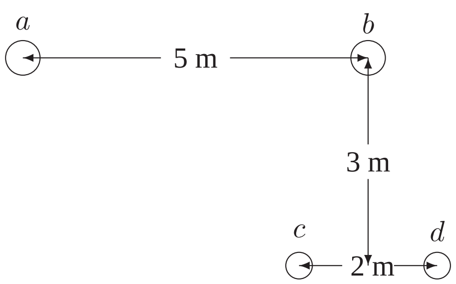


FIGURE 44
Conductor layout for Problem 4.13.

$$D_{ac} = \sqrt{(4)^2 + (3)^2} = 5.0000 \text{ m}$$
$$D_{ad} = \sqrt{(6)^2 + (3)^2} = 6.7082 \text{ m}$$

$$\lambda_{cdI_a} = (0.2)(226)\ln\frac{6.7082}{5.0000} = 13.28 \text{ Wb/Km}$$
$$\lambda_{cdI_b} = 0.2I_b \ln\frac{3.1622}{3.1622} = 0$$

The total flux linkage is

$$\lambda_{cd} = 13.28 \text{ mWb/Km}$$

The voltage induced in the telephone line per Km is

$$V = \omega\lambda_{cd} = 2\pi 60(13.28)10^{-3} = 5 \text{ V/Km}$$

4.14. A three-phase, 60-Hz, untransposed transmission line runs in parallel with a telephone line for 20 km. The power line carries a balanced three-phase rms current of $I_a = 320\angle 0^\circ$ A, $I_b = 320\angle{-120^\circ}$ A, and $I_c = 320\angle{-240^\circ}$ A
The line configuration is as shown in Figure 45. Assume zero current flows in the ungrounded telephone wires. Find the magnitude of the voltage induced in the telephone line.

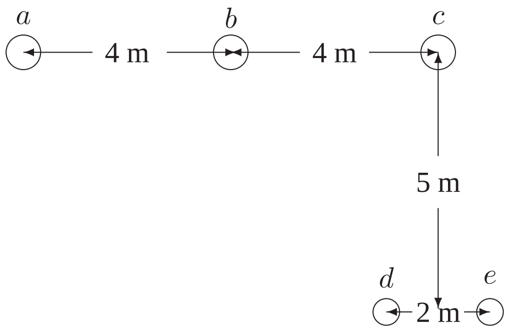


FIGURE 45
Conductor layout for Problem 4.14.

$$D_{ad} = \sqrt{(7)^2 + (5)^2} = 8.6023 \text{ m}$$
$$D_{ae} = \sqrt{(9)^2 + (5)^2} = 10.2956 \text{ m}$$
$$D_{bd} = \sqrt{(3)^2 + (5)^2} = 5.8309 \text{ m}$$
$$D_{be} = \sqrt{(5)^2 + (5)^2} = 7.0711 \text{ m}$$

$$\lambda_{deI_a} = 0.2I_a \ln \frac{10.2956}{8.6023} = 0.03594I_a$$
$$\lambda_{deI_b} = 0.2I_b \ln \frac{7.0711}{5.8309} = 0.03856I_b$$
$$\lambda_{deI_c} = 0.2I_c \ln \frac{5.0249}{5.0249} = 0$$

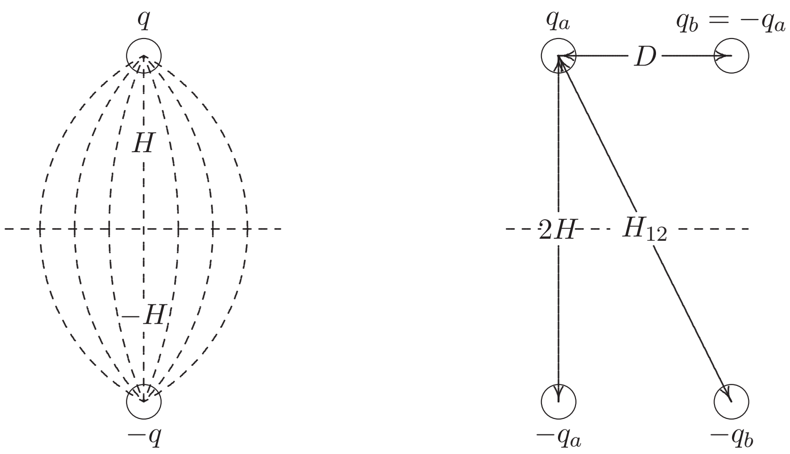
The total flux linkage is

$$\lambda_{de} = (0.03594)320\angle 0^\circ + (0.03856)320\angle{-120^\circ} = 11.943\angle{-63.48^\circ} \text{ mWb/Km}$$

The voltage induced in the 20 Km telephone line is

$$V = j\omega\lambda_{de} = j2\pi 60(11.943\angle{-63.48^\circ})(10^{-3})(20) = 90\angle 26.52^\circ \text{ V}$$

4.15. Since earth is an equipotential plane, the electric flux lines are forced to cut the surface of the earth orthogonally. The earth effect can be represented by placing an oppositely charged conductor a depth H below the surface of the earth as shown in Figure 4-13(a). This configuration without the presence of the earth will produce the same field as a single charge and the earth surface. This imaginary conductor is called the image conductor. Figure 4-13(b) shows a single-phase line with its image conductors.



(a) Earth plane replaced by image conductor

(b) Single-phase line and its image

FIGURE 46
Conductor layout for Problem 4.15.

Find the potential difference V_{ab} and show that the equivalent capacitance to neutral is given by

$$C_{an} = C_{bn} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\frac{2H}{H_{12}}\right)}$$

$$V_{ab} = \frac{1}{2\pi\epsilon}\left[q_a \ln\frac{D}{r} - q_a \ln\frac{H_{12}}{2H} + q_b \ln\frac{r}{D} - q_b \ln\frac{2H}{H_{12}}\right]$$

Substituting for $q_b = -q_a$, results in

$$V_{ab} = \frac{q_a}{\pi\epsilon}\left[\ln\left(\frac{D}{r}\frac{2H}{H_{12}}\right)\right]$$

$$C_{ab} = \frac{q_a}{V_{ab}} = \frac{\pi\epsilon}{\ln\left(\frac{D}{r}\frac{2H}{H_{12}}\right)}$$

Therefore, the equivalent capacitance to neutral is

$$C_{an} = C_{bn} = 2C_{ab} = \frac{2\pi\epsilon}{\ln\left(\frac{D}{r}\frac{2H}{H_{12}}\right)}$$

CHAPTER 5 PROBLEMS

5.1. A 69-kV, three-phase short transmission line is 16 km long. The line has a per phase series impedance of $0.125 + j0.4375\ \Omega$ per km. Determine the sending end voltage, voltage regulation, the sending end power, and the transmission efficiency when the line delivers

(a) 70 MVA, 0.8 lagging power factor at 64 kV.
(b) 120 MW, unity power factor at 64 kV.

Use **lineperf** program to verify your results.

The line impedance is

$$Z = (0.125 + j0.4375)(16) = 2 + j7\ \Omega$$

The receiving end voltage per phase is

$$V_R = \frac{64\angle 0^\circ}{\sqrt{3}} = 36.9504\angle 0^\circ \quad \text{kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 70\angle \cos^{-1} 0.8 = 70\angle 36.87^\circ = 56 + j42 \ \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{70000\angle - 36.87^\circ}{3 \times 36.9504\angle 0^\circ} = 631.477\angle - 36.87^\circ \quad \text{A}$$

The sending end voltage is

$$\begin{aligned} V_S = V_R + ZI_R &= 36.9504\angle 0^\circ + (2 + j7)(631.477\angle - 36.87^\circ)(10^{-3}) \\ &= 40.708\angle 3.9137^\circ \ \ \text{kV} \end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 70.508 \ \ \text{kV}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 40.708\angle 3.9137 \times 631.477\angle 36.87^\circ \times 10^{-3} \\ &= 58.393 \text{ MW} + j50.374 \text{ Mvar} \\ &= 77.1185\angle 40.7837^\circ \ \ \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{70.508 - 64}{64} \times 100 = 10.169\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{56}{58.393} \times 100 = 95.90\%$$

(b) The complex power at the receiving end is

$$S_{R(3\phi)} = 120\angle 0^\circ = 120 + j0 \ \ \text{MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V_R^*} = \frac{120000\angle 0^\circ}{3 \times 36.9504\angle 0^\circ} = 1082.53\angle 0^\circ \ \ \text{A}$$

The sending end voltage is

$$\begin{aligned} V_S = V_R + ZI_R &= 36.9504\angle 0^\circ + (2 + j7)(1082.53\angle 0^\circ)(10^{-3}) \\ &= 39.8427\angle 10.9639^\circ \ \ \text{kV} \end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 69.0096 \ \ \text{kV}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 39.8427\angle 10.9639 \times 1082.53\angle 0^\circ \times 10^{-3} \\ &= 127.031 \text{ MW} + j24.609 \text{ Mvar} \\ &= 129.393\angle 10.9639^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{69.0096 - 64}{64} \times 100 = 7.8275\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{120}{127.031} \times 100 = 94.465\%$$

The above computations are performed efficiently using the **lineperf** program.

The command:

```
lineperf
```

displays the following menu

```
Type of parameters for input                    Select
Parameters per unit length
r (Ω), g (Siemens), L (mH), C (μF)                 1

Complex z and y per unit length
r + j*x (Ω), g + j*b (Siemens)                     2

Nominal π or Eq. π model                           3

A, B, C, D constants                               4

Conductor configuration and dimension              5

To quit                                            0
Select number of menu → 2
Enter line length = 16
Enter frequency in Hz = 60
Enter series impedance r + j*x in ohm per unit length
z = 0.125+j*0.4375
Enter shunt admittance g + j*b in siemens per unit length
y = 0+j*0
```

Short line model
Z = 2 + j 7 ohms

$$\text{ABCD} = \begin{bmatrix} 1 & + & j0 & 2 & + & j7 \\ 0 & + & j0 & 1 & + & j0 \end{bmatrix}$$

```
      Transmission line performance
             Analysis                                Select

To calculate sending end quantities
for specified receiving end MW, Mvar                   1

To calculate receiving end quantities
for specified sending end MW, Mvar                     2

To calculate sending end quantities
when load impedance is specified                       3

Open-end line and reactive compensation                4

Short-circuited line                                   5

Capacitive compensation                                6

Receiving end circle diagram                           7

Loadability curve and voltage profile                  8

To quit                                                0

Select number of menu → 1

Enter receiving end line-line voltage kV = 64
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 56
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 42

Line performance for specified receiving end quantities

Vr = 64 kV (L-L) at 0°
Pr = 56 MW Qr = 42 Mvar
```

```
Ir = 631.477 A at -36.8699° PFr = 0.8 lagging
Vs = 70.5082 kV (L-L) at 3.91374°
Is = 631.477 A at -36.8699° PFs = 0.757182 lagging
Ps = 58.393 MW     Qs = 50.374 Mvar
PL = 2.393 MW     QL = 8.374 Mvar
Percent Voltage Regulation = 10.169
Transmission line efficiency = 95.9026
```

At the end of this analysis the **listmenu** (Analysis Menu) is displayed. Selecting option 1 and entering data for part (b), will result in

```
Line performance for specified receiving end quantities

Vr = 64 kV (L-L) at 0°
Pr = 120 MW Qr = 0 Mvar
Ir = 1082.53 A at 0° PFr = 1
Vs = 69.0096 kV (L-L) at 10.9639°
Is = 1082.53 A at 0° PFs = 0.981747
Ps = 127.031 MW     Qs = 24.069 Mvar
PL = 7.031 MW     QL = 24.069 Mvar
Percent Voltage Regulation = 7.82754
Transmission line efficiency = 94.4649
```

5.2. Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.1. The line delivers 70 MVA, 0.8 lagging power factor at 64 kV. Determine the total Mvar and the capacitance per phase of the Y-connected capacitors when the sending end voltage is

(a) 69 kV.
(b) 64 kV.
Hint: Use (5.85) and (5.86) to compute the power angle δ and the receiving end reactive power.
(c) Use **lineperf** to obtain the compensated line performance.

The complex load at the receiving end is

$$S_{R(3\phi)} = 70\angle \cos^{-1} 0.8 = 56 \text{ MW} + j42 \text{ Mvar}$$

$$Z = 2 + j7 = 7.28\angle 74.0546^\circ$$

(a) For $V_{R(LL)} = 69$ kV, from (5.85), we have

$$56 = \frac{(69)(64)}{7.28}\cos(74.0564 - \delta) - \frac{(1.0)(64)^2}{7.28}\cos 74.0546^\circ$$

Therefore,

$$\cos(74.0546 - \delta) = 0.3471 \qquad \text{or} \qquad \delta = 4.3646$$

Now from (5.86), we have

$$Q_{R(3\phi)} = \frac{(69)(64)}{7.28}\sin(74.0564 - 4.3646) - \frac{(1.0)(64)^2}{7.28}\sin 74.0546^\circ$$
$$= 27.883 \ \text{Mvar}$$

Therefore, the required capacitor Mvar is

$$Q_C = 42 - 27.883 = 14.117 \ \text{Mvar}$$

$$Z_C = \frac{|V_R|^2}{S_C^*} = \frac{(64/sqrt3)^2}{j14.117/3} = -j290.147\ \Omega$$

The required shunt capacitance per phase is

$$C = \frac{10^6}{(2\pi)(60)(290.147)} = 9.1422 \ \mu\text{F}$$

(b) For $V_{R(LL)} = 64$ kV, from (5.85), we have

$$56 = \frac{(64)(64)}{7.28}\cos(74.0564 - \delta) - \frac{(1.0)(64)^2}{7.28}\cos 74.0546^\circ$$

Therefore,

$$\cos(74.0546 - \delta) = 0.37425 \qquad \text{or} \qquad \delta = 6.0327^\circ$$

Now from (5.86), we have

$$Q_{R(3\phi)} = \frac{(64)(64)}{7.28}\sin(74.0564 - 6.0327) - \frac{(1.0)(64)^2}{7.28}\sin 74.0546^\circ$$
$$= \ - 19.2405 \ \text{Mvar}$$

Therefore, the required capacitor Mvar is

$$Q_C = 42 - (-19.2405) = 61.2405 \ \text{Mvar}$$

$$Z_C = \frac{|V_R|^2}{S_C^*} = \frac{(64/sqrt3)^2}{j61.2405/3} = -j66.8838\ \Omega$$

The required shunt capacitance per phase is

$$C = \frac{10^6}{(2\pi)(60)(66.8838)} = 39.6596 \ \mu\text{F}$$

With **lineperf** program, we continue the analysis in Problem 5.1 by selecting option 6. This will display the **compmenu**

```
        Capacitive compensation
              Analysis                        Select

Shunt capacitive compensation                   1

Series capacitive compensation                  2

Series and shunt capacitive compensation        3

To quit                                         0
```

Selecting option 1 of the **compmenu** results in

```
Enter sending end line-line voltage kV = 69
Enter desired receiving end line-line voltage kV = 64
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 56
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 42

Shunt capacitive compensation

Vs = 69 kV (L-L) at 4.36672°
Vr = 64 kV (L-L) at 0°
Pload = 56 MW,    Qload = 42 Mvar
Load current = 631.477 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 290.147 Ω, 9.14222 μF, 14.117 Mvar
Shunt capacitor current = 127.351 A at 90°
Pr = 56.000 MW,     Qr = 27.883 Mvar
Ir = 564.339 A at -26.4692°, PFr = 0.895174 lagging
Is = 564.339 A at -26.4692° PFs = 0.858639 lagging
Ps = 57.911 MW,     Qs = 34.571 Mvar
PL = 1.911 MW,    QL = 6.688 Mvar
Percent Voltage Regulation = 7.8125
Transmission line efficiency = 96.7003
```

Repeating for part (b), we have

```
Shunt capacitive compensation

Vs = 64 KV (L-L) at 6.03281°
Vr = 64 KV (L-L) at 0°
Pload = 56 MW,    Qload = 42 Mvar
Load current = 631.477 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 66.8837Ω, 39.6596 μF, 61.2406 Mvar
```

```
Shunt capacitor current = 552.457 A at 90°
Pr = 56.000 MW,     Qr = -19.241 Mvar
Ir = 534.168 A at 18.9618°, PFr = 0.945735 leading
Is = 534.168 A at 18.9618° PFs = 0.974648 leading
Ps = 57.712 MW,     Qs = -13.249 Mvar
PL = 1.712 MW,     QL = 5.992 Mvar
Percent Voltage Regulation = 2.22045e-14
Transmission line efficiency = 97.0335
```

5.3. A 230-kV, three-phase transmission line has a per phase series impedance of $z = 0.05 + j0.45\ \Omega$ per Km and a per phase shunt admittance of $y = j3.4 \times 10^{-6}$ siemens per km. The line is 80 km long. Using the nominal π model, determine

(a) The transmission line ABCD constants. Find the sending end voltage and current, voltage regulation, the sending end power and the transmission efficiency when the line delivers
(b) 200 MVA, 0.8 lagging power factor at 220 kV.
(c) 306 MW, unity power factor at 220 kV.

Use **lineperf** program to verify your results.

The line impedance and shunt admittance are

$$Z = (0.05 + j0.45)(80) = 4 + j36\ \Omega$$

$$Y = (j3.4 \times 10^{-6})(80) = j0.272 \times 10^{-3} \text{ siemens}$$

The ABCD constants of the nominal π model are

$$A = (1 + \frac{ZY}{2}) = (1 + \frac{(4 + j36)(j0.272 \times 10^{-3})}{2}) = 0.9951 + j0.000544$$
$$B = Z = 4 + j36$$
$$C = Y(1 + \frac{ZY}{4}) = j0.0002713$$

The receiving end voltage per phase is

$$V_R = \frac{220\angle 0^\circ}{\sqrt{3}} = 127\angle 0^\circ \ \text{ kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 200\angle \cos^{-1} 0.8 = 200\angle 36.87^\circ = 160 + j120 \ \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{200000\angle-36.87^\circ}{3\times 127\angle 0^\circ} = 524.864\angle-36.87^\circ \quad \text{A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = 0.9951 + j0.000544)(127\angle 0^\circ) + (4 + j36)$$
$$(524.864\times 10^{-3}\angle{-36.87^\circ}) = 140.1051\angle 5.704^\circ \;\; \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 242.67 \quad \text{kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0002713)(127000\angle 0^\circ) + (0.9951 + j0.000544)$$
$$(524.864\angle{-36.87^\circ}) = 502.38\angle{-33.69^\circ} \quad \text{A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3\times 140.1051\angle 5.704 \times 502.38\angle 33.69^\circ \times 10^{-3} \\ &= 163.179 \text{ MW} + j134.018 \text{ Mvar} \\ &= 211.16\angle 39.396^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{242.67}{0.9951} - 220}{220} \times 100 = 10.847\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{160}{163.179} \times 100 = 98.052\%$$

(b) The complex power at the receiving end is

$$S_{R(3\phi)} = 306\angle 0^\circ = 306 + j0 \;\; \text{MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{306000\angle 0^\circ}{3\times 127\angle 0^\circ} = 803.402\angle 0^\circ \quad \text{A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = 0.9951 + j0.000544)(127\angle 0^\circ) + (4 + j36)$$
$$(803.402 \times 10^{-3}\angle 0^\circ) = 132.807\angle 12.6^\circ \quad \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 230.029 \quad \text{kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0002713)(127000\angle 0^\circ) + (0.9951 + j0.000544)$$
$$(803.402\angle 0^\circ) = 799.862\angle 2.5^\circ \quad \text{A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 132.807\angle 12.6 \times 799.862\angle -2.5^\circ \times 10^{-3} \\ &= 313.742 \text{ MW} + j55.9 \text{ Mvar} \\ &= 318.68\angle 10.1^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{230.029}{0.9951} - 220}{220} \times 100 = 5.073\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{306}{313.742} \times 100 = 97.53\%$$

The above computations are performed efficiently using the **lineperf** program. The command:

```
lineperf
```

displays the following menu

```
Type of parameters for input                  Select
Parameters per unit length
r (Ω), g (Siemens), L (mH), C (μF)              1

Complex z and y per unit length
r + j*x (Ω), g + j*b (Siemens)                  2
```

```
Nominal π or Eq. π model                    3

A, B, C, D constants                        4

Conductor configuration and dimension       5

To quit                                     0
Select number of menu → 2
Enter line length = 80
Enter frequency in Hz = 60
Enter series impedance r + j*x in ohm per unit length z =
0.05 + j0.45
Enter shunt admittance g + j*b in siemens per unit length y =
0+j*3.4e-6

Short line model
Z = 4 + j 36 ohms
Y = 0 + j 0.000272 Siemens
```

$$\text{ABCD} = \begin{bmatrix} 0.9951 & + & j0.000544 & 4 & + & j36 \\ -7.3984e-008 & + & j0.00027133 & 0.9951 & + & j0.000544 \end{bmatrix}$$

```
      Transmission line performance
             Analysis                              Select

To calculate sending end quantities
for specified receiving end MW, Mvar                 1

To calculate receiving end quantities
for specified sending end MW, Mvar                   2

To calculate sending end quantities
when load impedance is specified                     3

Open-end line and reactive compensation              4

Short-circuited line                                 5

Capacitive compensation                              6
```

```
Receiving end circle diagram                          7

Loadability curve and voltage profile                 8

To quit                                               0

Select number of menu → 1

Enter receiving end line-line voltage kV = 220
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 160
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 120

Line performance for specified receiving end quantities

Vr = 220 kV (L-L) at 0°
Pr = 160 MW Qr = 120 Mvar
Ir = 524.864 A at -36.8699° PFr = 0.8 lagging
Vs = 242.67 kV (L-L) at 5.7042°
Is = 502.381 A at -33.6919° PFs = 0.77277 lagging
Ps = 163.179 MW    Qs = 134.018 Mvar
PL = 3.179 MW    QL = 14.018 Mvar
Percent Voltage Regulation = 10.847
Transmission line efficiency = 98.052
```

At the end of this analysis the **listmenu** (Analysis Menu) is displayed. Selecting option 1 and entering data for part (b), will result in

```
Line performance for specified receiving end quantities

Vr = 220 kV (L-L) at 0°
Pr = 306 MW Qr = 0 Mvar
Ir = 803.042 A at 0° PFr = 1
Vs = 230.029 kV (L-L) at 12.6033°
Is = 799.862 A at 2.5008° PFs = 0.984495
Ps = 313.742 MW    Qs = 55.900 Mvar
PL = 7.742 MW    QL = 55.900 Mvar
Percent Voltage Regulation = 5.0732
Transmission line efficiency = 97.532
```

5.4. Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.3. The line delivers 200 MVA, 0.8 lagging power factor at 220 kV.

(a) Determine the total Mvar and the capacitance per phase of the Y-connected capacitors when the sending end voltage is 220 kV. *Hint*: Use (5.85) and (5.86) to compute the power angle δ and the receiving end reactive power.
(b) Use **lineperf** to obtain the compensated line performance.

(a) The complex load at the receiving end is

$$S_{R(3\phi)} = 200\angle 36.87^\circ = 160 \text{ MW} + j120 \text{ Mvar}$$

$$Z = 4 + j36 = 36.2215\angle 83.6598^\circ$$

(a) For $V_{R(LL)} = 220$ kV, from (5.85), we have

$$160 = \frac{(220)(220)}{36.2215}\cos(83.6598^\circ - \delta) - \frac{(0.9951)(220)^2}{36.2215}\cos(83.6598^\circ - 0.0313^\circ)$$

Therefore,

$$\cos(83.6598 - \delta) = 0.23017$$

or

$$\delta = 6.967^\circ$$

Now from (5.86), we have

$$Q_{R(3\phi)} = \frac{(220)(220)}{36.2215}\sin(83.6598^\circ - 6.967^\circ) - \frac{(0.9951)(220)^2}{36.2215}$$
$$\sin(83.6598^\circ - 6.967)^\circ = -21.12 \text{ Mvar}$$

Therefore, the required capacitor Mvar is

$$Q_C = 21.12 + 120 = 141.12 \text{ Mvar}$$

$$Z_C = \frac{|V_R|^2}{S_C^*} = \frac{(220/sqrt3)^2}{j141.12/3} = -j342.964\ \Omega$$

The required shunt capacitance per phase is

$$C = \frac{10^6}{(2\pi)(60)(342.964)} = 141.123 \ \mu\text{F}$$

With **lineperf** program, we continue the analysis in Problem 5.1 by selecting option 6. This will display the **compmenu**

```
        Capacitive compensation
              Analysis                       Select

Shunt capacitive compensation                  1

Series capacitive compensation                 2

Series and shunt capacitive compensation       3

To quit                                        0
```

Selecting option 1 of the **compmenu** results in

```
Enter sending end line-line voltage kV = 220
Enter desired receiving end line-line voltage kV = 220
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 160
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 120

Shunt capacitive compensation

Vs = 220 kV (L-L) at 6.96702°
Vr = 220 kV (L-L) at 0°
Pload = 160 MW,    Qload = 120 Mvar
Load current = 524.864 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 342.964 Ω, 7.73428 μF, 141.123 Mvar
Shunt capacitor current = 370.351 A at 90°
Pr = 160.000 MW,    Qr = -21.123 Mvar
Ir = 423.534 A at 7.52047°, PFr = 0.991398 leading
Is = 427.349 A at 12.1375° PFs = 0.995931 leading
Ps = 162.179 MW,    Qs = -14.675 Mvar
PL = 2.179 MW,    QL = 6.447 Mvar
Percent Voltage Regulation = 0.49199
Transmission line efficiency = 98.6563
```

5.5. A three-phase, 345-kV, 60-Hz transposed line is composed of two ACSR 1,113,000, 45/7 Bluejay conductors per phase with flat horizontal spacing of 11 m. The conductors have a diameter of 3.195 cm and a GMR of 1.268 cm. The bundle spacing is 45 cm. The resistance of each conductor in the bundle is $0.0538\ \Omega$ per km and the line conductance is negligible. The line is 150 Km long. Using the nominal π model, determine the ABCD constant of the line. Use **lineperf** and option 5 to verify your results.

$$GMD = \sqrt[3]{(11)(11)(22)} = 13.859\ \text{ m}$$

$$GMR_L = \sqrt{(45)(1.268)} = 7.5538 \ \ \text{cm}$$

$$GMR_C = \sqrt{(45)(3.195/2)} = 8.47865 \ \ \text{cm}$$

$$L = 0.2 \frac{13.859}{7.5538 \times 10^{-2}} = 1.0424 \ \ \text{mH/Km}$$

$$C = \frac{0.0556}{\ln \frac{13.859}{8.47865 \times 10^{-2}}} = 0.010909 \ \ \mu\text{F/Km}$$

$$Z = (\frac{0.0538}{2} + j2\pi \times 60 \times 1.0424 \times 10^{-3})(150) = 4.035 + j58.947$$

$$Y = j(2\pi 60 \times 0.9109 \times 10^{-6})(150) = j0.0006169$$

The ABCD constants of the nominal π model are

$$A = \left(1 + \frac{ZY}{2}\right) = \left(1 + \frac{(4.035 + j38.947)(j0.0006169)}{2}\right)$$
$$= 0.98182 + j0.0012447$$
$$B = Z = 4.035 + j58.947$$
$$C = Y\left(1 + \frac{ZY}{4}\right) = j0.00061137$$

Using **lineperf** and option 5, result in

Type of parameters for input	Select
Parameters per unit length r (Ω), g (Siemens), L (mH), C (μF)	1
Complex z and y per unit length r + j*x (Ω), g + j*b (Siemens)	2
Nominal π or Eq. π model	3
A, B, C, D constants	4
Conductor configuration and dimension	5
To quit	0

Select number of menu → 5

When the line configuration and conductor specifications are entered, the following results are obtained

```
GMD = 13.85913 m
GMRL = 0.07554 m        GMRC = 0.08479 m
L = 1.04241 mH/Km        C = 0.0109105 micro F/Km

Short line model
Z = 4.035 + j 58.947 ohms
Y = 0 + j 0.000616976 Siemens
```

$$\text{ABCD} = \begin{bmatrix} 0.98182 & + & j0.0012447 & 4.035 & + & j58.947 \\ -3.8399e-007 & + & j0.00061137 & 0.98182 & + & j0.0012447 \end{bmatrix}$$

5.6. The ABCD constants of a three-phase, 345-kV transmission line are

$$A = D = 0.98182 + j0.0012447$$
$$B = 4.035 + j58.947$$
$$C = j0.00061137$$

The line delivers 400 MVA at 0.8 lagging power factor at 345 kV. Determine the sending end quantities, voltage regulation, and transmission efficiency.

The receiving end voltage per phase is

$$V_R = \frac{345\angle 0^\circ}{\sqrt{3}} = 199.186\angle 0^\circ \quad \text{kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 400\angle \cos^{-1} 0.8 = 400\angle 36.87^\circ = 320 + j240 \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{400000\angle - 36.87^\circ}{3 \times 199.186\angle 0^\circ} = 669.392\angle - 36.87^\circ \quad \text{A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = 0.98182 + j0.0012447)(199.186\angle 0^\circ) + (4.035 + j58.947)$$
$$(668.392 \times 10^{-3}\angle{-36.87^\circ}) = 223.449\angle 7.766^\circ \quad \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 387.025 \ \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.00061137)(199.186\angle 0^\circ) + (0.98182 + j0.0012447)$$
$$(669.392\angle -36.87^\circ) = 592.291\angle -27.3256^\circ \ \text{ A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 223.449\angle 7.766 \times 592.291\angle 27.3256^\circ \times 10^{-3} \\ &= 324.872 \text{ MW} + j228.253 \text{ Mvar} \\ &= 397.041\angle 35.0916^\circ \text{ MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{387.025}{0.98182} - 345}{345} \times 100 = 14.2589\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{320}{324.872} \times 100 = 98.500\%$$

Using **lineperf** and option 4, result in

```
Select number of menu → 4
Enter the complex constant A = 0.98182+j*0.0012447
Enter the complex constant B = 4.035+j*58.947
Enter the complex constant C = j*0.00061137
Two port model, ABCD constants
Z = 4.035 + j 58.947 ohm
Y = 3.87499e-007 + j 0.000616978 siemens
```

$$\texttt{ABCD} = \begin{bmatrix} 0.98182 & + & j0.0012447 & 4.035 & + & j58.947 \\ 0 & + & j0.00061137 & 0.98182 & + & j0.0012447 \end{bmatrix}$$

```
      Transmission line performance
             Analysis                           Select

To calculate sending end quantities
for specified receiving end MW, Mvar              1
```

```
To calculate receiving end quantities
for specified sending end MW, Mvar                  2

To calculate sending end quantities
when load impedance is specified                    3

Open-end line and reactive compensation             4

Short-circuited line                                5

Capacitive compensation                             6

Receiving end circle diagram                        7

Loadability curve and voltage profile               8

To quit                                             0

Select number of menu → 1

Enter receiving end line-line voltage kV = 345
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 320
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 240

Line performance for specified receiving end quantities
-------------------------------------------------------

Vr = 345 kV (L-L) at 0°
Pr = 320 MW Qr = 240 Mvar
Ir = 669.392 A at -36.8699° PFr = 0.8 lagging
Vs = 387.025 kV (L-L) at 7.76603°
Is = 592.291 A at -27.3222° PFs = 0.818268 lagging
Ps = 324.872 MW    Qs = 228.253 Mvar
PL = 4.872 MW    QL = -11.747 Mvar
Percent Voltage Regulation = 14.2589
Transmission line efficiency = 98.5002
```

5.7. Write a *MATLAB* function named **[ABCD] = abcdm(z, y, Lngt)** to evaluate and return the ABCD transmission matrix for a medium-length transmission line where **z** is the per phase series impedance per unit length, **y** is the shunt admittance per unit length, and **Lngt** is the line length. Then, write a program that uses the above function and computes the receiving end quantities, voltage regulation, and transmission efficiency when sending end quantities are specified. The program should prompt the user to enter the following quantities:

The sending end line-to-line voltage magnitude in kV
The sending end voltage phase angle in degrees
The three-phase sending end real power in MW
The three-phase sending end reactive power in Mvar

Use your program to obtain the solution for the following case:

A three-phase transmission line has a per phase series impedance of $z = 0.03 + j0.4\ \Omega$ per Km and a per phase shunt admittance of $y = j4.0 \times 10^{-6}$ siemens per Km. The line is 125 Km long. Obtain the ABCD transmission matrix. Determine the receiving end quantities, voltage regulation, and the transmission efficiency when the line is sending 407 MW, 7.833 Mvar at 350 kV.

The following function named **abcdm** returns the line ABCD constants

```
function [ABCD] = abcdm(z, y, Lngt);
Z = z*Lngt;
Y = y*Lngt;
A = 1 + Z*Y/2;  B = Z;
C = Y*(1 + Z*Y/4);  D = A;
ABCD = [A B; C D];
```

The following program saved as **ch5p7.m** computes the receiving end quantities from the specified sending end quantities.

```
z=input('Line series impedance per phase per unit length z=');
y=input('Line shunt admittance per phase per unit length y=');
Lngt = input('Transmission line length = ');
ABCD = abcdm(z, y,Lngt)
VL_s=input('Sending end line-to-line voltage magnitude in kV=');
AngV_s=input('Sending end voltage phase angle in degree = ');
P_s =input('Three-phase sending end real power in MW ');
Q_s =input('Three-phase sending end reactive power in Mvar ');
S_s = P_s + j*Q_s;         % MVA
AngV_srd = AngV_s*pi/180;  % Radian
V_s = VL_s/sqrt(3)*(cos(AngV_srd) + j*sin(AngV_srd));      %kV
I_s = conj(S_s)/(3*conj(V_s));   % kA
IL_s= abs(I_s)*1000; AngI_srd = angle(I_s);
AngI_s = AngI_srd*180/pi;
VI_r = inv(ABCD)*[V_s; I_s];
V_r = VI_r(1);  VL_r = sqrt(3)*abs(V_r);
AngV_rrd = angle(V_r); AngV_r = AngV_rrd*180/pi;
```

```
I_r =  VI_r(2);  IL_r = abs(I_r)*1000; AngI_rd = angle(I_r);
AngI_r = AngI_rd*180/pi;
S_r = 3*V_r*conj(I_r); P_r = real(S_r);
Q_r = imag(S_r); A = abs(ABCD(1,1));
Reg = (VL_s/A - VL_r)/VL_r*100;
Eff = P_r/P_s*100;
fprintf('Sending end line-to-line voltage =%g KV\n', VL_s)
fprintf('Sending end voltage phase angle =%g Degree\n',AngV_s)
fprintf('Sending end real power =  %g MW\n', P_s)
fprintf('Sending end reactive Power = %g Mvar\n', Q_s)
fprintf('Sending end current  =  %g A\n', IL_s)
fprintf('Sending end current phase angle=%g Degree \n',AngI_s)
fprintf('receiving end line-to-line voltage =  %g KV\n',VL_r)
fprintf('receiving end voltage phase angle=%g Degree\n',AngV_r)
fprintf('receiving end real power =  %g MW\n', P_r)
fprintf('receiving end reactive Power =  %g Mvar\n', Q_r)
fprintf('receiving end current  =  %g A\n', IL_r)
fprintf('receiving end current phase angle=%g Degree\n',AngI_r)
fprintf('Voltage regulation = %g percent \n',Reg)
fprintf('Transmission efficiency = %g percent \n',Eff)
```

typing **ch5p7** at the *MATLAB* prompt result in

```
Line series impedance per phase per unit length z = 0.03+j*0.4
Line shunt admittance per phase per unit length y = j*4.0e-6
Transmission line length = 125

    ABCD =

       0.9875+ 0.0009i   3.7500+50.0000i
       0.0000+ 0.0005i   0.9875+ 0.0009i

    Sending end line-to-line voltage magnitude in kV = 350
    Sending end voltage phase angle in degree = 0
    Three-phase sending end real power in MW 407
    Three-phase sending end reactive power in Mvar 7.883
    Sending end line-to-line voltage =  350 kV
    Sending end voltage phase angle  =  0 Degree
    Sending end real power =  407 MW
    Sending end reactive Power =  7.883 Mvar
    Sending end current  =  671.502 A
    Sending end current phase angle =  -1.1096 Degree
    Receiving end line-to-line voltage =  345.003 kV
    Receiving end voltage phase angle  =  -9.63278 Degree
    Receiving end real power =  401.884 MW
```

```
Receiving end reactive Power =  0.0475969 Mvar
Receiving end current  =  672.539 A
Receiving end current phase angle =  -9.63957 Degree
Voltage regulation = 2.73265 percent
Transmission efficiency = 98.7429 percent
```

5.8. Obtain the solution for Problems 5.8 through 5.13 using the **lineperf** program. Then, solve each Problem using hand calculations.

A three-phase, 765-kV, 60-Hz transposed line is composed of four ACSR 1,431,000, 45/7 Bobolink conductors per phase with flat horizontal spacing of 14 m. The conductors have a diameter of 3.625 cm and a GMR of 1.439 cm. The bundle spacing is 45 cm. The line is 400 Km long, and for the purpose of this problem, a lossless line is assumed.

(a) Determine the transmission line surge impedance Z_c, phase constant β, wavelength λ, the surge impedance loading SIL, and the ABCD constant.
(b) The line delivers 2000 MVA at 0.8 lagging power factor at 735 kV. Determine the sending end quantities and voltage regulation.
(c) Determine the receiving end quantities when 1920 MW and 600 Mvar are being transmitted at 765 kV at the sending end.
(d) The line is terminated in a purely resistive load. Determine the sending end quantities and voltage regulation when the receiving end load resistance is $264.5\ \Omega$ at 735 kV.

Use the command **lineperf** to obtain the solution for problem 5.8 through 5.13.

(a) For hand calculation we have

$$GMD = \sqrt[3]{(14)(14)(28)} = 17.6389 \text{ m}$$

$$GMR_L = 1.09\sqrt[4]{(45)^3(1.439)} = 20.75 \text{ cm}$$

$$GMR_C = 1.09\sqrt[4]{(45)^3(3.625/2)} = 21.98 \text{ cm}$$

$$L = 0.2\frac{17.6389}{20.75 \times 10^{-2}} = 0.88853 \text{ mH/Km}$$

$$C = \frac{0.0556}{\ln\frac{17.6389}{21.98\times10^{-2}}} = 0.01268 \ \mu\text{F/Km}$$

$$\beta = \omega\sqrt{LC} = 2\pi \times 60\,\sqrt{0.88853 \times 0.01268 \times 10^{-9}} = 0.001265 \text{ Radian/Km}$$

$$\beta\ell = (0.001265 \times 400)(180/\pi) = 29^\circ$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.001265} = 4967 \text{ Km}$$

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.88853 \times 10^{-3}}{0.01268 \times 10^{-6}}} = 264.7\ \Omega$$

$$SIL = \frac{(KV_{Lrated})^2}{Z_c} = \frac{(765)^2}{264.7} = 2210.89 \text{ MW}$$

The $ABCD$ constants of the line are

$$\begin{aligned}
A &= \cos\beta\ell = cos29^\circ = 0.8746\\
B &= jZ_c \sin\beta\ell = j264.7\sin 29^\circ = j128.33\\
C &= j\frac{1}{Z_c}\sin\beta\ell = j\frac{1}{264.7}\sin 29^\circ = j0.0018315\\
D &= A
\end{aligned}$$

(b) The complex power at the receiving end is

$$S_{R(3\phi)} = 2000\angle 36.87^\circ = 1600 \text{ MW} + j1200 \text{ Mvar}$$

$$V_R = \frac{735\angle 0^\circ}{\sqrt{3}} = 424.352\angle 0^\circ \text{ kV}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{2000000\angle - 36.87^\circ}{3 \times 424.352\angle 0^\circ} = 1571.02\angle - 36.87^\circ \text{ A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = (0.8746)(424.352\angle 0^\circ) + (j128.33)\\ (1571.02 \times 10^{-3}\angle{-36.87^\circ}) = 517.86\angle 18.147^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 896.96 \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0018315)(424352\angle 0^\circ) + (0.8746)$$
$$(1571.02\angle{-36.87^\circ}) = 1100.23\angle{-2.46^\circ} \quad \text{A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 517.86\angle 18.147 \times 1100.23\angle 2.46^\circ \times 10^{-3} \\ &= 1600 \text{ MW} + j601.59 \text{ Mvar} \\ &= 1709.3\angle 20.6^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{896.96}{0.8746} - 735}{735} \times 100 = 39.53\%$$

(c) The complex power at the sending end is

$$S_{S(3\phi)} = 1920 \text{ MW} + j600 \text{ Mvar} = 2011.566\angle 17.354^\circ \text{ MVA}$$

$$V_S = \frac{765\angle 0^\circ}{\sqrt{3}} = 441.673\angle 0^\circ \quad \text{kV}$$

The sending end current per phase is given by

$$I_S = \frac{S^*_{S(3\phi)}}{3\,V_S^*} = \frac{2011566\angle - 17.354^\circ}{3 \times 441.673\angle 0^\circ} = 1518.14\angle - 17.354^\circ \quad \text{A}$$

The receiving end voltage is

$$V_R = DV_S - BI_S = (0.8746)(441.673\angle 0^\circ) - (j128.33)$$
$$(1518.14 \times 10^{-3}\angle{-17.354^\circ}) = 377.2\angle{-29.537^\circ} \quad \text{kV}$$

The receiving end line-to-line voltage magnitude is

$$|V_{R(L-L)}| = \sqrt{3}\,|V_S| = 653.33 \quad \text{kV}$$

The receiving end current is

$$I_R = -CV_S + AI_S = (-j0.0018315)(441673\angle 0^\circ) + (0.8746)$$
$$(1518.14\angle{-17.354^\circ}) = 1748.73\angle{-43.55^\circ} \quad \text{A}$$

The receiving end power is

$$\begin{aligned} S_{R(3\phi)} = 3V_R I_R^* &= 3 \times 377.2\angle - 29.537^\circ \times 1748.73\angle 43.55^\circ \times 10^{-3} \\ &= 1920 \text{ MW} + j479.2 \text{ Mvar} \\ &= 1978.86\angle 14.013^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{765}{0.8746} - 653.33}{653.33} \times 100 = 33.88\%$$

(d)

$$V_R = \frac{735\angle 0^\circ}{\sqrt{3}} = 424.352\angle 0^\circ \ \text{ kV}$$

The receiving end current per phase is given by

$$I_R = \frac{V_R}{Z_L} = \frac{424352\angle 0^\circ}{264.5} = 1604.357\angle 0^\circ \ \text{ A}$$

The complex power at the receiving end is

$$S_{R(3\phi)} = 3V_R I_R{}^* = 3(424.352\angle 0^\circ)(1604.357\angle 0^\circ) \times 10^{-3} = 2042.44 \ \text{ MW}$$

The sending end voltage is

$$\begin{aligned} V_S = AV_R + BI_R &= (0.8746)(424.352\angle 0^\circ) + (j128.33) \\ &= (1604.357 \times 10^{-3}\angle 0^\circ) = 424.42\angle 29.02^\circ \ \text{ kV} \end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 735.12 \ \text{ kV}$$

The sending end current is

$$\begin{aligned} I_S = CV_R + DI_R &= (j0.0018315)(424352\angle 0^\circ) + (0.8746) \\ &\quad (1604.357\angle 0^\circ) = 1604.04\angle 28.98^\circ \ \text{ A} \end{aligned}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 424.42\angle 29.02 \times 1604.04\angle - 28.98^\circ \times 10^{-3} \\ &= 2042.4 \text{ MW} + j1.4 \text{ Mvar} \\ &= 2042.36\angle 0.04^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{735.12}{0.8746} - 735}{735} \times 100 = 14.36\%$$

5.9. The transmission line of Problem 5.8 is energized with 765 kV at the sending end when the load at the receiving end is removed.

(a) Find the receiving end voltage.
(b) Determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 735 kV.
(a) The sending end voltage per phase is

$$V_S = \frac{765\angle 0^\circ}{\sqrt{3}} = 441.673\angle 0^\circ \quad \text{kV}$$

When line is open $I_R = 0$ and from (5.71) the no-load receiving end voltage is given by

$$V_{R(nl)} = \frac{V_S}{\cos\beta\ell} = \frac{441.673}{0.8746} = 505 \quad \text{kV}$$

The no-load receiving end line-to-line voltage is

$$V_{R(L-L)(nl)} = \sqrt{3}\, V_{R(nl)} = 874.68 \quad \text{kV}$$

(b) For $V_{RLL} = 735$ kV, the required inductor reactance given by (5.100) is

$$X_{Lsh} = \frac{\sin(29^\circ)}{\frac{765}{735} - \cos(29^\circ)}(264.7) = 772.13 \quad \Omega$$

The three-phase shunt reactor rating is

$$Q_{3\phi} = \frac{(KV_{Lrated})^2}{X_{Lsh}} = \frac{(735)^2}{772.13} = 699.65 \quad \text{Mvar}$$

5.10. The transmission line of Problem 5.8 is energized with 765 kV at the sending end when a three-phase short-circuit occurs at the receiving end. Determine the receiving end current and the sending end current.

For a solid short at the receiving end, $V_R = 0$ and from (5.71) and (5.72) we have

$$V_s = jZ_c \sin\beta\ell I_R$$

or

$$I_R = \frac{765000/\sqrt{3}}{j264.7 \sin 29^\circ} = 3441.7\angle - 90^\circ \ \text{ A}$$

$$I_S = \cos \beta\ell I_R = (\cos 29^\circ)(3441.7\angle - 90^\circ) = 3010\angle - 90^\circ \ \text{ A}$$

5.11. Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.8. The line delivers 2000 MVA, 0.8 lagging power factor. Determine the total Mvar and the capacitance per phase of the Y-connected capacitors to keep the receiving end voltage at 735 kV when the sending end voltage is 765 kV. *Hint*: Use (5.93) and (5.94) to compute the power angle δ and the receiving end reactive power. Find the sending end quantities and voltage regulation for the compensated line.

(a) The equivalent line reactance for a lossless line is given by

$$X' = Z_c \ \sin \beta\ell = 264.7 \sin(29^\circ) = 128.33 \ \ \Omega$$

The receiving end power is

$$S_{R(3\phi)} = 2000\angle \cos^{-1}(0.8) = 1600 + j1200 \quad \text{MVA}$$

For the above operating condition, the power angle δ is obtained from (5.93)

$$1600 = \frac{(765)(735)}{128.33} \ \sin \delta$$

which results in $\delta = 21.418^\circ$. Using the approximate relation given by (5.94), the net reactive power at the receiving end is

$$Q_{R(3\phi)} = \frac{(765)(735)}{128.33} \ \cos(21.418^\circ) - \frac{(735)^2}{128.33} \ \cos(29^\circ) = 397.05 \quad \text{Mvar}$$

Thus, the required capacitor Mvar is $S_C = j397.05 - j1200 = -j802.95$. The capacitive reactance is given by

$$X_C = \frac{|V_L|^2}{S_C^*} = \frac{(735)^2}{j802.95} = -j672.8 \ \ \Omega$$

or

$$C = \frac{10^6}{2\pi(60)(672.8)} = 3.9426 \ \ \mu\text{F}$$

The net receiving end complex power is

$$S_{R(3\phi)} = 1600 + j397.05 = 1648.53\angle 13.936^\circ \ \text{ MVA}$$

The receiving end current is

$$I_R = \frac{1600 - j397.05)10^3}{3(424.352\angle 0^\circ)} = 1294.94\angle{-13.9368^\circ} \ \text{ A}$$

We can check the sending end voltage

$$V_S = AV_R + BI_R = (0.8746)(424.352\angle 0^\circ) + (j128.33)$$
$$(1294.94 \times 10^{-3}\angle - 13.9368^\circ) = 441.67\angle 21.418^\circ \ \text{ KV}$$

or

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 765 \ \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0018315)(424352\angle 0^\circ) + (0.8746)$$
$$(1294.94\angle - 13.9368^\circ) = 1209.46\angle 24.65^\circ \ \text{ A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 441.67\angle 21.418 \times 1209.46\angle - 24.65^\circ \times 10^{-3} \\ &= 1600 \text{ MW} - j90.35 \text{ Mvar} \\ &= 1602.55\angle - 3.23^\circ \ \text{ MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{765}{0.8746} - 735}{735} \times 100 = 19.00\%$$

5.12. Series capacitors are installed at the midpoint of the line of Problem 5.8, providing 40 percent compensation. Determine the sending end quantities and the voltage regulation when the line delivers 2000 MVA at 0.8 lagging power factor at 735 kV.

For 40 percent compensation, the series capacitor reactance per phase is

$$X_{ser} = 0.4 \times X' = 0.4(128.33) = 51.33 \ \ \Omega$$

or

$$C = \frac{10^6}{2\pi(60)(51.33)} = 51.67 \ \ \mu\text{F}$$

The new equivalent π circuit parameters are given by

$$Z' = j(X' - X_{ser}) = j(128.33 - 51.33) = j77 \ \ \Omega$$
$$Y' = = j\frac{2}{Z_c}\tan(\beta\ell/2) = j\frac{2}{264.7}\tan(29^\circ/2) = j0.001954 \ \text{ siemens}$$

The new B constant is $B = j77$ and the new A and C constants are given by

$$A = 1 + \frac{Z'Y'}{2} = 1 + \frac{(j77)(j0.001954)}{2} = 0.92476$$
$$C = Y'\left(1 + \frac{Z'Y'}{4}\right) = j0.001954\left(1 + \frac{(j77)(j0.001954)}{4}\right) = 0.0018805$$

The receiving end voltage per phase is

$$V_R = \frac{735}{\sqrt{3}} = 424.352 \quad \text{kV}$$

and the receiving end current is

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} = \frac{2000\angle{-36.87^\circ}}{3 \times 424.35\angle 0^\circ} = 1.57102\angle{-36.87^\circ} \quad \text{kA}$$

Thus, the sending end voltage is

$$\begin{aligned} V_S = AV_R + BI_R &= 0.92476 \times 424.352 + j77 \times 1.57102\angle{-36.87^\circ} \\ &= 474.968\angle 11.756^\circ \ \text{ kV} \end{aligned}$$

and the line-to-line voltage magnitude is $|V_{S(L-L)}| = \sqrt{3}\,V_S = 822.67$ kV. The sending end current is

$$\begin{aligned} I_S = CV_R + DI_R &= (j0.0018805)(424352\angle 0^\circ) + (0.92476) \\ &\quad (1571.02\angle - 36.87^\circ) = 1164.59\angle{-3.628^\circ} \ \text{ A} \end{aligned}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I^*_S &= 3 \times 474.968\angle 11.756 \times 1164.59\angle 3.628^\circ \times 10^{-3} \\ &= 1600 \text{ MW} + j440.2 \text{ Mvar} \\ &= 1659.4\angle 15.38^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{822.67/0.92476 - 735}{735} \times 100 = 21.035\%$$

5.13. Series capacitors are installed at the midpoint of the line of Problem 5.8, providing 40 percent compensation. In addition, shunt capacitors are installed at the receiving end. The line delivers 2000 MVA, 0.8 lagging power factor. Determine the total Mvar and the capacitance per phase of the series and shunt capacitors to keep the receiving end voltage at 735 kV when the sending end voltage is 765 kV. Find the sending end quantities and voltage regulation for the compensated line.

The receiving end power is

$$S_{R(3\phi)} = 2000\angle\cos^{-1}(0.8) = 1600 + j1200 \quad \text{MVA}$$

With the series reactance $X' = 77\ \Omega$, and $\cos\beta\ell = A = 0.92476$, the power angle δ is obtained from (5.93)

$$1600 = \frac{(765)(735)}{77}\ \sin\delta$$

which results in $\delta = 12.657°$. Using the approximate relation given by (5.94), the net reactive power at the receiving end is

$$Q_{R(3\phi)} = \frac{(765)(735)}{77}\ \cos(12.657°) - \frac{(735)^2}{77}\ 0.92476 = 636.75 \quad \text{Mvar}$$

Thus, the required shunt capacitor Mvar is $S_C = j636.75 - j1200 = -j563.25$. The shunt capacitive reactance is given by

$$X_C = \frac{|V_L|^2}{S_C^*} = \frac{(735)^2}{j563.25} = -j959.12\ \ \Omega$$

or

$$C = \frac{10^6}{2\pi(60)(959.12)} = 2.765\ \ \mu\text{F}$$

The net receiving end complex power is

$$S_{R(3\phi)} = 1600 + j636.75 = 1722.05\angle 21.7°\ \ \text{MVA}$$

The receiving end current is

$$I_R = \frac{(1600 - j636.75)10^3}{3(424.352\angle 0°)} = 1352.69\angle{-21.7°}\ \ \text{A}$$

We can check the sending end voltage

$$V_S = AV_R + BI_R = (0.92476)(424.352\angle 0°) + (j77)$$
$$(1352.69 \times 10^{-3}\angle - 21.7°) = 441.67\angle 12.657°\ \ \text{kV}$$

or

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 765 \ \ \text{kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0018805)(424352\angle 0^\circ) + (0.92476)$$
$$(1352.69\angle -21.7^\circ) = 1209.7\angle 16.1^\circ \ \ \text{A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 441.67\angle 12.657 \times 1209.7\angle -16.1^\circ \times 10^{-3} \\ &= 1600 \text{ MW} - j96.3 \text{ Mvar} \\ &= 1602.9\angle -3.44^\circ \ \ \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{765}{0.92476} - 735}{735} \times 100 = 12.55\%$$

5.14. The transmission line of Problem 5.8 has a per phase resistance of $0.011\ \Omega$ per km. Using the **lineperf** program, perform the following analysis and present a summary of the calculation along with your conclusions and recommendations.

(a) Determine the sending end quantities for the specified receiving end quantities of $735\angle 0^\circ$ kV, 1600 MW, 1200 Mvar.
(b) Determine the receiving end quantities for the specified sending end quantities of $765\angle 0^\circ$ kV, 1920 MW, 600 Mvar.
(c) Determine the sending end quantities for a load impedance of $282.38 + j0\ \Omega$ at 735 kV.
(d) Find the receiving end voltage when the line is terminated in an open circuit and is energized with 765 kV at the sending end. Also, determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 765 kV. Obtain the voltage profile for the uncompensated and the compensated line.
(e) Find the receiving end and the sending end current when the line is terminated in a three-phase short circuit.
(f) For the line loading of part (a), determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 735 kV when line is energized with 765 kV. Obtain the line performance of the compensated line.
(g) Determine the line performance when the line is compensated by series capacitor for 40 percent compensation with the load condition in part (a) at 735 kV.

(h) The line has 40 percent series capacitor compensation and supplies the load in part (a). Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 735 kV when line is energized with 765 kV at the sending end.
(i) Obtain the receiving end circle diagram.
(j) Obtain the line voltage profile for a sending end voltage of 765 kV.
(k) Obtain the line loadability curves when the sending end voltage is 765 kV, and the receiving end voltage is 735 kV. The current-carrying capacity of the line is 5000 A per phase.

Use **lineperf** to perform all the above analyses.

5.15. The ABCD constants of a lossless three-phase, 500-kV transmission line are

$$A = D = 0.86 + j0$$
$$B = 0 + j130.2$$
$$C = j0.002$$

(a) Obtain the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

To improve the line performance, series capacitors are installed at both ends in each phase of the transmission line. As a result of this, the compensated ABCD constants become

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix}$$

where X_c is the total reactance of the series capacitor. If $X_c = 100\ \Omega$
(b) Determine the compensated ABCD constants.
(c) Determine the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

(a) The receiving end voltage per phase is

$$V_R = \frac{500\angle 0^\circ}{\sqrt{3}} = 288.675\angle 0^\circ \quad \text{kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 1000\angle \cos^{-1} 0.8 = 1000\angle 36.87^\circ = 800 + j600 \ \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V_R^*} = \frac{1000000\angle -36.87^\circ}{3 \times 288.675\angle 0^\circ} = 1154.7\angle -36.87^\circ \quad \text{A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = (0.86)(288.675\angle 0^\circ) + (j130.2)$$
$$(1154.7 \times 10^{-3}\angle{-36.87^\circ}) = 359.2\angle 19.5626^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 622.153 \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.002)(288675\angle 0^\circ) + (0.86)$$
$$(1154.7\angle{-36.87^\circ}) = 794.647\angle{-1.3322^\circ} \text{ A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 359.2\angle 19.5626 \times 794.647\angle 1.3322^\circ \times 10^{-3} \\ &= 800 \text{ MW} + j228.253 \text{ Mvar} \\ &= 831.925\angle 15.924^\circ \text{ MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{622.153}{0.86} - 500}{500} \times 100 = 44.687\%$$

(b) The compensated $ABCD$ constants are

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}j100 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.86 & j139.2 \\ j0.002 & 0.86 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2}j100 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.96 & j39.2 \\ j0.002 & 0.96 \end{bmatrix}$$

(c) Repeating the analysis for the new $ABCD$ constants result in
The sending end voltage is

$$V_S = AV_R + BI_R = (0.96)(288.675\angle 0^\circ) + (j39.2)$$
$$(1154.7 \times 10^{-3}\angle{-36.87^\circ}) = 306.434\angle 6.7865^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 530.759 \text{ kV}$$

$$I_S = CV_R + DI_R = (j0.002)(288675\angle 0^\circ) + (0.96)$$
$$(1154.7\angle{-36.87^\circ}) = 891.142\angle{-5.6515^\circ} \text{ A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 306.434\angle 6.7865 \times 891.142\angle 5.6515^\circ \times 10^{-3} \\ &= 800 \text{ MW} + j176.448 \text{ Mvar} \\ &= 819.227\angle 12.438^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{530.759}{0.96} - 500}{500} \times 100 = 10.5748\%$$

We can use **lineperf** and option 4 to obtain the above solutions.

5.16. A three-phase 420-kV, 60-HZ transmission line is 463 km long and may be assumed lossless. The line is energized with 420 kV at the sending end. When the load at the receiving end is removed, the voltage at the receiving end is 700 kV, and the per phase sending end current is $646.6\angle 90^\circ$ A.

(a) Find the phase constant β in radians per Km and the surge impedance Z_c in Ω.
(b) Ideal reactors are to be installed at the receiving end to keep $|V_S| = |V_R| = 420$ kV when load is removed. Determine the reactance per phase and the required three-phase Mvar.

(a) The sending end and receiving end voltages per phase are

$$V_S = \frac{420}{\sqrt{3}} = 242.487 \text{ kV}$$
$$V_{Rnl} = \frac{700}{\sqrt{3}} = 404.145 \text{ kV}$$

With load removed $I_R = 0$, from (5.71) we have

$$242.487 = (\cos \beta\ell)(404.145)$$

or

$$\beta\ell = 53.13^\circ = 0.927295 \text{ Radian}$$

and from (5.72), we have

$$j646.6 = j\frac{1}{Z_c}(\sin 53.13^\circ)(404.145)10^3$$

or

$$Z_c = 500 \ \Omega$$

(b) For $V_S = V_R$, the required inductor reactance given by (5.100) is

$$X_{Lsh} = \frac{\sin(53.13°)}{1 - \cos(53.13°)}(500) = 1000 \quad \Omega$$

The three-phase shunt reactor rating is

$$Q_{3\phi} = \frac{(KV_{Lrated})^2}{X_{Lsh}} = \frac{(420)^2}{1000} = 176.4 \quad \text{Mvar}$$

5.17. A three-phase power of 3600 MW is to be transmitted via four identical 60-Hz transmission lines for a distance of 300 Km. From a preliminary line design, the line phase constant and surge impedance are given by $\beta = 9.46 \times 10^{-4}$ radian/Km and $Z_c = 343\ \Omega$, respectively.
Based on the practical line loadability criteria determine the suitable nominal voltage level in kV for each transmission line. Assume $V_S = 1.0$ pu, $V_R = 0.9$ pu, and the power angle $\delta = 36.87°$.

$$\beta\ell = (9.46 \times 10^{-4})(300)(\frac{180}{\pi}) = 16.26°$$

The real power per transmission circuit is

$$P = \frac{3600}{4} = 900 \;\; \text{MW}$$

From the practical line loadability given by(5.97), we have

$$900 = \frac{(1.0)(0.9)(SIL)}{\sin(16.26°)} \;\; \sin(36.87°)$$

Thus

$$SIL = 466.66 \quad \text{MW}$$

From (5.78)

$$KV_L = \sqrt{(Z_c)(SIL)} = \sqrt{(343)(466.66)} = 400 \quad \text{kV}$$

5.18. Power system studies on an existing system have indicated that 2400 MW are to be transmitted for a distance of 400 Km. The voltage levels being considered include 345 kV, 500 kV, and 765 kV. For a preliminary design based on the practical line loadability, you may assume the following surge impedances

345 kV $Z_C = 320\ \Omega$
500 kV $Z_C = 290\ \Omega$
765 kV $Z_C = 265\ \Omega$

The line wavelength may be assumed to be 5000 Km. The practical line loadability may be based on a load angle δ of $35°$. Assume $|V_S| = 1.0$ pu and $|V_R| = 0.9$ pu. Determine the number of three-phase transmission circuits required for each voltage level. Each transmission tower may have up to two circuits. To limit the corona loss, all 500-kV lines must have at least two conductors per phase, and all 765-kV lines must have at least four conductors per phase. The bundle spacing is 45 cm. The conductor size should be such that the line would be capable of carrying current corresponding to at least 5000 MVA. Use **acsr** command in *MATLAB* to find a suitable conductor size. Following are the minimum recommended spacings between adjacent phase conductors at various voltage levels.

Voltage level, kV	Spacing meter
345	7.0
500	9.0
765	12.5

(a) Select a suitable voltage level, and conductor size, and tower structure. Use **lineperf** program and option 1 to obtain the voltage regulation and transmission efficiency based on a receiving end power of 3000 MVA at 0.8 power factor lagging at the selected rated voltage. Modify your design and select a conductor size for a line efficiency of at least 94 percent for the above specified load.
(b) Obtain the line performance including options 4–8 of the **lineperf** program for your final selection. Summarize the line characteristics and the required line compensation.

For each voltage level SIL is computed from (5.78). From the practical line loadability equation given by (5.97), the real power per circuit is computed, and number of circuits is established and tabulated in the following table.

Voltage KV	Surge Imp., Ω	SIL MW	Power/ circuit	No.of circuit
345	320	371.95	398.56	$\frac{2400}{398.56} = 6$
500	290	862.07	923.74	$\frac{2400}{923.74} = 3$
765	265	2208.39	2366.39	$\frac{2400}{2366.39} = 1$

For the 345 kV level we require six three-phase transmission circuit, and for the 500 kV line we need three circuits. Considering the cost of the transmission towers, right of ways and associated equipment, it can be concluded that one circuit at 765 kV is the most economical and practical choice.

Using the 765 kV for transmission line voltage, the per phase current corresponding to 5000 MVA maximum is

$$I = \frac{5000 \times 10^3}{\sqrt{3}\,765} = 3773.5 \ \ \text{A}$$

Since we have four conductor per phase, then current per conductor is

$$I_{cond} = \frac{3773.5}{4} = 943.38 \ \ \text{A}$$

From the **acsr** file we select the CRANE conductor with a current-carrying capacity of 950 A. Typing acsr at the *MATLAB* prompt and selecting 'crane', result in

```
Enter ACSR code name within single quotes -> 'crane'
Al Area Strand  Diameter GMR  Resistance Ohm/km  Ampacity
 cmil   Al/St      cm      cm   60Hz 25C  60Hz 50C  Ampere
874500  54/7    2.9108   1.1765  0.06712   0.07632  950
```

Using **lineperf** result in

```
                    TRANSMISSION LINE MODEL
Type of parameters for input              Select

Parameters per unit length
r(ohm), g(seimens) L(mH) & C (micro F)       1

Complex z and y per unit length
r+j*x (ohm/length), g+j*b (seimens/length)   2

Nominal pi or Eq. pi model                   3

A, B, C, D constants                         4

Conductor configuration and dimension        5

To quit                                      0

Select number of menu --> 5
Enter spacing unit within quotes 'm' or 'ft'-> 'm'
Enter row vector [D12, D23, D13] = [12.5  12.5  25]
Cond. size, bundle spacing unit: Enter 'cm' or 'in'-> 'cm'
Conductor diameter in cm = 2.9108
Geometric Mean Radius in cm = 1.1765
No. of bundled cond. (enter 1 for single cond.) = 4
Bundle spacing in cm = 45
```

```
GMD  = 15.74901 m
GMRL =  0.19733 m   GMRC =  0.20811 m
L = 0.875934 mH/km     C = 0.0128525 micro F/km
Enter Line length = 400
Enter Frequency in Hz = 60
Enter line resistance/phase in ohms per unit length = 0.06712/4
Enter line conductance/phase in siemens per unit length = 0
Is the line model Short? Enter 'Y' or 'N' within quotes --> 'n'

Equivalent pi model
 --------------------
Z' = 6.15014 + j 126.538 ohms
Y' = 2.21291e-006 + j 0.00198054 siemens
Zc  = 261.145 + j -6.63073 ohms
alpha l = 0.0128511 neper beta l = 0.506128 radian = 28.999

        0.8747      + j 0.0062303    6.1501 + j 126.54
ABCD =
       -4.0954e-006 + j 0.0018565    0.8747 + j 0.0062303

    TRANSMISSION LINE PERFORMANCE
     ----------Analysis----------             Select
 To calculate sending end quantities
 for specified receiving end MW, Mvar            1

 To calculate receiving end quantities
 for specified sending end MW, Mvar              2

 To calculate sending end quantities
 when load impedance is specified                3

 Open-end line & inductive compensation          4

 Short-circuited line                            5

 Capacitive compensation                         6

 Receiving end circle diagram                    7

 Loadability curve and voltage profile           8

 To quit                                         0
 Select number of menu -->
```

Selecting options 1 through 8 result in

```
Line performance for specified receiving end quantities
-------------------------------------------------------
Vr = 765 kV (L-L)  at 0
Pr = 2400 MW   Qr = 1800 Mvar
Ir = 2264.12 A  at -36.8699  PFr = 0.8 lagging
Vs = 1059.49 kV (L-L)  at 21.4402
Is = 1630.56 A  at -12.6476  PFs = 0.82818 lagging
Ps = 2478.108   MW   Qs = 1677.037   Mvar
PL = 78.108     MW   QL = -122.963   Mvar
Percent Voltage Regulation   = 58.3312
Transmission line efficiency = 96.8481

Open line and shunt reactor compensation
----------------------------------------
Vs = 765 kV (L-L)  at 0
Vr = 874.563 kV (L-L)  at -0.00712268
Is = 937.387 A  at 89.7183  PFs = 0.00491663 leading

Desired no load receiving end voltage = 765 kV
Shunt reactor reactance = 1009.82 ohm
Shunt reactor rating = 579.531 Mvar
Hit return to continue

Line short-circuited at the receiving end
-----------------------------------------
Vs = 765 kV (L-L)  at 0
Ir = 3486.33 A  at -87.2174
Is = 3049.57 A  at -86.8093

Shunt capacitive compensation
-----------------------------
Vs = 765 kV (L-L)  at 31.8442
Vr = 765 kV (L-L)  at 0
Pload = 2400 MW   Qload = 1800 Mvar
Load current = 2264.12 A  at -36.8699  PFl = 0.8 lagging
Required shunt capcitor:287.826ohm,9.21593microF,2033.26 Mvar
Shunt capacitor current = 1534.51 A  at 90
Pr = 2400.000   MW   Qr = -233.261   Mvar
Ir = 1819.83 A  at 5.55126  PFr = 0.99531 leading
Is = 1863.22 A  at 31.9226  PFs = 0.999999 leading
Ps = 2468.802   MW   Qs = -3.378     Mvar
PL = 68.802     MW   QL = 229.882    Mvar
Percent Voltage Regulation   = 14.322
```

```
Transmission line efficiency = 97.2131

Series capacitor compensation
-----------------------------
Vr = 765 kV (L-L)  at 0
Pr = 2400 MW   Qr = 1800 Mvar
Required series capacitor:50.6151ohm,52.4069microF, 209.09Mvar
Subsynchronous resonant frequency = 37.9473 Hz
Ir = 2264.12 A  at -36.8699  PFr = 0.8 lagging
Vs = 933.799 kV (L-L)  at 14.1603
Is = 1729.43 A  at -13.485  PFs = 0.885837 lagging
Ps = 2477.831   MW   Qs = 1297.875   Mvar
PL = 77.831     MW   QL = -502.125   Mvar
Percent Voltage Regulation   = 31.9848
Transmission line efficiency = 96.8589

Series and shunt capacitor compensation
---------------------------------------
Vs = 765 kV (L-L)  at 18.5228
Vr = 765 kV (L-L)  at 0
Pload = 2400 MW   Qload = 1800 Mvar
Load current = 2264.12 A  at -36.8699  PFl = 0.8 lagging
Required shunt capcitor: 322.574ohm, 8.22319microF,1814.24Mvar
Shunt capacitor current = 1369.22 A  at 90
Required series capacitor:50.6151ohm,52.4069microF,176.311Mvar
Subsynchronous resonant frequency = 37.9473 Hz
Pr = 2400 MW   Qr = -14.2373 Mvar
Ir = 1811.33 A  at 0.339886  PFr = 0.999982 leading
Is = 1882.74 A  at 27.2821  PFs = 0.988337 leading
Ps = 2465.565   MW   Qs = -379.900   Mvar
PL = 65.565     MW   QL = -365.662   Mvar
Percent Voltage Regulation   = 8.12641
Transmission line efficiency = 97.3408
```

To transmit 2400 MW for a distance of 400 Km, we select a horizontal tower at 765 kV with four ACSR crane conductor per phase. The recommended compensators are:

- A three-phase shunt reactor for the open-ended line with a maximum rating of 580 Mvar at 765 kV.
- A three-phase shunt capacitor with a maximum rating of 1815 Mvar at 765 kV.
- A three-phase series capacitor with a maximum rating of 176 Mvar, 765 kV.