Quantum Electronics

1. Introduction

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Overview

1. Marking Scheme
2. References
3. Syllabus (Contents)
4. Motivation: (Chapter 1: Introduction)

Marking

- Seminar: 15%
- Assignments (9): 25%
- Final Exam: 60%

References

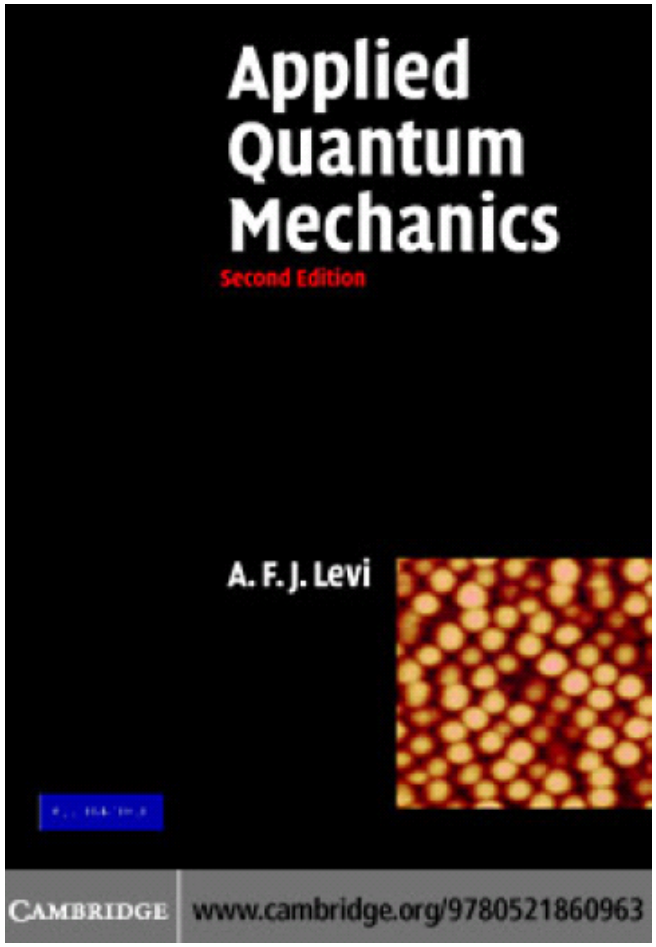
❑ Main references

➢ Textbook:

- **A. F. J. Levi**,

" ***Applied Quantum Mechanics,***"
Second Edition,
Cambridge University Press, 2006

- **Lecture notes**



References

Other Useful references:

1. **Jasprit Singh, "*Quantum Mechanics: Fundamentals and Applications to Technology*", John Wiley & sons, Inc., 2001.**
2. **Amnon Yarive, " An Introduction to Theory and Applications of Quantum Mechanics", John Wiley & sons, Inc., 1982.**
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5. م. س. روگالسکی، س. ب. پالمر، ترجمه دکتر محمدابراهیم زمردیان، *فیزیک کوانتومی*، انتشارات دانشگاه فردوسی مشهد، 1381.

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پراکندگی: (scattering)

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Appendix F: The Greek alphabet

A	α	alpha	= a
B	β	beta	= b
Γ	γ	gamma	= g
Δ	δ	delta	= d
E	ε	epsilon	= e
Z	ζ	zeta	= z
H	η	eta	= e
Θ	θ	theta	= th(th)
I	ι	iota	= i
K	κ	kappa	= k
Λ	λ	lambda	= l

M	μ	mu	= m
N	ν	nu	= n
Ξ	ξ	xi	= x(ks)
Π	π	pi	= p
P	ρ	rho	= r
Σ	σ	sigma	= s
T	τ	tau	= t
Y	υ	upsilon	= u
Φ	ϕ	phi	= pf(f)
X	χ	chi	= kh(hh)
Ψ	ψ	psi	= ps
Ω	ω	omega	= o

Introduction: **1.1 Motivation**

❖**why we needs to know about quantum mechanics?**

❑**Because we live in a quantum world!**

➢**Engineers** would like to make and control electronic, opto-electronic, and optical devices on an atomic scale.

➢In **biology** there are molecules and cells we wish to understand and modify on an atomic scale.

➢In **chemistry**, where an important goal is the synthesis of both organic and inorganic compounds with precise atomic composition and structure.

❖Quantum mechanics gives the engineer, the biologist, and the chemist the tools with which to study and control objects on an atomic scale.

1.1 Motivation (Cont.)

❖**In the electrical engineering, there are important reasons to study quantum mechanics.**

➢one simple motivation is the fact that transistor dimensions will soon approach a size where single-electron and quantum effects determine device performance.

➢**Moore's Law**: The number of transistors in silicon ICs increases by a factor of two every eighteen months.

➢**Moore's law** predicts that DRAM cell size will be *less than that of an atom by the year 2030.*

➢We need to learn to use quantum mechanics to make sure that we can create the smallest, highest-performance devices possible.

1.1 Motivation (Cont.)

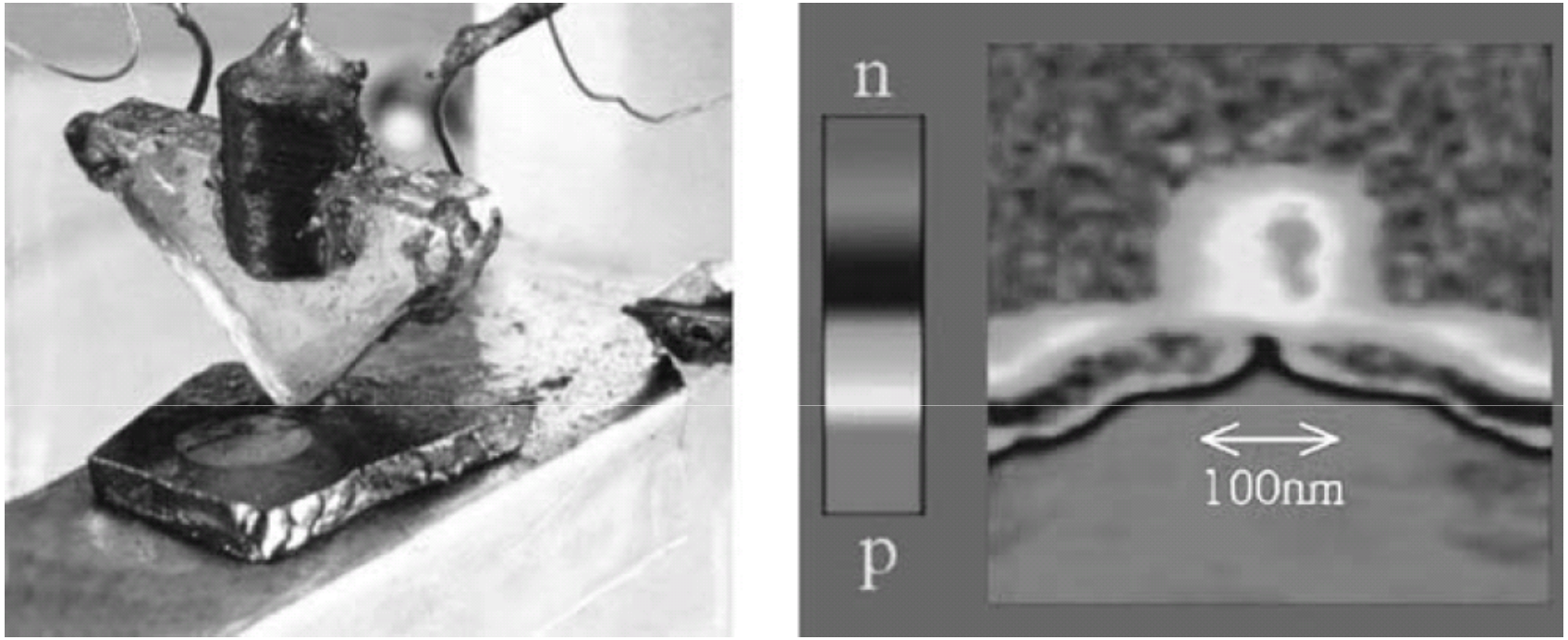


Fig. 1.2 Photograph (left) of the first transistor. Brattain and Bardeen's p-n-p point-contact germanium transistor on December 23, 1947. The device is **a few mm in size**. On the right is a scanning capacitance microscope cross-section image of a silicon p-type P-MOSFET with an effective channel length of **about 20 nm**, or **about 60 atoms.**

1.2 Classical mechanics

1.2.1 Introduction

correspondence principle: classical mechanics is often assumed to be the macroscopic (large-scale) limit of quantum mechanics. *Formally, one requires that the results of classical mechanics be obtained in* the limit $\hbar \to 0$. $h = 2\pi\hbar$

Newotonian Mechanics:

classical or newtonian mechanics allows a continuous spectrum of energies and allows continuous spatial distribution of matter.

Motion of macroscopic material bodies is usually described by classical mechanics.

1.2.1 Introduction: (*Newotonian Mechanics*)

A particle with **mass m** is fully described by the particle's position $[x(t), y(t), z(t)]$ and its momentum $[p_x(t), p_y(t), p_z(t)]$.

Newton's first law: $\mathbf{p(t) = m\, v(t) = m\, dx/dt = const.}$ **(1.a)**

where **v =dx/dt** is the **velocity** of the object moving in the direction of the unit vector $x\tilde{} = x/|x|$.

Units: Time is seconds (s), and distance is meters (m).

The momentum is (kg m s^{-1}), and the velocity (speed) is (m s^{-1}).

The motion of the object is described by the differential equation:

Newton's second law: $\mathbf{F = dp/dt = m\, d^2x/dt^2 = ma}$ **(1.b)**

where the vector F is the force. The magnitude of force is measured in units of newtons (N).

1.2.1 Introduction: (*Newotonian Mechanics*)

The **work done (Energy)** moving the object from point 1 to point 2 in space along a path is *defined as:*

$$W_{12} = \int_{\mathbf{r}=\mathbf{r}_1}^{\mathbf{r}=\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r} \qquad (2)$$

where r is a spatial vector coordinate.

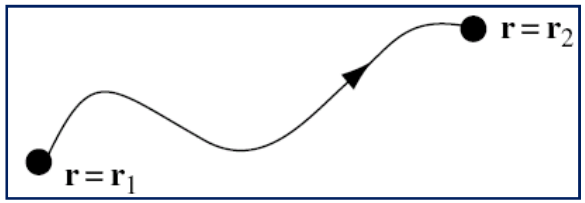


Fig. 1.3 Illustration of a classical particle trajectory from position r1 to r2.

For a *conservative force field,* use of the fact ***F = dp/dt =m dv/dt,*** one may write:

$$W_{12} = \int_{\mathbf{r}=\mathbf{r}_1}^{\mathbf{r}=\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r} = m \int d\mathbf{v}/dt \cdot \mathbf{v} dt = \frac{m}{2} \int \frac{d}{dt}(v^2) dt \qquad (3)$$

$$W_{12} = m(v_2^2 - v_1^2)/2 = T_2 - T_1$$

where $v^2 = \mathbf{v} \cdot \mathbf{v}$

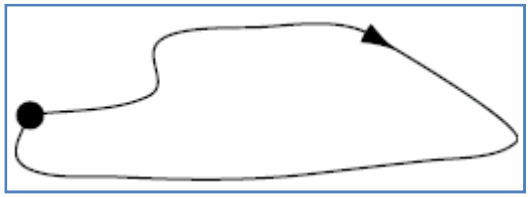
$T=E_{kin} = mv^2/2=p^2/2m$

and the scalar $T = mv^2/2$ is called the kinetic energy of the object.

1.2.1 Introduction: (*Newotonian Mechanics*)

For conservative forces, for any, the work done around any *closed path*, is always zero, or:

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0 \qquad (4)$$



$$\mathbf{F} = -\nabla V(\mathbf{r}) \qquad (5)$$

since $\oint \mathbf{F} \cdot d\mathbf{r} = -\oint \nabla V \cdot d\mathbf{r} = -\oint dV = 0.$

For a nonconservative force, such as a particle subject to frictional forces, the work done around any closed path is not zero, and $\oint \mathbf{F} \cdot d\mathbf{r} \neq 0$

V(r) is called the potential. Potential is measured in **volts (V),** and **potential energy** :in **joules (J)** or **electron volts (eV).**

Then **total energy**, which is the sum of **kinetic** and **potential energy,** is a **constant of the motion**.

In other words, total energy **T +V is conserved.**

Conservative: پایستار
Frictional:اصطکاکی

1.2.1 Introduction: (*Newotonian Mechanics*)

Hamiltonian function: H = T +V **(6.a)**

T*: kinetic energy** and ***V:* potential energy** can be expressed as functions of the variable's **position** and **time**, it is possible to define a ***Hamiltonian function *for* dynamics of particles in *the system.*

The *Hamiltonian function H(x,p)* is defined as energy of a system:

$$H(x,p) = \frac{p^2}{2m} + V(X) = T + V(x) \qquad \textbf{(6.b)}$$

1.2.1 Hamiltonian formulation: (*Newotonian Mechanics*)

$$H(x,p) = \frac{p^2}{2m} + V(X) = T + V(x) \qquad (6.b)$$

$$\frac{\partial}{\partial x} H(x,p) = \frac{d}{dx} V(x) \qquad (6.c)$$

$$\frac{\partial}{\partial p} H(x,p) = \frac{p}{m} \qquad (6.d)$$

$$p(t) = mv(t) = m\frac{dx(t)}{dt} = const. \qquad (6.e)$$

$$F(x) = -\frac{d}{dx} V(x) \qquad (6.f)$$

Hamiltonian equations of motions:

$$\frac{dx}{dt} = \frac{\partial}{\partial p} H(x,p) \qquad \frac{dp}{dt} = -\frac{\partial}{\partial x} H(x,p) \qquad (6.g)$$

1.2.2 The one-dimensional simple harmonic oscillator

Hooke's law: $F = -kx$, *where* k is the spring constant.

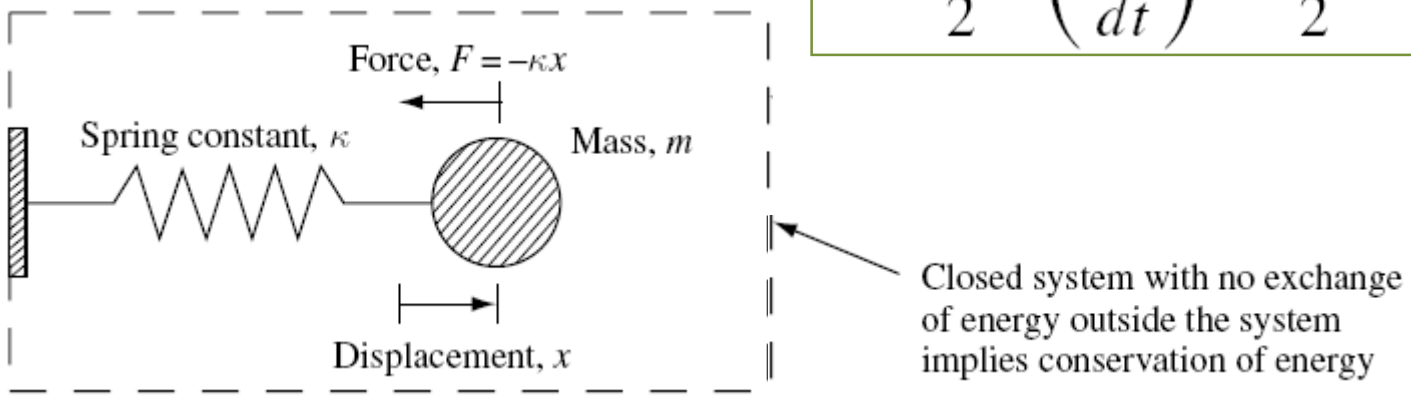
Total energy function or Hamiltonian for the system is H=T+V. (6)

potential energy: $V = \frac{1}{2}\kappa x^2 = \int_0^x \kappa x' dx'$ (6-V)

kinetic energy: $T = m(dx/dt)^2/2$ (6-T)

so that:

$$H = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\kappa x^2 \quad (7)$$



1.2.2 The one-dimensional simple harmonic oscillator

The system is *closed, so there is no exchange of energy outside the system. There is* no dissipation, total energy in the system is a constant, and:

$$H = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\kappa x^2 \qquad (7)$$

$$\frac{dH}{dt} = 0 = m\frac{dx}{dt}\frac{d^2x}{dt^2} + \kappa x\frac{dx}{dt} \qquad (8)$$

so that the *equation of motion can be written as*

$$\kappa x + m\frac{d^2x}{dt^2} = 0 \qquad (9)$$

The solutions for this second-order linear differential equation are:

$$x(t) = A\cos(\omega_0 t + \phi) \qquad (10)$$

$$\frac{dx(t)}{dt} = -\omega_0 A\sin(\omega_0 t + \phi) \qquad (11)$$

$$\frac{d^2x(t)}{dt^2} = -\omega_0^2 A\cos(\omega_0 t + \phi) \qquad (12)$$

A: amplitude of oscillation,
ω_0: angular frequency of oscillation (rad s^{-1}),
$\varnothing$: fixed phase.

1.2.2 The one-dimensional simple harmonic oscillator

potential energy and kinetic energy:

$$V = \frac{1}{2}\kappa^2 A^2 \cos^2(\omega_0 t + \phi) \quad (13) \qquad T = \frac{1}{2} m\omega_0^2 A^2 \sin^2(\omega_0 t + \phi) \quad (14)$$

Total energy:

$$\boxed{E = T + V = m\omega_0^2 A^2/2 = \kappa A^2/2} \quad (15)$$

since $\sin^2(\theta) + \cos^2(\theta) = 1$ and $\kappa = m\omega_0^2$.

An increase in total energy, increases amplitude, and an increase in *k*, corresponding to an increase in the stiffness of the spring, decreases A.

$$A = \sqrt{2E/\kappa} = \sqrt{2E/m\omega_0^2},$$

The theory gives us the relationships among all the parameters of the classical harmonic oscillator: *k*, m, A, and total energy.

1.2.2 The one-dimensional simple harmonic oscillator

- We have shown that the classical simple harmonic oscillator vibrates in a single *mode* with frequency ω_0.

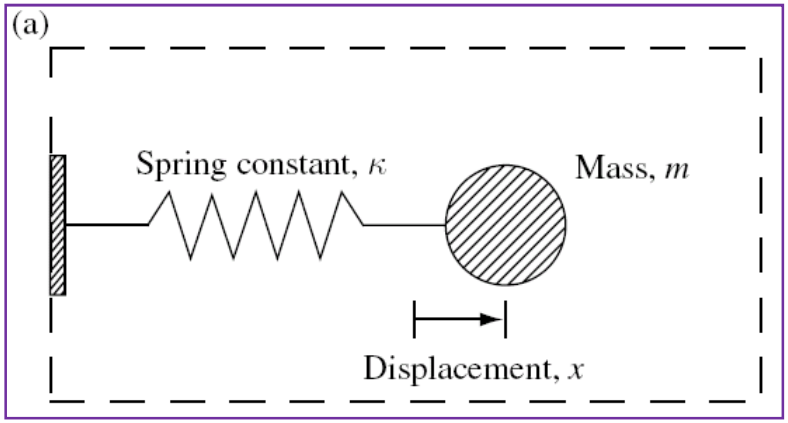
- The vibrational energy stored in the mode can be changed continuously by varying the amplitude of vibration, A.

$$E = T + V = m\omega_0^2 A^2/2 = \kappa A^2/2 \qquad (15)$$

$$A = \sqrt{2E/\kappa} = \sqrt{2E/m\omega_0^2},$$

Example for the classical one-dimensional harmonic oscillator

Example:
Spring constant, $\kappa = 360\ \mathrm{N\ m^{-1}}$
Particle mass, $m = 0.1\ \mathrm{kg}$
Oscillation amplitude, $A = 0.01\ \mathrm{m}$
Kinetic energy, $T(x) = \kappa(A^2 - x^2)/2$
Potential energy, $V(x) = \kappa x^2/2$

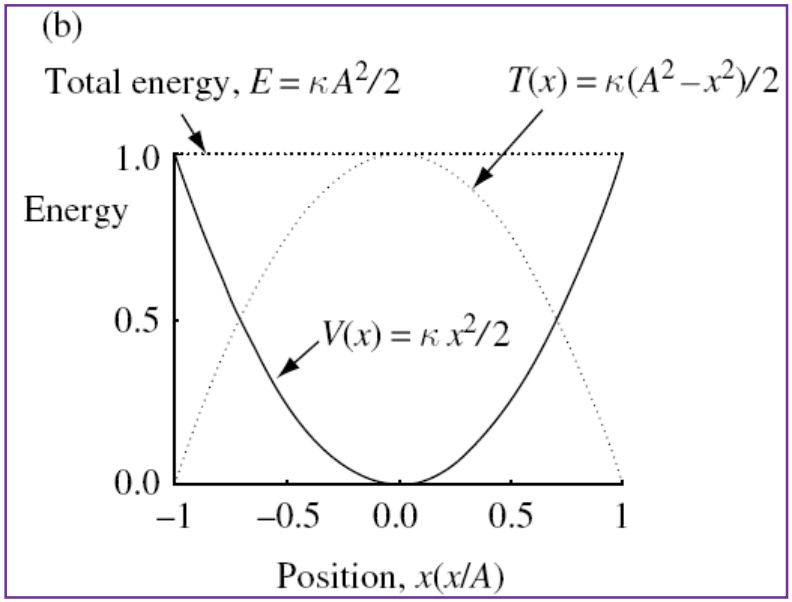


$$\omega_0 = \sqrt{\kappa/m} = 60\ \mathrm{rad\ s^{-1}}$$

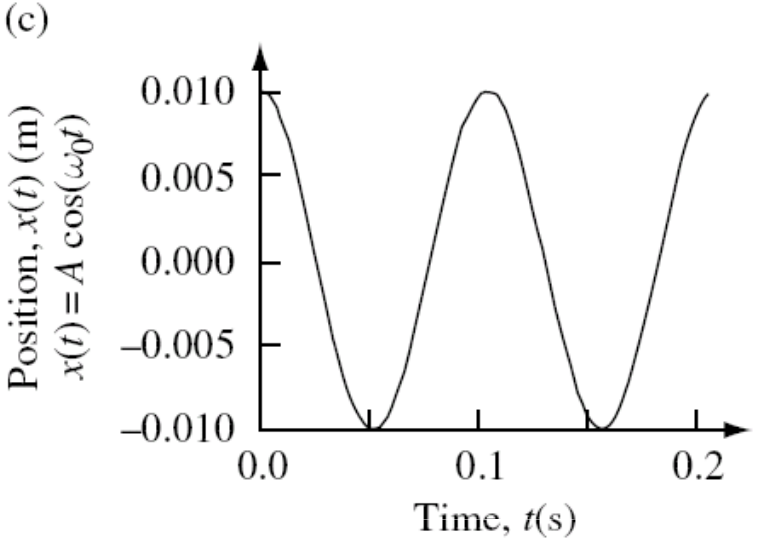
$$\omega = 2\pi\nu$$

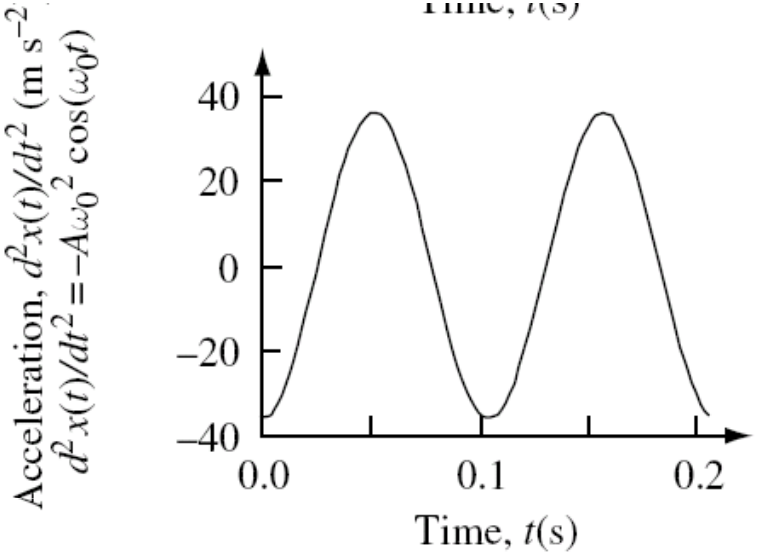
$$\nu \sim 9.5\ \mathrm{Hz} \text{ and } \tau \sim 0.1\ \mathrm{s}$$

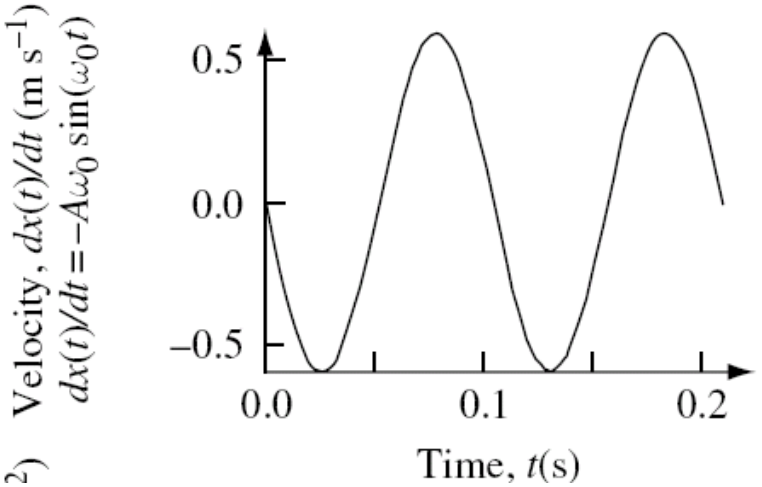
$$E = \kappa A^2/2 = 18\ \mathrm{mJ}$$



Example for the classical one-dimensional harmonic oscillator







$$\omega_0 = \sqrt{\kappa/m} = 60\,\text{rad s}^{-1}$$

$$\omega = 2\pi\nu$$

$$\nu \sim 9.5\,\text{Hz and } \tau \sim 0.1\,\text{s}$$

$$E = \kappa A^2/2 = 18\,\text{mJ}$$

1.2.3 Harmonic oscillation of a diatomic molecule

We will show that the **Hamiltonian** can be separated into **center of mass motion** and **relative motion of the two atoms.**

Frequency of oscillation:

$$\omega = \sqrt{k/m_r}$$

where *k* is the **spring constant** and m_r is the *reduced mass such that:*

$$1/m_r = 1/m_1 + 1/m_2.$$

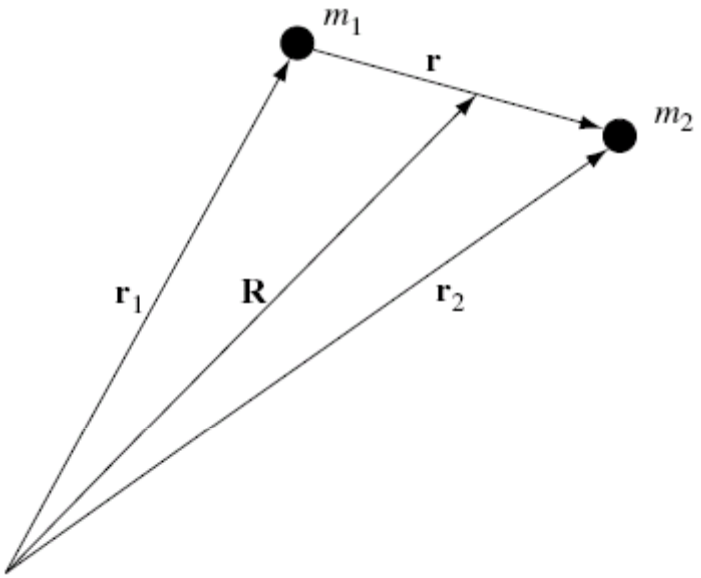
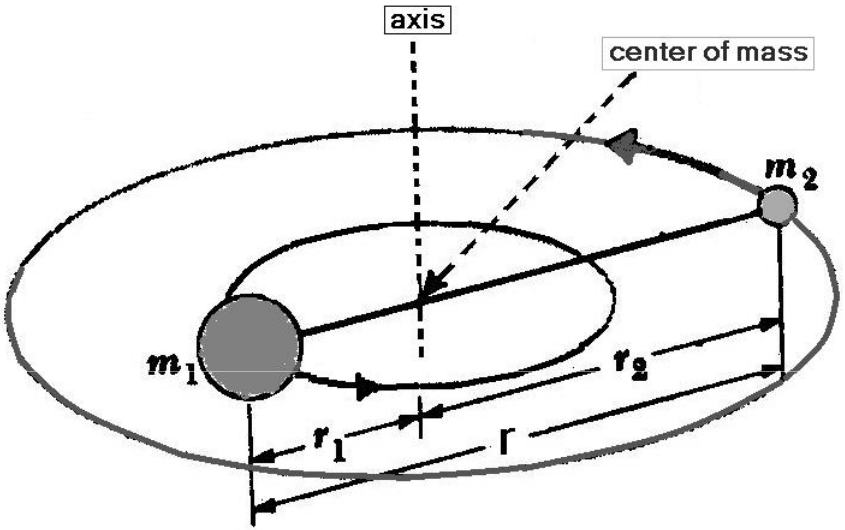


Fig. 1.7 Illustration of a diatomic molecule consisting of two atoms with mass m_1 and m_2 and position r_1 and r_2 respectively. The relative position vector is $r = r_2 - r_1$ and the center of mass coordinate is R.

1.2.3 Harmonic oscillation of a diatomic molecule



یادآوری: هسته و الکترون هر دو به دور مرکز جرمشان که خیلی به هسته نزدیک است، می چرخند، زیرا جرم هسته خیلی بیشتر از جرم الکترون است. سیستمی از این نوع معادل یک ذره منفرد با جرم m_r است که به دور موضع ذره سنگین تر می چرخد. اگر m_1 جرم هسته و m_2 جرم الکترون باشد، m_r به طریق زیر بدست می آید:

$$m_r = \frac{m_1 m_2}{m_1 + m_2}$$

کمیت m_r را جرم کاهش یافته الکترون می نامند، زیرا مقدار آن از m_2 کمتر است.

با مطالعه و بررسی این مسایل، ما می خواهیم با مفهوم رابطه پاشیدگی (**dispersion**) آشنا شویم.

$$\omega = \omega(q) \qquad q = \frac{2\pi}{\lambda} \qquad v(p) = \frac{\omega}{q} \qquad v_g = \frac{\partial \omega}{\partial q}\Big|_{q=q_0}$$

1.2.3 Harmonic oscillation of a diatomic molecule

We assume that the forces, and hence the potential, governing relative motion depend only on the magnitude of the difference vector so that

$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$.

If we choose the origin as the center of mass then $m_1\mathbf{r}_1 + m_2\mathbf{r}_2 = 0$.

$$\mathbf{r}_1 = \frac{-m_2}{m_1}\mathbf{r}_2 = \frac{-m_2}{m_1}(\mathbf{r}_1 - \mathbf{r}) \quad (18)$$

$$\mathbf{r}_1\left(1 + \frac{m_2}{m_1}\right) = \frac{m_2}{m_1}\mathbf{r} \quad (19)$$

$$\mathbf{r}_1(m_1 + m_2) = m_2\mathbf{r} \quad (20)$$

$$\mathbf{r}_1 = \frac{m_2}{(m_1 + m_2)}\mathbf{r} \quad (21)$$

$$\mathbf{r}_1 = \frac{m_2}{(m_1 + m_2)}\mathbf{r} \quad (16)$$

and

$$\mathbf{r}_2 = \frac{m_1}{(m_1 + m_2)}\mathbf{r} \quad (17)$$

1.2.3 Harmonic oscillation of a diatomic molecule

Now, combining center of mass motion and relative motion, the **Hamiltonian** is the sum of kinetic and potential energy terms **T** and **V** respectively, so:

$$H = T + V$$

$$= \frac{1}{2}m_1\left(\frac{d\mathbf{R}}{dt} - \frac{m_2}{(m_1+m_2)}\frac{d\mathbf{r}}{dt}\right)^2 + \frac{1}{2}m_2\left(\frac{d\mathbf{R}}{dt} + \frac{m_1}{(m_1+m_2)}\frac{d\mathbf{r}}{dt}\right)^2 + V(|\mathbf{r}|) \qquad (22)$$

where the total kinetic energy is

$$T = \frac{1}{2}(m_1+m_2)\left(\frac{d\mathbf{R}}{dt}\right)^2 + \frac{1}{2}\frac{m_1m_2^2}{(m_1+m_2)^2}\left(\frac{d\mathbf{r}}{dt}\right)^2 + \frac{1}{2}\frac{m_1^2m_2}{(m_1+m_2)^2}\left(\frac{d\mathbf{r}}{dt}\right)^2 \qquad (23)$$

$$= \frac{1}{2}(m_1+m_2)\left(\frac{d\mathbf{R}}{dt}\right)^2 + \frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}\left(\frac{d\mathbf{r}}{dt}\right)^2$$

or

$$T = \frac{1}{2}M\left(\frac{d\mathbf{R}}{dt}\right)^2 + \frac{1}{2}m_r\left(\frac{d\mathbf{r}}{dt}\right)^2 \qquad (24)$$

$$\mathbf{r}_1 = \frac{m_2}{(m_1+m_2)}\mathbf{r}$$

and

$$\mathbf{r}_2 = \frac{m_1}{(m_1+m_2)}\mathbf{r}$$

1.2.3 Harmonic oscillation of a diatomic molecule

$$T = \frac{1}{2}M\left(\frac{d\mathbf{R}}{dt}\right)^2 + \frac{1}{2}m_r\left(\frac{d\mathbf{r}}{dt}\right)^2 \quad (24)$$

In Eq. (1.24) the total mass is

$$M = m_1 + m_2 \quad (25)$$

and the *reduced mass* is

$$m_r = \frac{m_1 m_2}{(m_1 + m_2)} \quad (26)$$

1.2.3 Harmonic oscillation of a diatomic molecule

If we assume the potential is harmonic then the frequency of oscillation of the molecule is given by $\omega = \sqrt{k/m_r}$

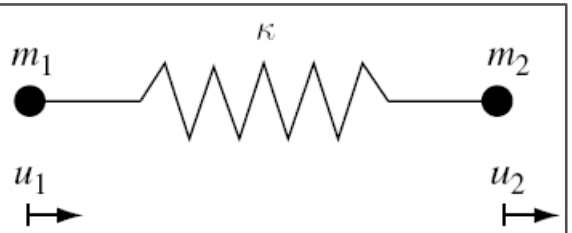


Fig. 1.8 diatomic molecule restricted to motion in one dimension.

The equations of motion for the system are

$$m_1 \frac{d^2 u_1}{dt^2} = \kappa(u_2 - u_1) \qquad (27)$$

$$m_2 \frac{d^2 u_2}{dt^2} = \kappa(u_1 - u_2) \qquad (28)$$

which has solution of the form $e^{-i\omega t}$ giving

$$(\kappa - m_1\omega^2)u_1 - \kappa u_2 = 0 \qquad (29)$$

$$-\kappa u_1 + (\kappa - m_2\omega^2)u_2 = 0 \qquad (30)$$

1.2.3 Harmonic oscillation of a diatomic molecule

$$\boxed{\begin{aligned}(\kappa - m_1\omega^2)u_1 - \kappa u_2 &= 0 \\ -\kappa u_1 + (\kappa - m_2\omega^2)u_2 &= 0\end{aligned}} \quad (29)\ (30)$$

$$\begin{vmatrix} \kappa - m_1\omega^2 & -\kappa \\ -\kappa & \kappa - m_2\omega^2 \end{vmatrix} = (\kappa - m_1\omega^2)(\kappa - m_2\omega^2) - \kappa^2 = 0 \quad (31)$$

Hence,

$$\kappa = \left(\frac{m_1 m_2}{m_1 + m_2}\right)\omega^2 = m_r\omega^2 \quad (32)$$

and, as before, the frequency of oscillation is just

$$\omega = \sqrt{\kappa\left(\frac{m_1 + m_2}{m_1 m_2}\right)} = \sqrt{\kappa/m_r} \quad (33)$$

1.2.4 The monatomic linear chain (1)

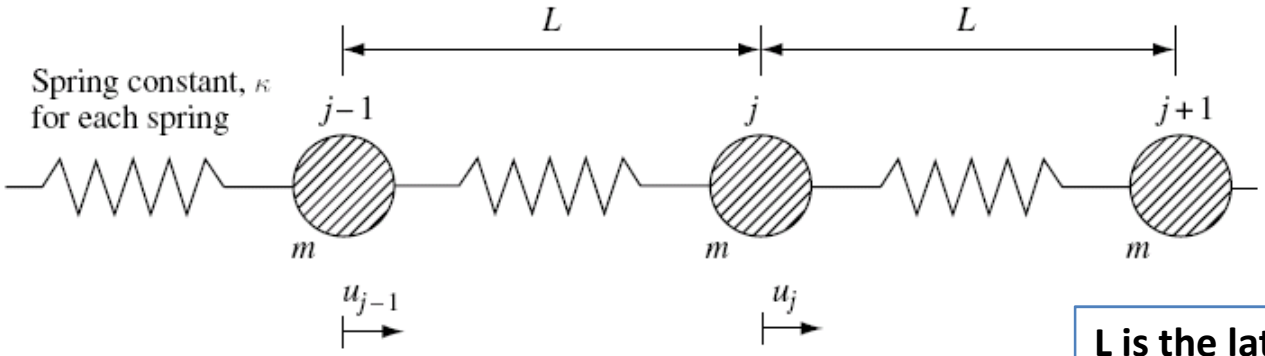
Assuming small deviations uj from equilibrium, the Hamiltonian is:

$$H = \sum_j \frac{m}{2}\left(\frac{du_j}{dt}\right)^2 + V_0(0) + \frac{1}{2!}\sum_{jk}\frac{\partial^2 V_0}{\partial u_j u_k}u_j u_k + \frac{1}{3!}\sum_{jkl}\frac{\partial^3 V_0}{\partial u_j u_k u_l}u_j u_k u_l + \cdots \quad (35)$$

The first term: sum over kinetic energy of each particle, and

$V_0(0)$: potential energy (when all particles are **stationary** in the equilibrium position.)

The remaining terms : Taylor expansion of the potential about the equilibrium positions. Each particle oscillates about its equilibrium position and is coupled to other oscillators via the potential.



L is the lattice constant

1.2.4 The monatomic linear chain (2)

In the harmonic approximation, the force constant is real and symmetric:

$$k = k_{jk} = \left(\frac{\partial^2 E_0}{\partial u_k}\right)|_0 = k_{kj}$$

Restricting the sum in Eq. (35) to nearest neighbors and setting $\mathbf{V}_0(0) = 0$, the Hamiltonian becomes:

$$H = \sum_j \frac{m}{2}\left(\frac{du_j}{dt}\right)^2 + \frac{\kappa}{2}\sum_j (2u_j^2 - u_j u_{j+1} - u_j u_{j-1}) \qquad (36)$$

The displacement from equilibrium at site ***j*** is $\boldsymbol{u}_j$ and is related to that of its nearest neighbor by:

$$u_{j\pm1} = u_j e^{\pm iqL} \qquad (37)$$

where ***q*** is the wave vector of a vibration of wavelength. $q = \frac{2\pi}{\lambda}$

1.2.4 The monatomic linear chain (3)

$$H = \sum_j \frac{m}{2}\left(\frac{du_j}{dt}\right)^2 + \frac{\kappa}{2}\sum_j (2u_j^2 - u_j u_{j+1} - u_j u_{j-1}) \qquad (36)$$

assuming **no dissipation** in the system, so that **dH/dt = 0**, the **equation of motion** is:

$$m\frac{d^2u_j}{dt^2} = \kappa(u_{j+1} + u_{j-1} - 2u_j) \qquad (38)$$

Second-order differential equations of this type have time dependence of the form: $e^{-i\omega t}$ which, on substitution into Eq. (38), gives:

$$-m\omega^2 u_j = \kappa(e^{iqL} + e^{-iqL} - 2)u_j = -4\kappa \sin^2\left(\frac{qL}{2}\right)u_j \qquad (39)$$

From Eq. (39) it follows that:

$$\omega(q) = \sqrt{\frac{4\kappa}{m}}\sin\left(\frac{qL}{2}\right) \qquad (40)$$

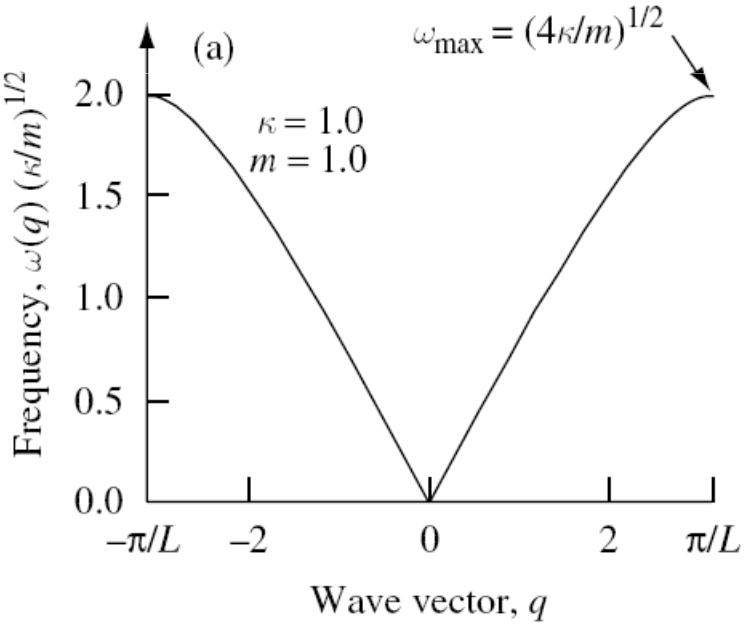
1.2.4 The monatomic linear chain (4)

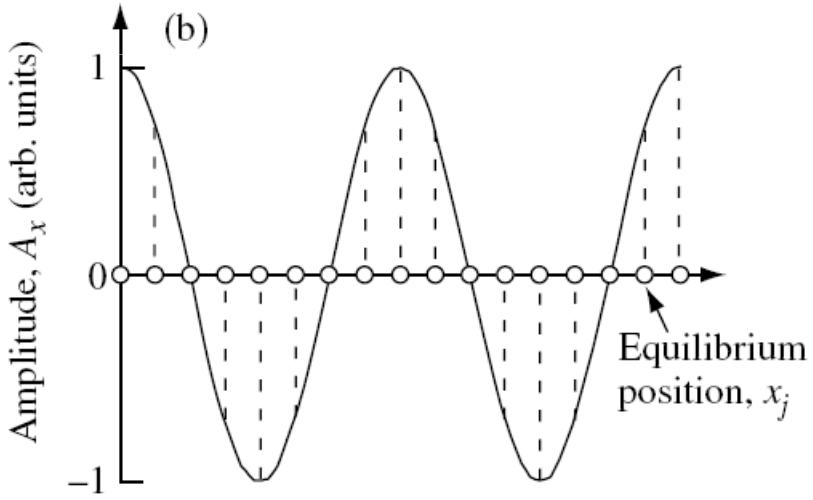
$$\omega = \omega(q) = \omega_{acoustic}(q)$$

$$\omega_{max} = \left(\frac{4k}{m}\right)^{1/2}$$

$$v_g = \partial\omega/\partial q$$

The acoustic branch dispersion relation describing lattice dynamics of a monatomic linear chain predicts that vibrational waves propagate at constant **group velocity v_g**.
This is the velocity of sound waves in the system.





1.2.5 The diatomic linear chain (1)

There are two atoms per *unit cell spaced* by L/2. One atom in the unit cell has mass m_1 and the other atom has mass m_2.

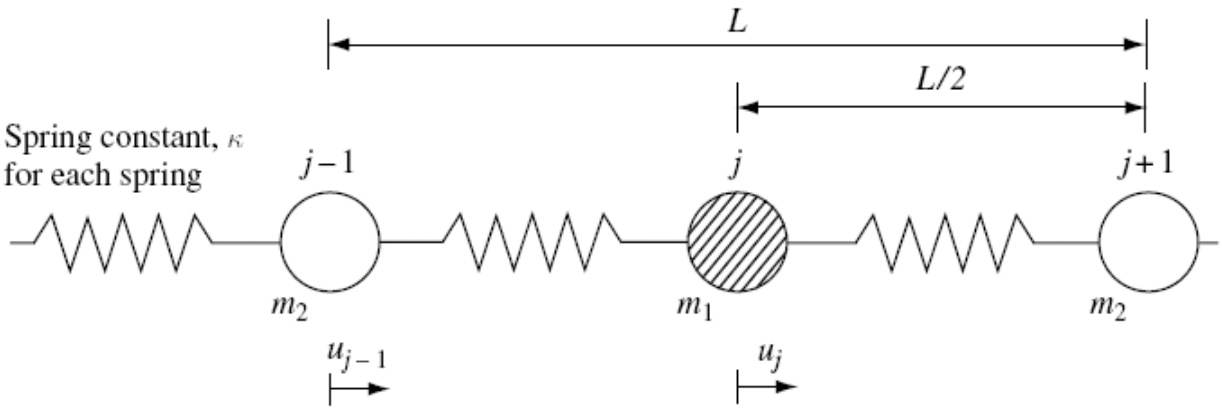
The motion of one atom :

$$u_{j\pm2} = u_j e^{\pm iqL} \qquad (41) \qquad q = 2\pi/\lambda$$

The equations of motion:

$$m_1 \frac{d^2u_j}{dy^2} = \kappa(u_{j+1} + u_{j-1} - 2u_j) \qquad (42)$$

$$m_2 \frac{d^2u_{j-1}}{dt^2} = \kappa(u_j + u_{j-2} - 2u_{j-1}) \qquad (43)$$



1.2.5 The diatomic linear chain (2)

or:

$$m_1 \frac{d^2 u_j}{dt^2} = \kappa(1 + e^{iqL})u_{j-1} - 2\kappa u_j \tag{44}$$

$$m_2 \frac{d^2 u_{j-1}}{dt^2} = \kappa(1 + e^{-iqL})u_j - 2\kappa u_{j-1} \tag{45}$$

Solutions for $\boldsymbol{u_j}$ *and* $\boldsymbol{u_{j-1}}$ *have time dependence of the form* $e^{-i\omega t}$ *, giving:*

$$-m_1\omega^2 u_j = \kappa(1 + e^{iqL})u_{j-1} - 2\kappa u_j \tag{46}$$

$$-m_2\omega^2 u_{j-1} = \kappa(1 + e^{-iqL})u_j - 2\kappa u_{j-1} \tag{47}$$

or

$$(2\kappa - m_1\omega^2)u_j - \kappa(1 + e^{iqL})u_{j-1} = 0 \tag{48}$$

$$-\kappa(1 + e^{-iqL})u_j + (2\kappa - m_2\omega^2)u_{j-1} = 0 \tag{49}$$

1.2.5 The diatomic linear chain (3)

$$\begin{vmatrix} 2\kappa - m_1\omega^2 & -\kappa(1+e^{iqL}) \\ -\kappa(1+e^{-iqL}) & 2\kappa - m_2\omega^2 \end{vmatrix} = 0 \qquad (50)$$

so that the characteristic polynomial is

$$\omega^4 - 2\kappa\left(\frac{m_1+m_2}{m_1 m_2}\right)\omega^2 + \frac{2\kappa^2}{m_1 m_2}(1-\cos(qL)) = 0 \qquad (51)$$

The roots of this polynomial give the characteristic values, or ***eigenvalues,*** $\boldsymbol{\omega_q}$.

In the *long wavelength limit q→0*

$$\omega^2\left(\omega^2 - 2\kappa\left(\frac{m_1+m_2}{m_1 m_2}\right)\right) = 0 \qquad (52)$$

In the *short wavelength limit q→π/L*

$$\omega^4 - 2\kappa\left(\frac{m_1+m_2}{m_1 m_2}\right)\omega^2 + \frac{4\kappa^2}{m_1 m_2} = 0 \qquad (53)$$

1.2.5 The diatomic linear chain (4)

In the *long wavelength limit* $q \to 0$

$$\omega^2\left(\omega^2 - 2\kappa\left(\frac{m_1+m_2}{m_1 m_2}\right)\right) = 0 \quad (52)$$

$$\omega = 0 \text{ and } \omega = \left(2\kappa\left(\frac{m_1+m_2}{m_1 m_2}\right)\right)^{1/2}$$

In the *short wavelength limit* $q \to \pi/L$

$$\omega^4 - 2\kappa\left(\frac{m_1+m_2}{m_1 m_2}\right)\omega^2 + \frac{4\kappa^2}{m_1 m_2} = 0 \quad (53)$$

$$\omega_1 = (2\kappa/m_1)^{1/2}, \qquad \omega_2 = (2\kappa/m_2)^{1/2}$$

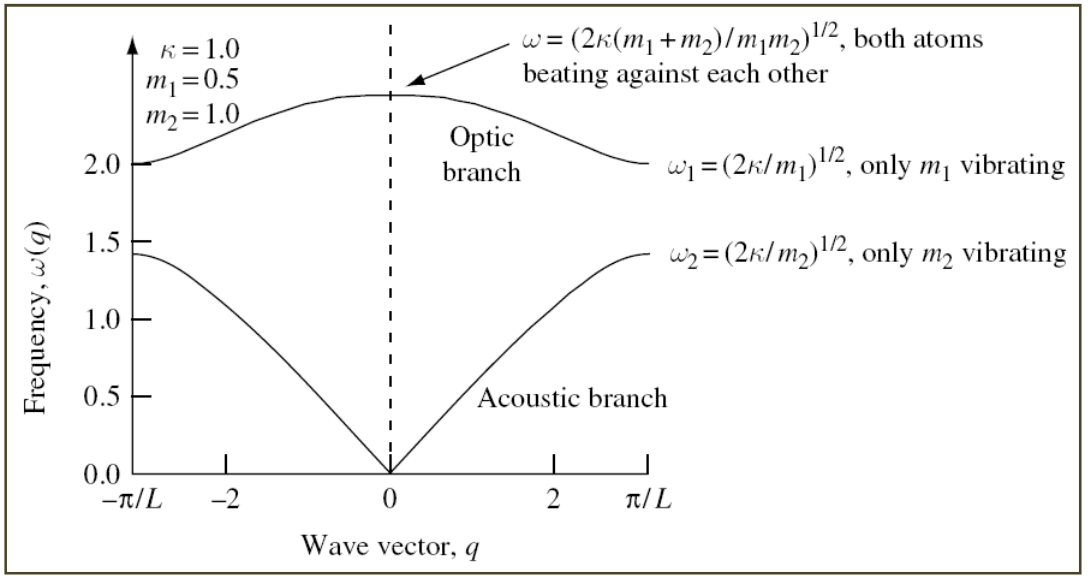
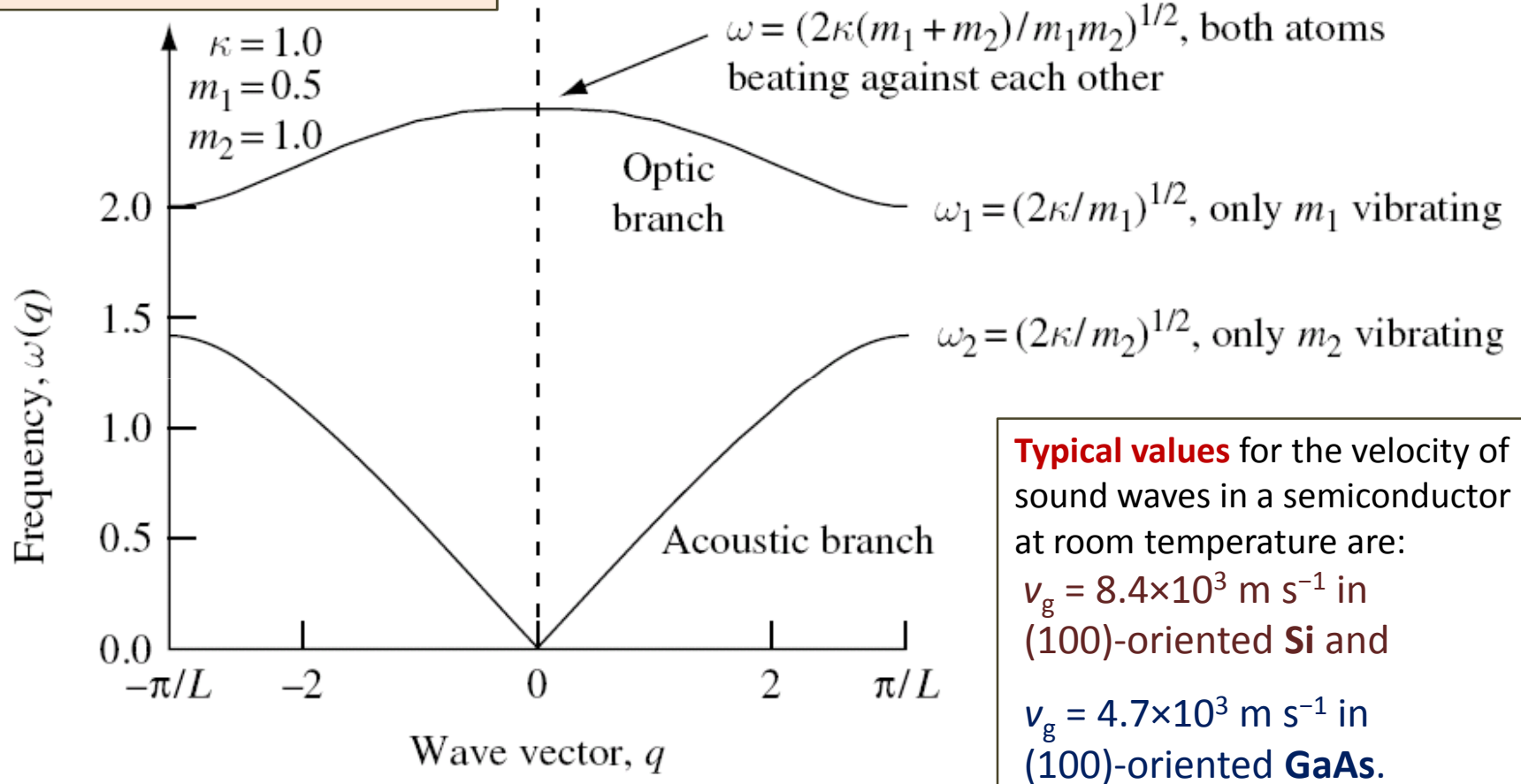


Fig. 1.12 Dispersion relation for lattice vibrations of a one- dimensional diatomic linear chain.
Particles are mass m_1 = 0.5 and m_2 = 1.0.
The spring constant is = 1.0.

1.2.5 The diatomic linear chain (5)

For the one-dimensional case



Typical values for the velocity of sound waves in a semiconductor at room temperature are:

$v_g = 8.4\times10^3$ m s^{-1} in (100)-oriented **Si** and

$v_g = 4.7\times10^3$ m s^{-1} in (100)-oriented **GaAs**.

Fig. 1.12 Dispersion relation for lattice vibrations of a one- dimensional diatomic linear chain.

1.2.5 The diatomic linear chain (6)

In three dimensions:
we add extra degrees of freedom, resulting in a total of three acoustic and three optic branches.

GaAs : III-V compound ,
used to make laser diodes and high-speed transistors.
Gallium Arsenic has the **zinc blende crystal** structure with a lattice constant of **L = 0.565nm**. Gallium and As atoms have **different atomic masses.**

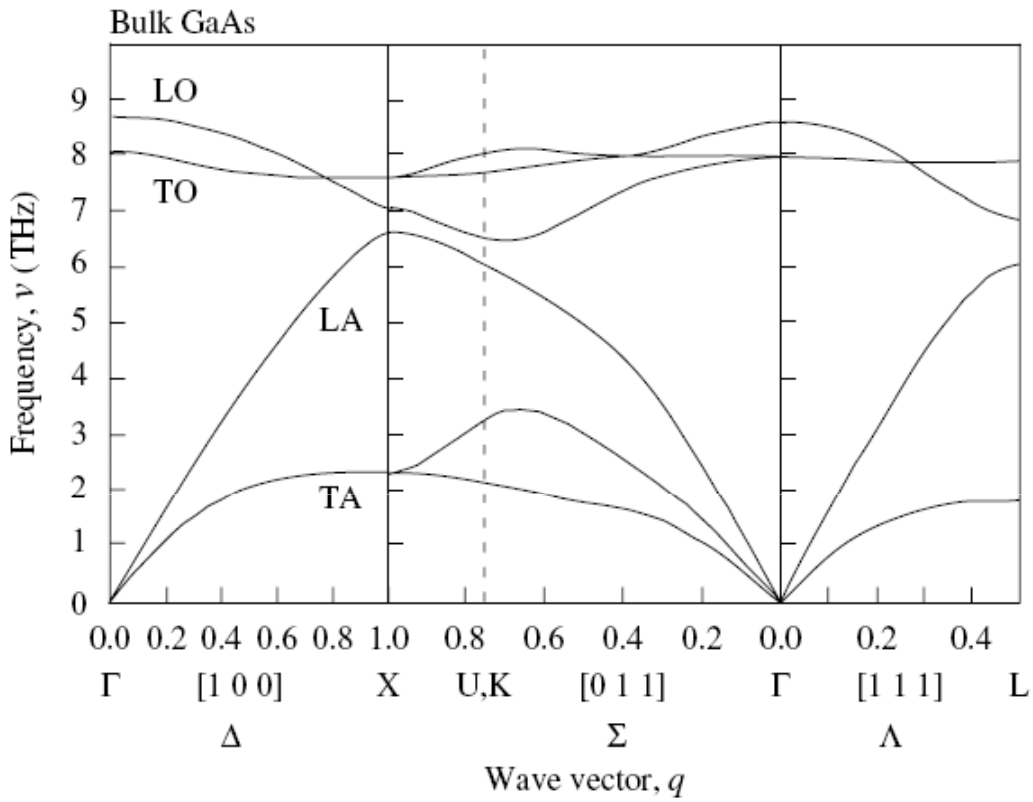


Fig. 1.13 Lattice vibration dispersion relation along principle crystal symmetry directions of bulk GaAs.
The longitudinal acoustic (LA), two transverse acoustic (TA), longitudinal optic (LO), and two **transverse optic (TO)** branches are indicated.

1.3 Classical electromagnetism:

1.3.1 Electrostatics 1.3.2 Electrodynamics

1.3.1 Electrostatics:

force: (due to Q and –Q)

$$\mathbf{F}(\mathbf{r}) = \frac{-Q^2}{4\pi\varepsilon_0 r^2}\tilde{\mathbf{r}} \qquad (54)$$

force: (due to charge e in an electric field)

$$\mathbf{F} = e\mathbf{E}$$

force: (relation to Potential)

$$\mathbf{F} = -\nabla V$$

Potential energy:

$$V = \int e\mathbf{E}\cdot d\tilde{\mathbf{x}} \qquad (55)$$

$$V(r) = \frac{-e^2}{4\pi\varepsilon_0 r} \qquad (56)$$

adiabatic:
بی دررو

Permittivity: (ε)
گذردهی الکتریکی

Permeability: (μ)
گذردهی مغناطیسی

Susceptibility: (χ)
ضریب حساسیت

Permittivity of free space	$\varepsilon_0 = 8.854\ 187\ 8 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Speed of light in free space	$c = 1/\sqrt{\varepsilon_0\mu_0}$

1.3 Classical electromagnetism:

1.3.1 Electrostatics:

E: electric field
D: displacement vector field

B: magnetic flux density
H: magnetic field

V: Potential

ρ: charge density

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \varepsilon_r \qquad (57)$$

$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (58)$$

$$\mathbf{H} = \mathbf{B} / \mu_0 \mu_r$$

$$\mathbf{E} = -\nabla V$$

Gauss's law:
(Stoke's theorem)

$$\int_V \nabla \cdot \mathbf{E}\, dV = \oint_S \mathbf{E} \cdot \mathbf{n}\tilde{}\, dS = \int_V (\rho / \varepsilon_0 \varepsilon_r) dV \qquad (59)$$

1.3 Classical electromagnetism

1.3.1 Electrostatics:

Capacitance:

$$C = \frac{Q}{V} \quad (60)$$

$$C = \frac{Q}{V} = \frac{\rho A}{\rho d/\varepsilon_0 \varepsilon_r} = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad (61)$$

stored energy:

$$\Delta E = \int_{t'=-\infty}^{t'=t} CV \frac{dV}{dt'} dt' = \int_{V'=0}^{V'=V} CV' dV' = \frac{1}{2} CV^2 \quad (62)$$

$$\Delta E = \frac{1}{2} CV^2 \quad (63)$$

stored *energy density:* ΔU

1. energy stored per unit volume in the electric field:

$$\Delta U = \frac{1}{2} \mathbf{E} \cdot \mathbf{D} \quad (64)$$

energy stored per unit volume in a magnetic field:

$$\Delta U = \frac{1}{2} \mathbf{B} \cdot \mathbf{H} \quad (65)$$

Inductance

$$L = \frac{1}{I} \int_S \mathbf{B} \cdot \mathbf{n}^{\sim} dS \quad (66)$$

1.3 Classical electromagnetism

Gauss's law:

$$\int_V \nabla \cdot \mathbf{E}\, dV = \oint_S \mathbf{E} \cdot \mathbf{n}\tilde{} dS = \int_V (\rho/\varepsilon_0\varepsilon_r) dV \quad (67)$$

electric flux:

$$E_r = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2} \quad (68)$$

Potential:

$$V = -\int_{r_2}^{r_1} E_r dr = -\int_{r_2}^{r_1} \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2} dr = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad (69)$$

Capacitance:

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0\varepsilon_r}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \quad (70) \qquad C = 4\pi\varepsilon_0\varepsilon_r r_1 \quad (71)$$

charging energy:

$$\Delta E = \int_{t'=-\infty}^{t'=1} CV\frac{dV}{dt'}dt' = \int_{V'=0}^{V'=V} CV'dV' = \frac{1}{2}CV^2 = \frac{Q^2}{2C} \quad (72) \qquad \Delta E = \frac{e^2}{2C} = \frac{e^2}{8\pi\varepsilon_0\varepsilon_r r_1} \quad (73)$$

1.3 Classical electromagnetism: (1.3.2 Electrodynamics)

Classical electrodynamics describes the spatial and temporal behavior of electric and magnetic fields.

Plane waves can be represented *spatially* as:

$$\sin(kx) = \frac{1}{2i}(e^{ikx} - e^{-ikx}) \quad (74)$$

$$\cos(kx) = \frac{1}{2}(e^{ikx} + e^{-ikx}) \quad (75)$$

$$e^{ikx} = \cos(kx) + i\sin(kx) \quad (76)$$

Plane waves can be represented *temporally* by:

$$e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t) \quad (77)$$

plane wave:

$$A e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} \quad (78)$$

A: amplitude of the wave,
$K=2\pi/\lambda$:**wave vector of magnitude,**
$\omega=2\pi f$: angular frequency,
$f=1/\tau$: frequency,
τ: periode

1.3 Classical electromagnetism: (1.3.2 Electrodynamics)

Table 1.1 *Maxwell equations*

$\nabla \cdot \mathbf{D} = \rho$	Coulomb's law
$\nabla \cdot \mathbf{B} = 0$	No magnetic monopoles
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	Modified Ampère's law

D :displacement vector field

E: electric field, or electric flux density

χ_e: electric susceptibility

P: electric polarization field

H and B: The magnetic field vector, or the magnetic flux density

$$\boldsymbol{D} = \varepsilon E = \varepsilon_0 \varepsilon_r \boldsymbol{E} = \varepsilon_0 (1 + \chi_e) \boldsymbol{E} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} = \mu_0 \mu_r \boldsymbol{H} = \mu_0 (1 + \chi_m) \boldsymbol{H}$$

$$= \mu_0 (\boldsymbol{H} + \boldsymbol{M})$$

μ: permeability,

μ_r: relative permeability,

χ_m : magnetic susceptibility,

M: magnetization

divergence theorem:

$$\int_V \nabla \cdot \mathbf{a}\, d^3 r = \int_S \mathbf{a} \cdot \mathbf{n}^{\sim}\, dS \qquad (84)$$

S. Stokes' theorem:

$$\int_S (\nabla \times \mathbf{a}) \cdot \mathbf{n}^{\sim}\, dS = \oint_C \mathbf{a} \cdot d\mathbf{l} \qquad (85)$$

V: volume
n∼: unit-normal vector to the surface S
J: current density
ρ: charge density

$$\nabla \cdot (\nabla \times \mathbf{H}) = \nabla \cdot \mathbf{J} + \nabla \cdot \frac{\partial \mathbf{D}}{\partial t} \qquad (86)$$

$$0 = \nabla \cdot \mathbf{J} + \nabla \cdot \frac{\partial \mathbf{D}}{\partial t} \qquad (87)$$

$$0 = \nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} \qquad (88)$$

$$\oint \mathbf{H} \cdot d\mathbf{I} = \int_S (\nabla \times \mathbf{H}) \cdot \mathbf{n}^{\sim} dS = \int_S \mathbf{J} \cdot \mathbf{n}^{\sim} dS = I \qquad (89)$$

1.3.2.1 Light propagation in a dielectric medium

In the **dielectric**, current density **J** = 0 because the dielectric has no mobile charge, and if $\mu_r = 1$ at optical frequencies then **H** = **B**/μ_0.

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) = -\mu_0 \frac{\partial^2}{\partial t^2}\mathbf{D} \tag{90}$$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 \frac{\partial^2}{\partial t^2}\mathbf{D} \tag{91}$$

$$\nabla^2 \mathbf{E} = \mu_0 \frac{\partial^2}{\partial t^2}\mathbf{D} \tag{92}$$

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) = \frac{\partial^2}{\partial t^2}\mu_0 \varepsilon \mathbf{E}(\mathbf{r}, t) \tag{93}$$

$$\nabla^2 \mathbf{E}(\mathbf{r}, \omega) = -\omega^2 \mu_0 \varepsilon_0 \varepsilon_r(\omega) \mathbf{E}(\mathbf{r}, \omega) \tag{94}$$

wave equation:

$$\nabla^2 \mathbf{E}(\mathbf{r}, \omega) = \frac{-\omega^2}{c^2} \varepsilon_r(\omega) \mathbf{E}(\mathbf{r}, \omega) \tag{95}$$

Solution: plane waves.

1.3.2.1 Light propagation in a dielectric medium

If $\varepsilon_r(\omega)$ *is real and positive, the solutions to this wave equation for an electric field* propagating in an isotropic medium are just plane waves. The **speed** of wave propagation is $c/n_r(\omega)$, where $n_r(\omega)=[\varepsilon_r(\omega)]^{1/2}$ is the ***refractive index of the material.*** *In the more* general case, when relative permeability $\mu_r \neq 1$, the refractive index is:

$$n_r(\omega) = \sqrt{\varepsilon_r(\omega)}\sqrt{\mu_r(\omega)} = \frac{\sqrt{\varepsilon(\omega)}\sqrt{\mu(\omega)}}{\sqrt{\varepsilon_0\mu_0}} \qquad (96)$$

If one of either ε or μ is negative, refractive index is imaginary and electromagnetic waves cannot propagate. It is common for metals to have negative values of ε.

In a metal, free electrons can collectively oscillate at a long-wavelength natural frequency called the **plasma frequency**, $\omega_p = (ne^2/\varepsilon_0 m)^{1/2}$.

$\varepsilon_r(\omega) = 1-\omega^2_p/\omega^2$: a good approximation for a metal at long wavelengths.

If $\omega \gg \omega_p$: ε=positive ,and electromagnetic waves can propagate through the metal.

For $\omega \ll \omega_p$: ε=negative ,*n* is imaginary, waves cannot propagate in the metal and are reflected.

why bulk metals are usually not transparent to electromagnetic radiation of frequency less than ω_p?

1.3.2.1 Light propagation in a dielectric medium

In a homogeneous dielectric medium: $\mu_r = 1$ and $\varepsilon(\omega)=\varepsilon_0\varepsilon_r = \varepsilon_0(\varepsilon'_r(\omega)+ \varepsilon''_r(\omega))$ where $\varepsilon'_r(\omega)$ and $\varepsilon''_r(\omega)$ are the real and imaginary parts. In this situation:

$$\mathbf{E}(\mathbf{r},\omega) = \mathbf{E}_0(\omega)e^{i\mathbf{k}(\omega)\cdot\mathbf{r}} = \mathbf{E}_0(\omega)e^{i(k'(\omega)+ik''(\omega))\tilde{\mathbf{k}}\cdot\mathbf{r}} \quad (97)$$

$$n_r(\omega) = \sqrt{\frac{1}{2}(\varepsilon'_r(\omega) + \sqrt{\varepsilon'^2_r(\omega) + \varepsilon''^2_r(\omega)})} \quad (98)$$

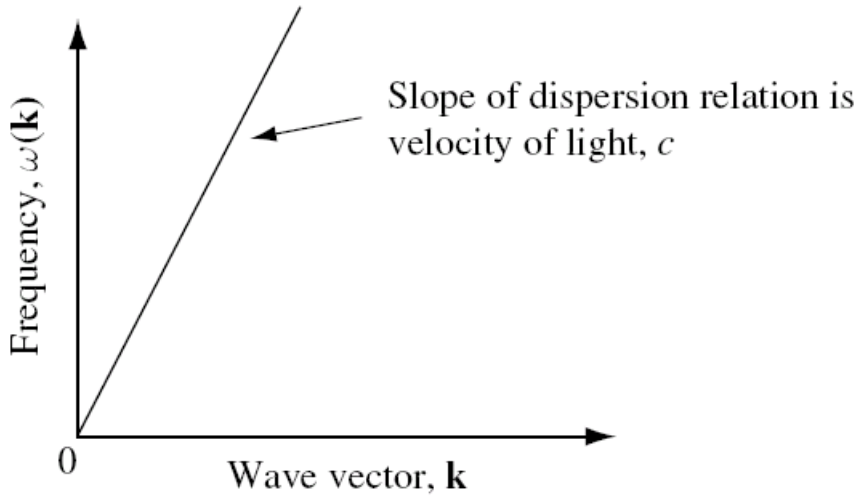


Fig. 1.17 Dispersion relation for an electromagnetic wave in **free space**. The slope of the line is the velocity of light.

1.3.2.1 Light propagation in a dielectric medium

For the case: $k''(\omega) = 0$ and $\mu_r = 1$, the refractive index is just $n_r(\omega) = [\varepsilon'_r(\omega)]^{1/2}$, and we have a simple oscillatory solution with no spatial decay in the electric and magnetic field vector:

$$\mathbf{E}(\mathbf{r}, \omega) = \mathbf{E}_0 e^{-i\omega t} e^{i\mathbf{k}(\omega)\cdot\mathbf{r}} \quad (99)$$

$$\mathbf{H}(\mathbf{r}, \omega) = \mathbf{H}_0 e^{-i\omega t} e^{i\mathbf{k}(\omega)\cdot\mathbf{r}} \quad (100)$$

Maxwell's equations in free space:

$$\nabla \cdot \mathbf{D} = 0 \quad (101)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (102)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (103)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad (104)$$

The first two equations are divergence equations that require that **$\mathbf{k} \cdot \mathbf{E} = 0$ and $\mathbf{k} \cdot \mathbf{B} = 0$.**
This means that **E and B** are **perpendicular (transverse)** to the **direction of propagation k~.**

in free space:

$$\nabla \times \mathbf{E}_0 e^{-i\omega t} e^{i\mathbf{k}(\omega)\cdot\mathbf{r}} = -\mu_0 \frac{\partial}{\partial t} \mathbf{H}_0 e^{-i\omega t} e^{i\mathbf{k}(\omega)\cdot\mathbf{r}} \quad (105)$$

$$i\mathbf{k} \times \mathbf{E}_0 e^{-i\omega t} e^{i\mathbf{k}(\omega)\cdot\mathbf{r}} = i\omega\, \mu_0 \mathbf{H}_0 e^{-i\omega t} e^{i\mathbf{k}(\omega)\cdot\mathbf{r}} \quad (106)$$

$$i\mathbf{k} \times \mathbf{E} = i\omega\, \mu_0 \mathbf{H} \quad (107)$$

1.3.2.1 Light propagation in a dielectric medium

Using the fact that the dispersion relation for plane waves in free space is $\omega = ck$ and the speed of light is $c = 1/[\varepsilon_0\mu_0]^{1/2}$, leads us directly to:

$$\mathbf{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\tilde{\mathbf{k}} \times \mathbf{E} \quad (108)$$

$$\mathbf{B}c = \tilde{\mathbf{k}} \times \mathbf{E},$$

where $\tilde{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$ is the unit vector for $\mathbf{k}$.

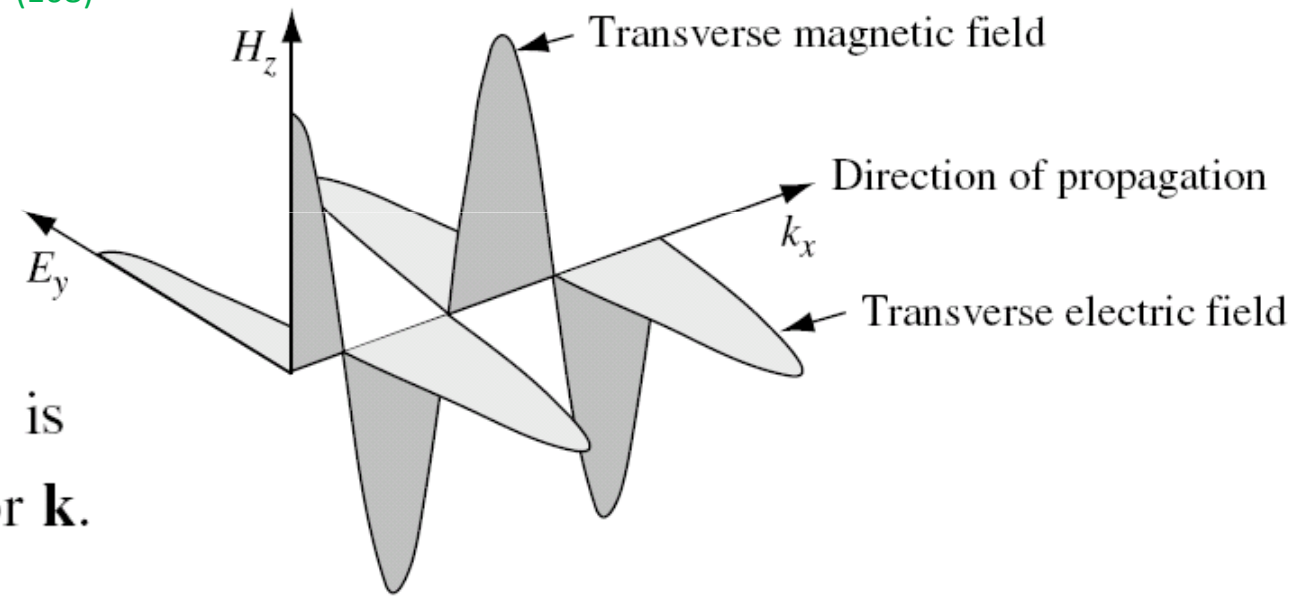


Fig. 1.18 Illustration of transverse magnetic field H_z and electric field E_y of a plane wave propagating in free space in the x direction.

1.3.2.1 Light propagation in a dielectric medium

Oscillating transverse electromagnetic **waves** can **decay** in **time** and in **space**.

$$\mathbf{E}(t) = \mathbf{y}^{\sim}|\mathbf{E}_0|\sin(\omega t)e^{-\gamma t}$$

$$\mathbf{E}(x) = \mathbf{y}^{\sim}|\mathbf{E}_0|\cos(kx)e^{-\gamma x}$$

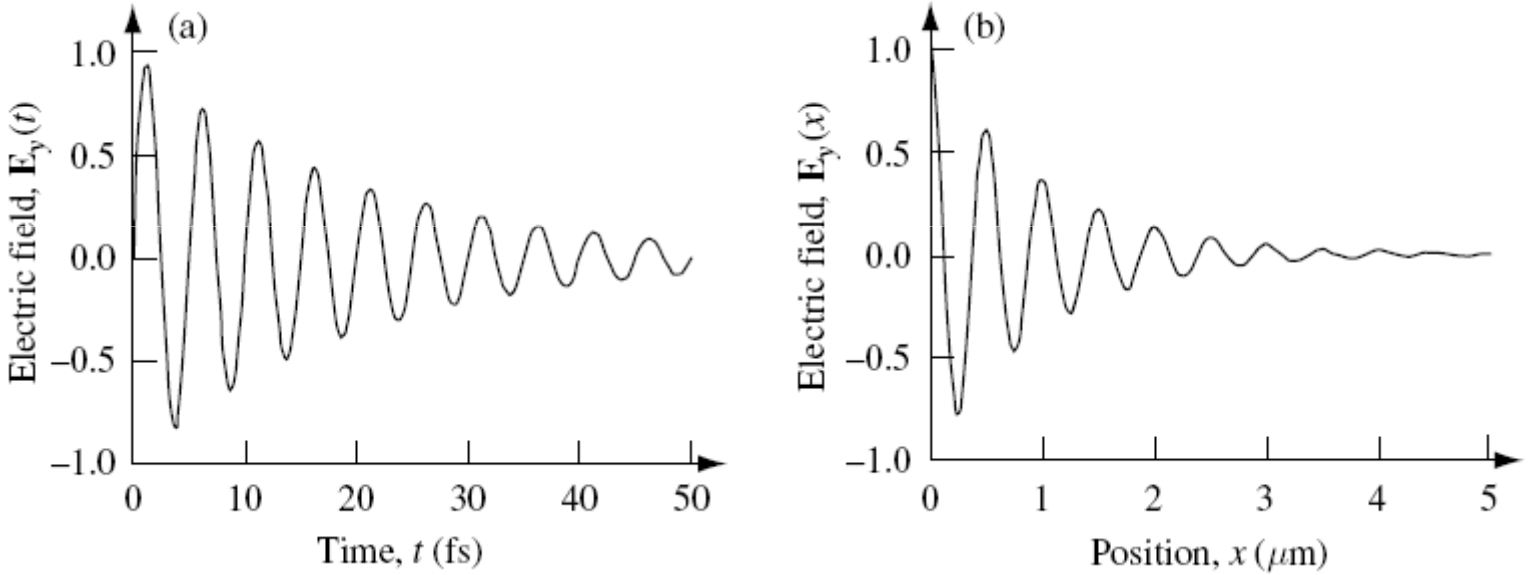


Fig. 1.19 (a) Illustration of **temporal decay** of an oscillating electric field**.**
(b) Illustration of **spatial decay** of an oscillating electric field.

1.3.2.2 Power and momentum in an electromagnetic wave

The power in an electromagnetic wave can be obtained by considering the response of a test **charge e** moving at **velocity *v*** in an external **electric field E.** The rate of work or power is just e*v* ·**E**, where e*v* is a current. The total power in a given volume is:

$$\int_{\text{Volume}} d^3r\,\mathbf{J}\cdot\mathbf{E} = \int_{\text{Volume}} \left(\mathbf{E}\cdot(\nabla\times\mathbf{H}) - \mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}\right) d^3r \qquad (109)$$

Because:

$$\nabla\times\mathbf{H} = \mathbf{J} + \partial\mathbf{D}/\partial t$$

From:

$$\mathbf{E}\cdot(\nabla\times\mathbf{H}) = \mathbf{H}\cdot(\nabla\times\mathbf{E}) - \nabla\cdot(\mathbf{E}\times\mathbf{H}) \text{ and } \nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t,$$

$$\int_{\text{Volume}} d^3r\,\mathbf{J}\cdot\mathbf{E} = -\int_{\text{Volume}} \left(\nabla\cdot(\mathbf{E}\times\mathbf{H}) + \mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t} + \mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}\right) d^3r \qquad (110)$$

Or on different form:

$$\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t} + \mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t} = -\mathbf{J}\cdot\mathbf{E} - \nabla\cdot(\mathbf{E}\times\mathbf{H}) \qquad (111)$$

1.3.2.2 Power and momentum in an electromagnetic wave

From (64), (65), (111): $\Delta U = \frac{1}{2}\mathbf{E}\cdot\mathbf{D}$ $\Delta U = \frac{1}{2}\mathbf{B}\cdot\mathbf{H}$ $\mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t}+\mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t}=-\mathbf{J}\cdot\mathbf{E}-\nabla\cdot(\mathbf{E}\times\mathbf{H})$

The total energy density:

$$U=\frac{1}{2}(\mathbf{E}\cdot\mathbf{D}+\mathbf{B}\cdot\mathbf{H}) \quad (112)$$

$$\frac{\partial U}{\partial t}=-\mathbf{J}\cdot\mathbf{E}-\nabla\cdot\mathbf{S} \quad (113)$$

***S:* Poynting vector:**

$$\mathbf{S}=\mathbf{E}\times\mathbf{H} \quad (114)$$

The Poynting vector is the energy flux density in the electromagnetic field.

In free space, The total energy density:

$$U=\frac{|S|}{c} \quad (115)$$

1.3.2.2 Power and momentum in an electromagnetic wave

In free space: $U = \frac{|S|}{c}$ (115)

$$\mathbf{H} = \sqrt{\varepsilon_0/\mu_0}\,\mathbf{k}^{\sim} \times \mathbf{E} \quad (108)$$

$$\mathbf{E}(\mathbf{r},\omega) = \mathbf{E}_0 e^{-i\omega t} e^{i\mathbf{k}(\omega)\cdot\mathbf{r}} \quad (99)$$

$$\mathbf{H}(\mathbf{r},\omega) = \mathbf{H}_0 e^{-i\omega t} e^{i\mathbf{k}(\omega)\cdot\mathbf{r}} \quad (100)$$

⇨ ***S: Poynting vector:***

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathbf{E} \times \mathbf{k}^{\sim} \times \mathbf{E} \quad (116)$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$$

$$\mathbf{S} = \sqrt{\frac{\varepsilon_0}{\mu_0}}((\mathbf{E}\cdot\mathbf{E})\mathbf{k}^{\sim} - (\mathbf{E}\cdot\mathbf{k}^{\sim})\mathbf{E})$$

(E·K^)=0 (117) →

$$\mathbf{S} = \sqrt{\frac{\varepsilon_0}{\mu_0}}(\mathbf{E}\cdot\mathbf{E})\mathbf{k}^{\sim} \quad (118)$$

Defining the ***impedance of free space***:

$$Z_0 \equiv \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 376.73\ \Omega$$

$$Z_0 = 120 \times \pi\ \Omega$$

(119) →

$$\mathbf{S} = \frac{(\mathbf{E}\cdot\mathbf{E})}{Z_0}\mathbf{k}^{\sim} \quad (120)$$

For **monochromatic plane waves** propagating in the **x direction**, the **Poynting vector:**

$$\mathbf{S} = \frac{|\mathbf{E}_0|^2}{Z_0}(\cos^2(k_x x - \omega t + \Delta_{\text{phase}}))\mathbf{k}^{\sim} \quad (121)$$

$$\langle \mathbf{S} \rangle = \frac{|\mathbf{E}_0|^2}{2Z_0}\mathbf{k}^{\sim} \quad (122)$$

1.3.2.2 Power and momentum in an electromagnetic wave

Momentum: p

Electromagnetic waves carry not only **energy**, but also **momentum**.

The classical Lorentz force on a test charge **e** moving at velocity **v** is:

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (123)$$

$$\mathbf{F} = d\mathbf{p}/dt \quad \Rightarrow \quad \mathbf{p} = \frac{\mathbf{E} \times \mathbf{H}}{c^2} = \frac{\mathbf{S}}{c^2} \qquad (124)$$

momentum can be expressed in terms of the energy density as:

$$\mathbf{p} = \frac{U}{c}\tilde{\mathbf{k}} \qquad (125)$$

The magnitude of the momentum is just:

$$|\mathbf{p}| = \frac{1}{c}\frac{|\mathbf{S}|}{c} = \frac{U}{c} \qquad (126)$$

$$\Rightarrow \quad p = \frac{U}{c} \qquad (127)$$

1.3.2.3 Choosing a potential

In general, Maxwell's equations allow electric and magnetic fields to be described in terms of a scalar potential V(**r, t**) **and a vector potential A(r, t).**

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0 \qquad \nabla \cdot \mathbf{B} = 0 \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\frac{\partial}{\partial t} \nabla \times \mathbf{A} \quad (128)$$

or:

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \quad (129)$$

Since the curl of the gradient of any scalar field is zero, we may equate the last equation with the gradient of a scalar field, V, where:

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\Delta V \quad (130)$$

$$\mathbf{E}(\mathbf{r}, t) = -\nabla V(\mathbf{r}, t) - \frac{\partial}{\partial t} \mathbf{A}(\mathbf{r}, t) \quad (131)$$

$$\mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t) \quad (132)$$

1.3.2.4 Dipole radiation

Oscillatory current in the wire: $I(t)=I_0 e^{j\omega t}$

dipole moment for the harmonic time-dependent source: $\mathbf{d} = Q\tilde{\mathbf{z}} r_0 = \frac{I r_0 \tilde{\mathbf{z}}}{i\omega}$ (133)

$$P_r = \frac{Z_0}{12\pi} I r_0 k^2 \quad (153)$$

$$P_r = \frac{Z_0}{12\pi} \frac{\omega^4 |\mathbf{d}|^2}{c^2} \quad (154)$$

P_r: The total time-averaged radiated power

$$Z_0 = 120 \times \pi\ \Omega$$

$$P_r = 10\,\omega^4 |\mathbf{d}|^2 / c^2$$

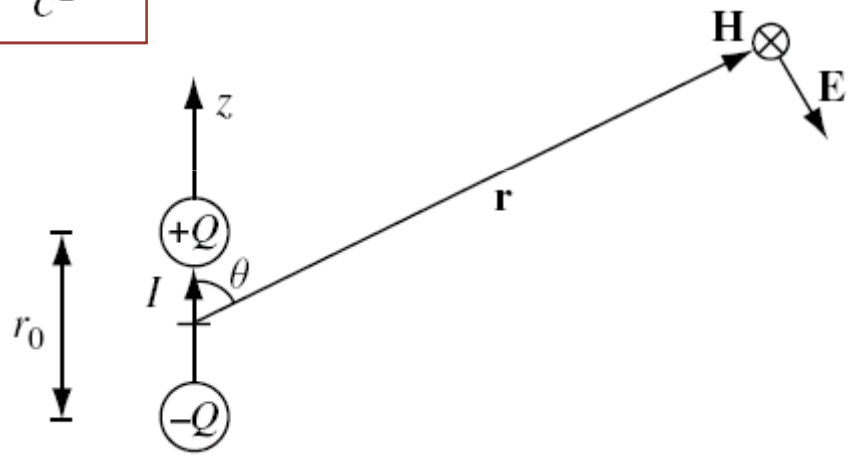


Fig. 1.21 A small length of **conducting wire** connects **two conducting spheres** oriented in the z direction that have center-to-center spacing of r_0. **Oscillatory current I** flows in the wire, charging and discharging the spheres. The magnetic and electric field at position **r** is indicated.

1.4 Example exercises