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Chap.12

Light Sources

Photonics and Lasers An Introduction, Richard S. Quimby,

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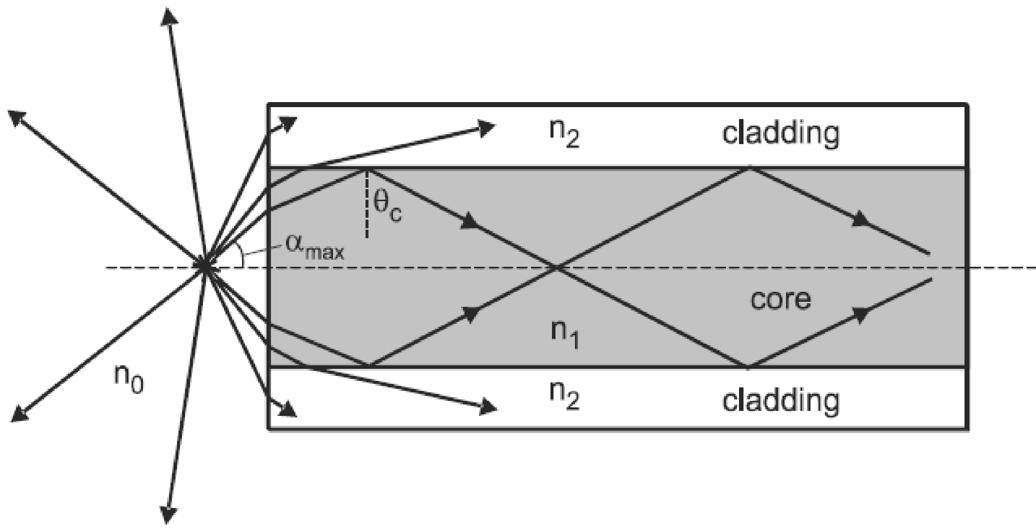
Introduction

- We saw in previous chapters how light propagates in a waveguide.
- And how the light can be generated by an LED or laser diode.
- A related issue is the efficiency which light can be coupled from a light source into a waveguide.
- In this chapter we examine this efficiency for sources with different degrees of directionality.
- It will be seen that the coupling efficiency is greater for more directional sources.

12.1. Point Source

- Consider first isotropic emission, in which light appears to emanate from a point.
- We like to calculate the fraction of emitted power that is collected by an optical fiber, as illustrated in Fig. 12-1.

Figure 12-1 Light rays from a point source making a sufficiently small angle with the fiber axis are coupled into the fiber core (shaded area).



12.1. Point Source -2

- If a point source emitting a total power **Ps** is embedded in a uniform medium of refractive index n_0 outside the fiber.
- It will radiate light equally into all 4π steradians of solid angle, with an emitted power per unit solid angle of $Ps/4\pi$.
- However, only those rays making an angle $\alpha < \alpha max$ with the fiber axis will be trapped by the fiber, where:

$$\sin \alpha_{max} = NA/n_0 .$$

- NA is the numerical aperture of the fiber.

12.1. Point Source-3

We know that light will be trapped is:

$$\begin{aligned}\Omega &= 2\pi(1 - \cos\alpha_{max}) \qquad (12\text{-}1)\\ &= 2\pi(1 - \sqrt{1 - \sin^2\alpha_{max}})\\ &= 2\pi(1 - \sqrt{1 - (NA/n_0)^2})\\ &\approx 2\pi\frac{1}{2}\left(\frac{NA}{n_0}\right)^2 = \pi\frac{NA^2}{n_0^2}\end{aligned}$$

- Where in the last step the expansion

$(1 + x)^n \approx 1 + nx$

- Has been used, along with the approximation (good for most optical fibers) $NA^2 \ll 1$.

12.1. Point Source-4

- Since the light is distributed uniformly with angle, the power coupled into the fiber is given by

$$P_{in} = \left(\frac{\Omega}{4\pi}\right) P_s \qquad (12\text{-}2)$$

- Which leads to the coupling efficiency

$$\eta_c \equiv \frac{P_{in}}{P_s} = \frac{\Omega}{4\pi} = \frac{NA^2}{n_0^2} \qquad (12\text{-}3)$$

- This result shows that higher numerical aperture fibers accept a larger fraction of the emitted light from a point source.

12.2. Lambertian Source

- Many practical light source such as the LED are best treated as extended sources, in which light is emitted over some surface area .
- The emitting surface is characterized by a brightness.
- Defined as the power emitted per unit solid angle, per unit surface area.
- This brightness is found in many cases to vary with direction as :

$$B(\theta) = B(0)\cos\theta \quad \text{(Lambert's law)} \qquad (12\text{-}4)$$

- Where θ is the angle to the surface normal.

12.2. Lambertian Source-2

- A light-emitting surface with an angular distribution given by Eq. (12-4) is known as a Lambertian source.
- The dependence to **cosθ** can be under stood by reffering to Fig. 12.2.

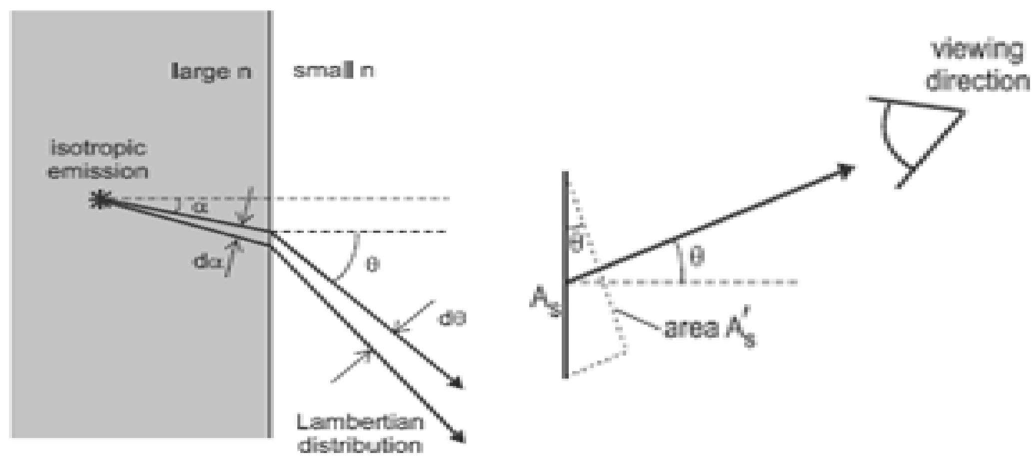


Figure 12-2 (a)Light emitted isotropically inside a high-index medium becomes distributed according to lambert's law after refraction at a boundary.

(b) when viewed off-axis, light from and emitting surface appears to come from the projected area .

12.2. Lambertian Source-3

- Which shows light being generated inside a high refractive index material and emitted into air.
- Inside the material, light is emitted isotropically, and equal amount of light radiated into each differential solid angle: $d\Omega = 2\pi \sin\alpha \, d\alpha$.
  .
- Because of refraction at the boundary, however, the $d\Omega = 2\pi \sin\theta \, d\theta$ corresponding solid angle on the air side, θ ,becomes proportionately larger as θ increases.
- Since the same optical power is spread out over a larger solid angle as θ increases, the brightness decreases.

12.2. Lambertian Source-4

- This argument can be made quantitative by using Snell's law to relate the two solid angles.
- The resulting brightness (including Fresnel reflection losses at the boundary) is:

$$B(\theta) = \frac{P_s}{\pi A_s n(n+1)^2} \cos\theta \qquad (12\text{-}5)$$

- Where P_s is the total power emitted inside the material, and A_s is the emition surface area.
- assumed in deriving this that the index of refraction **n** is large ($n > 2.5$), which is valid for most semiconductors.

12.2. Lambertian Source-5

- An alternative view of Lamber's law is to define an "apparent brightnss" B_{eff} as the power per solid angle emitted in a certain direction, divided by the projected area of the surface.
- From fig.12-2b, this projected area is $A_s' = A_s \cos\theta$, so the apparent brightness is:

$$B_{eff} = \frac{\Delta P}{A_s' \Delta\Omega} = \frac{\Delta P}{A_s \Delta\Omega \cos\theta} = \frac{B(\theta)}{\cos\theta} \qquad (12\text{-}6)$$

- If B_{eff} is independent of θ, then $B(\theta)\ \alpha \cos\theta$, which is Lamber's law.

12.2. Lambertian Source-6

- We can say, then, that a Lambertian surface is one for which the apparent brightness is independent of viewing angle.
- To determine coupling efficiency into a fiber, we first calculate the total power P_0 emitted from the Lambertian surface.
- Integrating Eq.(12-4) over one hemisphere, we have :

$$B(\theta) = B(0)\cos\theta \quad \text{(Lambert's law)} \qquad (12\text{-}4)$$

$$P_0 = A_s \int_0^{\pi/2} B(\theta)\, d\Omega \qquad (12\text{-}7)$$

$$= A_s \int_0^{\pi/2} [B(0)\cos\theta][2\pi \sin\theta\, d\theta]$$

$$= 2\pi A_s B(0) \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta$$

12.2. Lambertian Source-7

- Using the substitution $u = \sin\theta$, $du = \cos\theta\, d\theta$, this can be evaluated as :

$$P_0 = 2\pi A_s B(0) \int_0^1 u\, du \qquad (12\text{-}8)$$

$$= 2\pi A_s B(0) \frac{1}{2} u^2 |_0^1$$

$$= \pi A_s B(0)$$

- Combining this with Eq. (12-5) for $\theta = 0$ gives:

$$B(\theta) = \frac{P_s}{\pi A_s n(n+1)^2} \cos\theta \qquad (12\text{-}5)$$

$$P_0 \simeq \frac{P_s}{n(n+1)^2} \qquad (12\text{-}9)$$

12.2. Lambertian Source-8

- The ratio P_0/P_s corresponds to the LED external efficiency η_{ext} defined in Eq. (11-15).

$$\eta_{ext} = \frac{\Omega}{4\pi} T$$

- for GaAs (n=3.6), Eq. (12-9) gives $\eta_{ext} \simeq 0.013$.

$$P_0 \simeq \frac{P_s}{n(n+1)^2} \qquad (12\text{-}9)$$

- The power P_{in} coupled into the fiber is calculated in the same way as the total power, except that the upper limit on θ is $\sin \alpha_{max}$ rather than $\pi/2$. the upper limit on u in Eq. (12-8) is then sin rather than 1, giving:

$$P_0 = 2\pi A_s B(0) \int_0^1 u \, du \qquad (12\text{-}8)$$

$$= 2\pi A_s B(0) \frac{1}{2} u^2 |_0^1$$

$$= \pi A_s B(0)$$

12.2. Lambertian Source-9

- $P_{in} = \pi A_s B(0)\sin^2\alpha_{max}$ (12-10)

  $= P_0 \sin^2\alpha_{max}$

$\eta_c = \frac{P_{in}}{P_0} = \sin^2\alpha_{max} = \left(\frac{NA}{n_0}\right)^2$ efficiency for Lambertain source (12-11)

- This efficiency is the same as for a point source, except that it is four times higher.
- The difference comes from the more directional emission of the lambertian source.
- With a greater fraction of the emitted light lying within the angular acceptance range of the fiber.
- For an extended source like an LED, the coupling efficiency also depends on the size of the emitting area compared with the core area of the optical fiber.

12.2. Lambertian Source-10

➢ As seen fig.12-3:

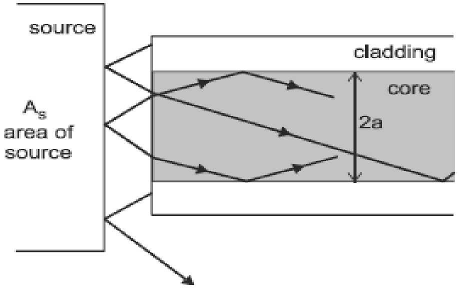


Figure 12-3 Light striking the fiber outside the core area is not coupled, regardless of angle. This decreases the coupling efficiency when the emission area is greater than the fiber core area.

➢light that is emitted outside of the core area will not be accepted by the fiber, regardless of emission angle.

12.2. Lambertian Source-11

- Only the fraction $\pi a^2 / A_s$ will be accepted by the fiber if $\pi a^2 < A_s$.
- In this case, the coupling efficiency will be

$$\eta_c = \frac{\pi a^2}{A_s} \left(\frac{NA}{n_0} \right)^2 \quad \text{(coupling efficiency when } \pi a^2 < A_s\text{) (12-12)}$$

- When $\pi a^2 > A_s$, the coupling efficiency depends only on emission angle, and Eq. (12-11) applies.
- Adge-emitting LEDs have a smaller emission area, making them more efficient for coupling into an optical fiber.

Example 12.1

❖ A GaAs LED with a square emitting area of side 0.2 mm emits light through air into an optical fiber with core index 1.5, fractional index difference $\Delta = 0.008$, and core diameter $50\ \mu\text{m}$. Determine (a) the coupling efficiency, and (b) total efficiency with which light generated inside the LED material is coupling into the fiber.

❑ Solution:

(a) the numerical aperture is given by Eq. (4-5) as:

$$\text{NA} \simeq n\sqrt{2\Delta} = (1.5)\sqrt{2(0.008)} = 0.190$$

Example-12.1-cont

- ✓ And the ratio of areas is:

$$\frac{\pi a^2}{A_s} = \frac{\pi(25 \times 10^{-6})^2}{(2 \times 10^{-4})^2} = 0.049$$

- ✓Eq. (12-12) then gives:

$$\eta_c = \frac{\pi a^2}{A_s}\left(\frac{NA}{n_0}\right)^2$$

$$\eta_c = (0.049)(0.19)^2 = 1.77 \times 10^{-3}$$

(b) Using Eq. (12-9),

$$\frac{P_0}{P_s} = \frac{1}{n(n+1)^2} = \frac{1}{3.6(4.6)^2} = 0.013$$

The total efficiency is then:

$$\eta_{tot} = \frac{P_{in}}{P_s} = (0.013)(1.77 \times 10^{-3}) = 2.32 \times 10^{-5}$$

12.3. Laser source

- In the previous section, we saw that light obeying Lamber's law is coupled more efficiently into an optical fiber than light from a point source.
- Due to increased directionality in the angular distribution.
- We would expect laser light to be even more efficiently coupled, due to its high degree of directionality (see fig. 11-10).

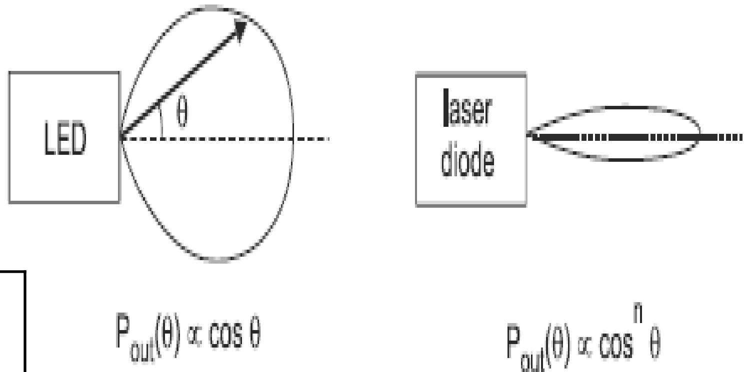


Figure 11-10 the angular distribution pattern is much more directional for a laser than for an LED.

12.3. Laser source-2

- To make a quantitative estimate of this improved coupling efficiency, the angular variation of a laser's brightness is taken to be: $B(\theta) = B(0)\cos^m\theta$ (12-13)
- Where m is large, and not necessarily an integer.
- Although this is not the actual angular distribution for a laser,
- It can be a close approximation when the proper value of m is chosen, and it makes the calculations simpler.
- As for the Lambertian emitter, we first calculate the total power emitted,

$$P_0 = A_s \int_0^{\pi/2} B(0)\cos^m\theta\, 2\pi \sin\theta\, d\theta \quad (12\text{-}14)$$

$$= 2\pi A_s B(0) \int_0^{\pi/2} \cos^m\theta \sin\theta\, d\theta$$

12.3. Laser source-3

- Which can be valuated using the substitution
  u=cos θ, $du = -\sin\theta\, d\theta$

As:
$$P_0 = 2\pi A_s B(0) \int_0^1 u^m\, du \qquad (12\text{-}15)$$
$$= 2\pi A_s B(0) \left[\frac{u^{m+1}}{m+1}\right]_0^1$$
$$= \frac{2\pi}{m+1} A_s B(0)$$

- The power coupled into the fiber is determined in the same way, except that the upper limit on θ is α_{max}, which leads to a lower limit on u.

- The power into the fiber is then

$$u_{min} = \cos\alpha_{max} = \sqrt{1-\sin^2\alpha_{max}} = \sqrt{1-(NA/n_0)^2}$$

$$P_{in} = \frac{2\pi}{m+1} A_s B(0)(1-u_{min}^{m+1}) \qquad (12\text{-}16)$$
$$= \frac{2\pi}{m+1} A_s B(0)\left[1-\left(1-\frac{NA^2}{n_0^2}\right)^{(m+1)/2}\right]$$

12.3. Laser source-4

- Which corresponds to a coupling efficiency:

$$\eta_c = \frac{P_{in}}{P_0} = 1 - \left(1 - \frac{NA^2}{n_0^2}\right)^{(m+1)/2} \qquad (12\text{-}17)$$

- For small NA and m not to large, the biomial expansion $(1+x)^n \approx 1+nx$ can be used to obtain the simplified result

$$\eta_c \simeq \frac{m+1}{2}\left(\frac{NA}{n_0}\right)^2 \text{ (laser coupling efficiency)} \quad (12\text{-}18)$$

❖ Equ.(12-18) is obviously not true for arbitrarily large m.

Example 12.2

- A laser beam has an angular distribution with a full width at half maximum (FWHM) of 20^o. Determine the value of m for this distribution, and calculate the coupling efficiency into a large core area optical fiber with NA = 0.22.

- Solution:

The half width is 10^o, so m is found from $\frac{1}{2} = \cos^m 10^o$

Taking the natural log of both sides yields

$\ln(0.5) = m \ln(\cos 10^o)$

so

Example -12.2-cont

$$m = \frac{\text{In }(0.5)}{\text{In}(\cos 10^{\circ})} = 45.3$$

putting this into the approximate expression Eq. (12-18) gives the result

$$\eta_c \simeq \frac{m+1}{2}\left(\frac{NA}{n_0}\right)^2 \text{ (laser coupling efficiency)} \qquad (12\text{-}18)$$

$$\eta_c \simeq \frac{46.3}{2}(0.22)^2 = 1.12$$

Which is clearly not exact since η_c must be < 1. To get an accurate result here, it is neccesary to use Eq. (12-7):

$$\eta_c = \frac{P_{in}}{P_0} = 1 - \left(1 - \frac{NA^2}{n_0^2}\right)^{(m+1)/2} \qquad (12\text{-}17)$$

$$\eta_c = 1 - [1 - (0.22)^2]^{46.3/2} = 0.683$$

This efficiency is, as expected, must higher than that for an LED.

Thank you for your attention