

OPTOELECTRONICS (I)

Chapter 3: Modulation of light

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3.1 Elliptical Polarization

$$\varepsilon_x = i\varepsilon_0 \cos(kz - \omega t)$$

$$\varepsilon_y = j\varepsilon_0 \sin(kz - \omega t)$$

$$\varepsilon = \varepsilon_x + \varepsilon_y$$

$$\varepsilon = \varepsilon_0 [i \cos(kz - \omega t) + j \sin(kz - \omega t)]$$

$$\phi = \phi_2 - \phi_1 = \pi / 2$$

A **single wave** in which the **electric vector** at a given point **in space** is **constant** in **amplitude** but **rotates** with angular frequency ω .

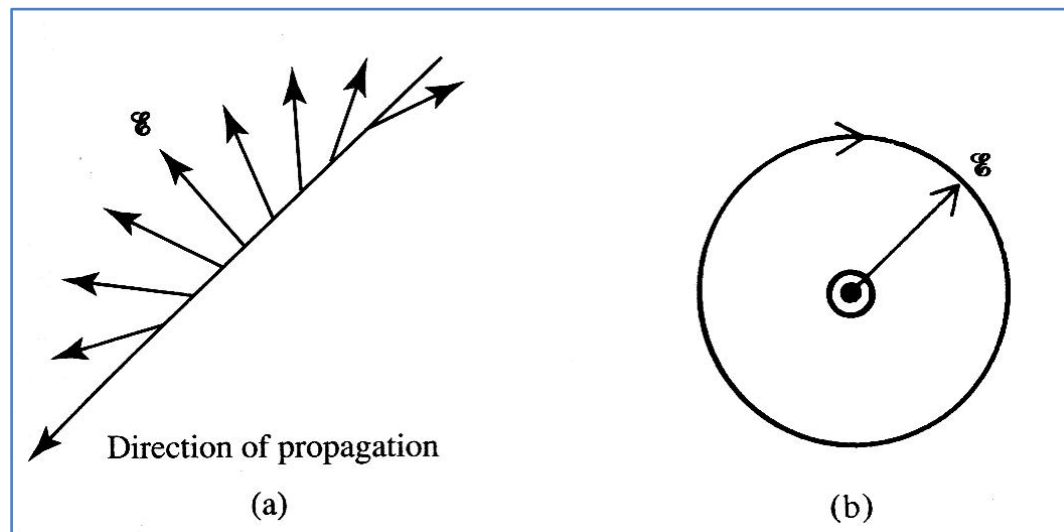
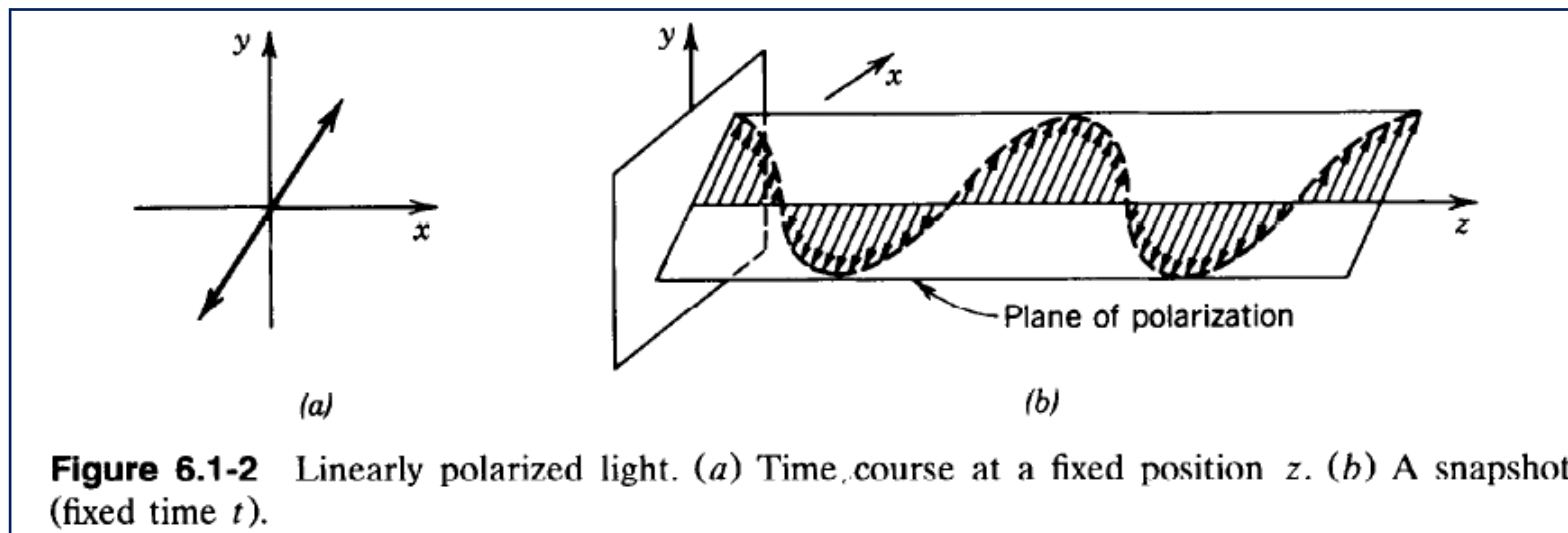
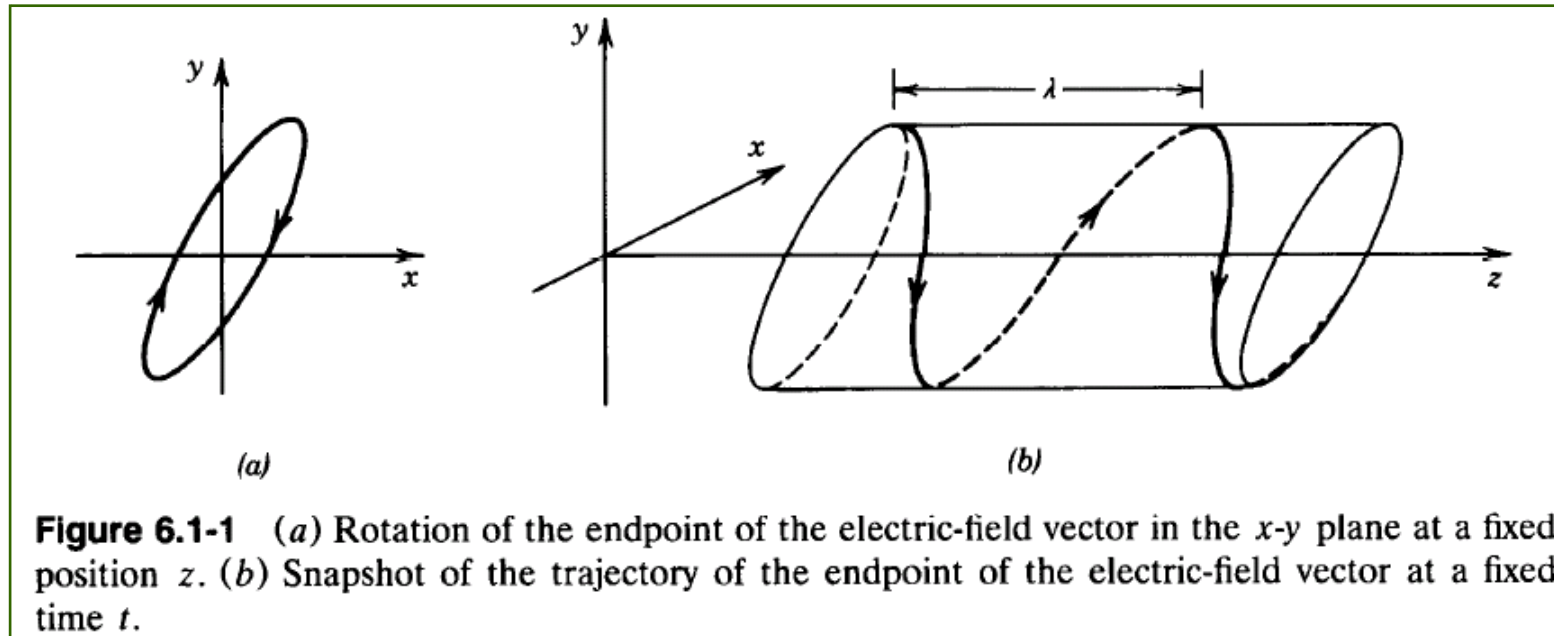


Fig. 3.1 **Right circularly** polarized light: (a) electric vectors at a given **instant** in **time** and (b) rotation of the vector at a given **position** in **space**.



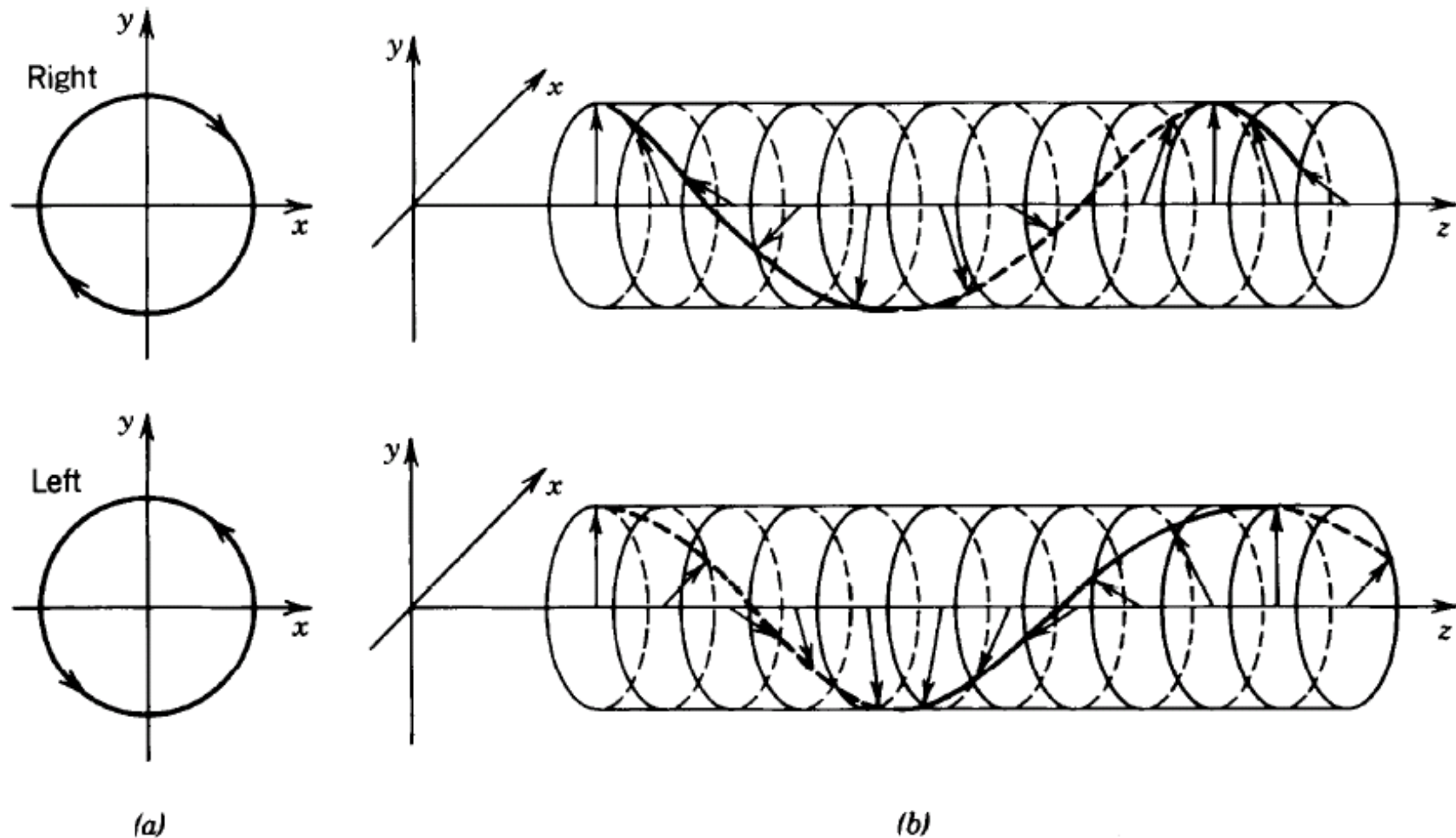


Figure 6.1-3 Trajectories of the endpoint of the electric-field vector of a circularly polarized plane wave. (a) Time course at a fixed position z . (b) A snapshot (fixed time t). The sense of rotation in (a) is opposite that in (b) because the traveling wave depends on $t - z/c$.

3.1 Elliptical Polarization

$$\varepsilon_x = i\varepsilon_0 \cos(kz - \omega t)$$

$$\varepsilon_y = j\varepsilon_0' \sin(kz - \omega t)$$

$$\varepsilon_0 \neq \varepsilon_0'$$

The resultant wave is said to be **elliptically polarized** and in fact **plane** and **circular polarization** are **special cases** of elliptical polarization.

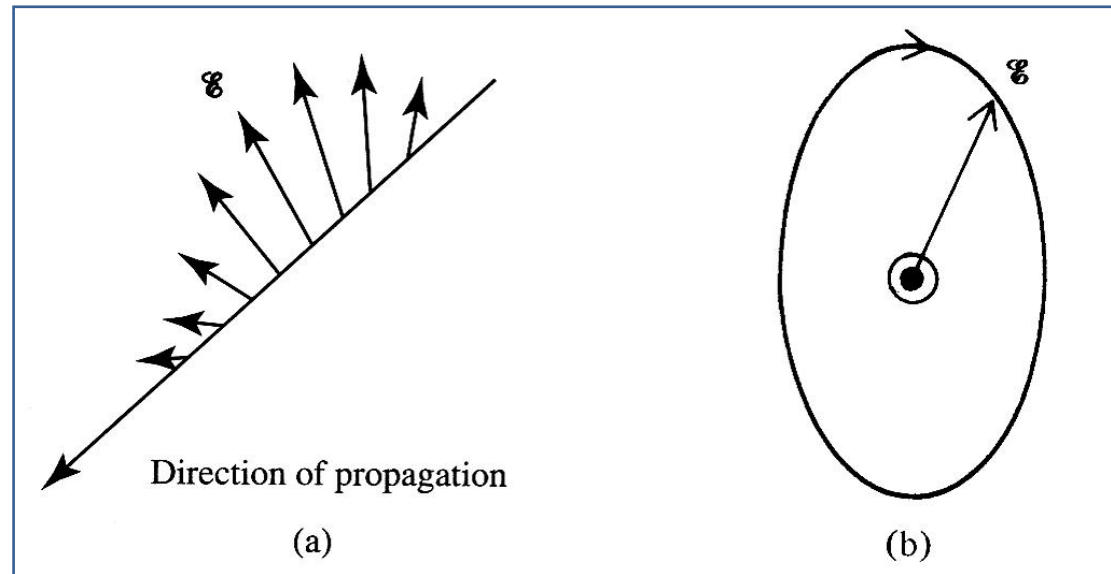


Fig. 3.1 **Right elliptically polarized light**: electric vectors at (a) a given **instant** in **time** and (b) a given **position** in **space**.

3.2 Birefringence (doubly refracting)

➤ Many important **crystalline optical** materials such as **calcite** (CaCO_3), **quartz** (SiO_2) and **KDP** (Potassium dihydrogen phosphate, KH_2PO_4) are **anisotropic**.

$$P = \varepsilon_0 \cdot \chi \cdot E$$

$$P = a_1 E + \frac{1}{2} a_2 E^2 + \frac{1}{6} a_3 E^3 + \dots$$

$$a_1 = \varepsilon_0 \cdot \chi^{(1)}$$

$$\chi \equiv \frac{P_x}{\varepsilon_0 E_x} \quad (\text{electric susceptibility})$$

$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} \mathbf{E}^2 + \chi^{(3)} \mathbf{E}^3 + \dots \right) \equiv \mathbf{P}^{(1)} + \mathbf{P}^{(2)} + \mathbf{P}^{(3)} + \dots$$

بلورهای دوشکستی: ضریب شکست، نه تنها به راستای انتشار امواج، بلکه به راستای قطبش بلور نیز بستگی دارد.

3.2 Birefringence (doubly refracting)

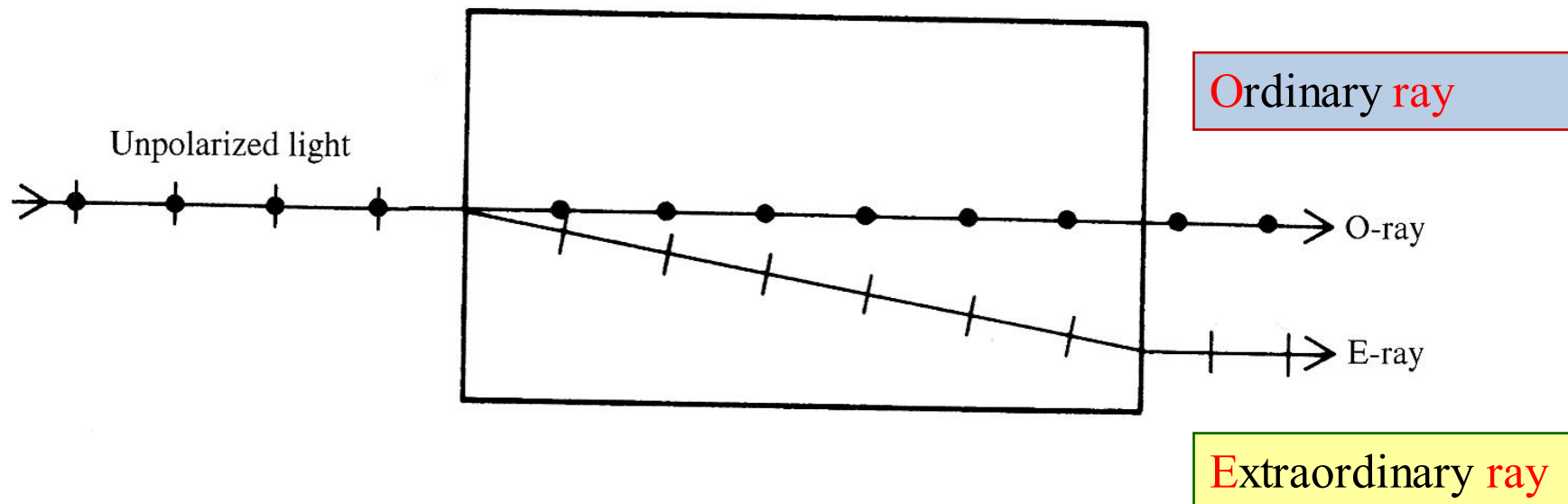
$$D = \varepsilon_0 \cdot E + P = \varepsilon_0 \cdot E + \varepsilon_0 \cdot \chi \cdot E = \varepsilon_0 (1 + \chi) E$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_0(1 + \chi_x) & 0 & 0 \\ 0 & \varepsilon_0(1 + \chi_y) & 0 \\ 0 & 0 & \varepsilon_0(1 + \chi_z) \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

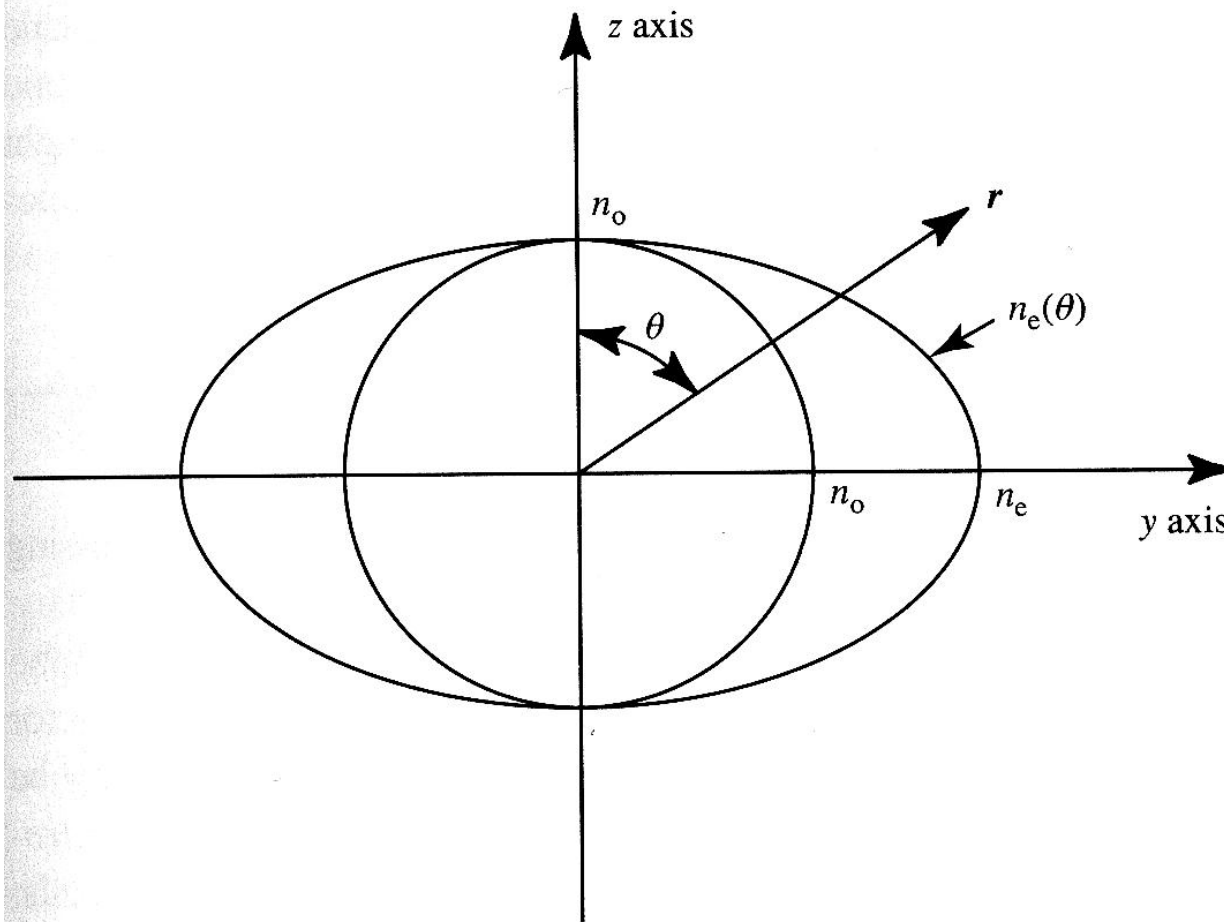
$$n_x = \sqrt{1 + \chi_x} \quad n_y = \sqrt{1 + \chi_y} \quad n_z = \sqrt{1 + \chi_z}$$

3.2 Birefringence

- ❖ وقتی نور **غیرقطبیده** (یا با قطبش وابسته به ساختار بلور)، بر یک **بلور دوشکستی** میتابد، نور منتشر شده از میان بلور را می توان ترکیبی از دو موج مستقل در نظر گرفت.
- ❖ پرتوهای معمولی و غیرمعمولی دارای جهات قطبش عمود بر یکدیگر است.



3.2 Birefringence



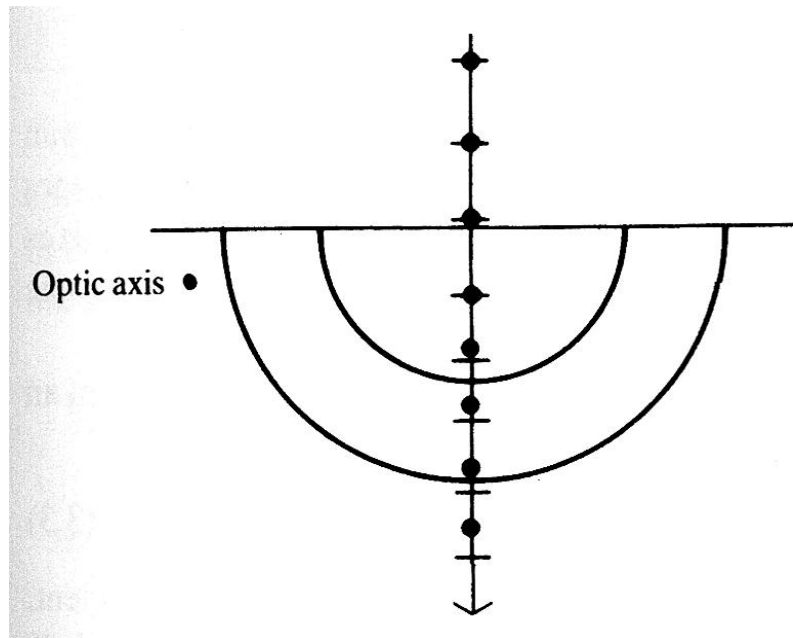
Normal (index) surfaces for a **positive uniaxial** crystal projected on the xz

3.2.1 Phase plates

Quarter-wave, half-wave and whole-wave plate

❖ تیغه های بلوری را میتوان برای ایجاد اثری دلخواه در باریکه نور گذرا از میان یک تیغه، در راستایی خاص نسبت به محور نوری برش داد.

❖ تیغه ها را میتوان با سطوحی بزرگ و موازی با محور نوری به منظور تغییر فاز معین بین پرتوهای O و E برش داد.



$$|n_o d - n_e d| = \lambda_0 / 4$$

❖ اگر ماده دوشکستی را طوری بسازیم که سطح تماس آن عمود بر محور نوری باشد، دو پرتو O و E در آنها از یکدیگر جدا نمی گردند، بلکه فقط شاهد تغییر فاز بین آن دو خواهیم بود.

❖ تیغه یک چهارم موج: تیغه ای با ضخامت d و اختلاف راه نوری $\lambda_0/4$ معادل با اختلاف فاز $\pi/2$ است.

3.3 Optical activity

- ❖ بلورهایی خاص (و نیز برخی از مایعات) میتوانند صفحه قطبش نور عبوری از داخل خود را **دوران** دهند. این گونه مواد دارای **کنش نوری** هستند.
- ❖ بلور کوارتز یکی از این نوع است.
- ❖ چرخش ایجاد شده، به ضخامت بلور و طول موج نور بستگی دارد.
- ❖ این بلورها دارای ضرایب شکست متفاوت برای نورهای قطبیده مختلف هستند.

3.4 Electro-optic effect

- when an **electric field** is **applied** across an **optical medium**, the **distribution** of electrons within it is **distorted** so that the **polarizability** and hence the **refractive index** of medium **changes anisotropically**.
- The result of this **electro-optic effect** may be to **introduce new optic axes** into naturally **doubly refracting** crystals, (KDP).
- Or to make naturally **isotropic** crystals **doubly refracting** (GaAS).
- The **change in refractive index** as a **function** of the **applied field**:

$$\Delta(1/n^2) = r\mathcal{E} + P\mathcal{E}^2$$

r.E: Pockels effect

P.E²: Kerr effect

r: **linear electro-optic** coefficient **P**: **quadratic electro-optic** coefficient

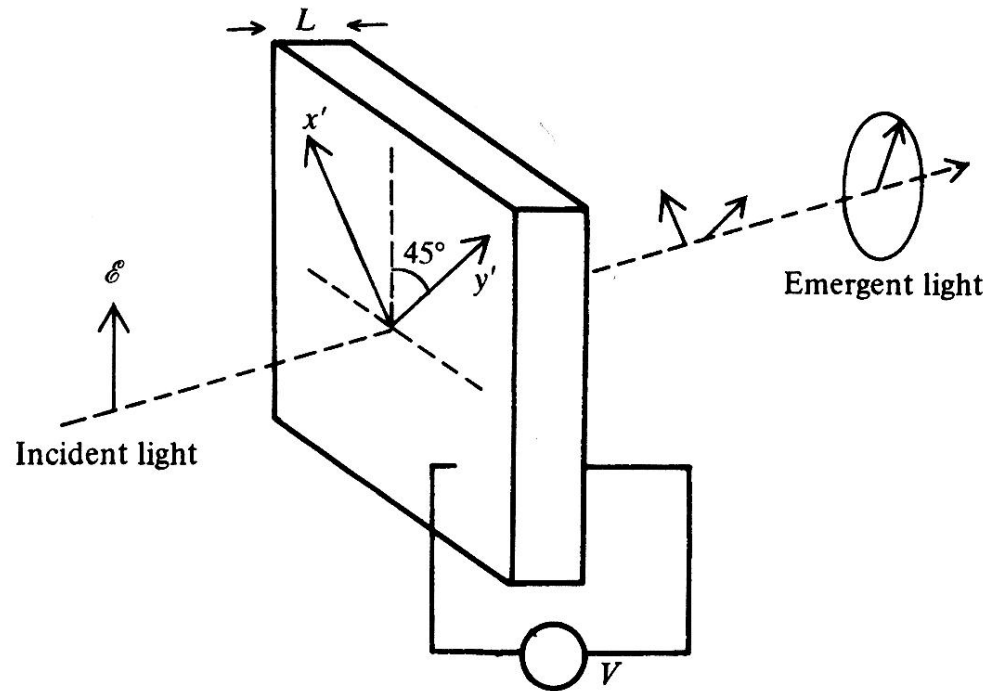
3.4.1 Pockels electro-optic modulator

❖ اگر در KDP، میدان الکتریکی در جهت محور Z اعمال شود، محورهای اصلی X, Y، ۴۵ درجه چرخش پیدا کرده و ضرایب شکست در این جهات جدید به صورت زیر نمایان می شود:

$$n_{x'} = n_0 + \frac{n_0^3}{2} r_{63} E_z$$

$$n_{y'} = n_0 - \frac{n_0^3}{2} r_{63} E_z$$

$$\Delta \left(\frac{1}{n^2} \right) = -\frac{2\Delta n}{n^3} = r_{63} E_z$$



3.4 Pockels electro-optic modulator

$$1 \quad \mathcal{E} = \mathcal{E}_0 \cos(\omega t - kz)$$

$$2 \quad \mathcal{E}_{x'} = \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t - kz)$$

$$\mathcal{E}_{y'} = \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t - kz)$$

$$3 \quad \phi_{x'} = \frac{2\pi}{\lambda_0} n_x L$$

$$\phi_{y'} = \frac{2\pi}{\lambda_0} n_y L$$

4

$$\phi_{x'} = \frac{2\pi}{\lambda_0} L n_o \left(1 + \frac{1}{2} r n_o^2 \mathcal{E}_z\right)$$

$$\phi_{y'} = \frac{2\pi}{\lambda_0} L n_o \left(1 - \frac{1}{2} r n_o^2 \mathcal{E}_z\right)$$

5

$$\phi_{x'} = \phi_0 + \Delta\phi$$

$$\phi_{y'} = \phi_0 - \Delta\phi$$

$$\Delta\phi = \frac{\pi}{\lambda_0} L r n_o^3 \mathcal{E}_z = \frac{\pi}{\lambda_0} r n_o^3 V$$

$$\Phi = \phi_{x'} - \phi_{y'} = 2\Delta\phi = \frac{2\pi}{\lambda_0} r n_o^3 V$$

$$\mathcal{E}_{x'} = \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t + \Delta\phi)$$

$$\mathcal{E}_{y'} = \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t - \Delta\phi)$$

❖ اگر پرتو تابشی از معادله ای مانند مقابل تبعیت کند، در این صورت مولفه های آن در امتداد محورهای y' , x' به صورت زیر خواهند بود.

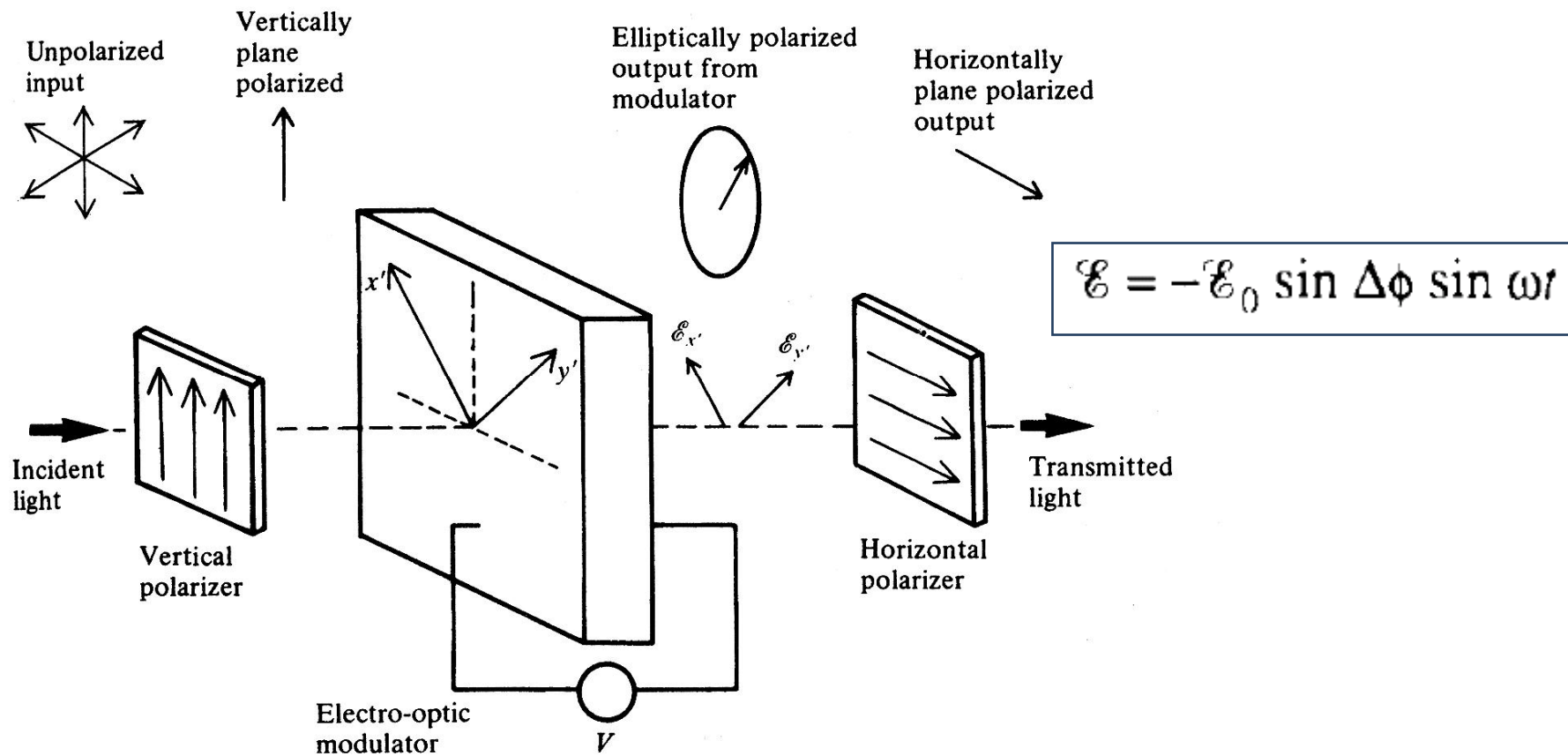
جابجایی فاز، $\Delta\Phi$ ، برای هر دو مولفه مستقیماً به ولتاژ، V ، بستگی دارد.

3.4 Pockels electro-optic modulator

$$E_x = \frac{E_0}{\sqrt{2}} \cos(\omega t + \Delta\phi)$$

$$E_y = \frac{E_0}{\sqrt{2}} \cos(\omega t - \Delta\phi)$$

$$E = -\frac{E_0}{2} [\cos(\omega t + \Delta\phi) - \cos(\omega t - \Delta\phi)]$$



3.4 Pockels electro-optic modulator

$$\mathcal{E} = -\mathcal{E}_0 \sin \Delta\phi \sin \omega t$$

$$I = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \mathcal{E}^2 dt$$

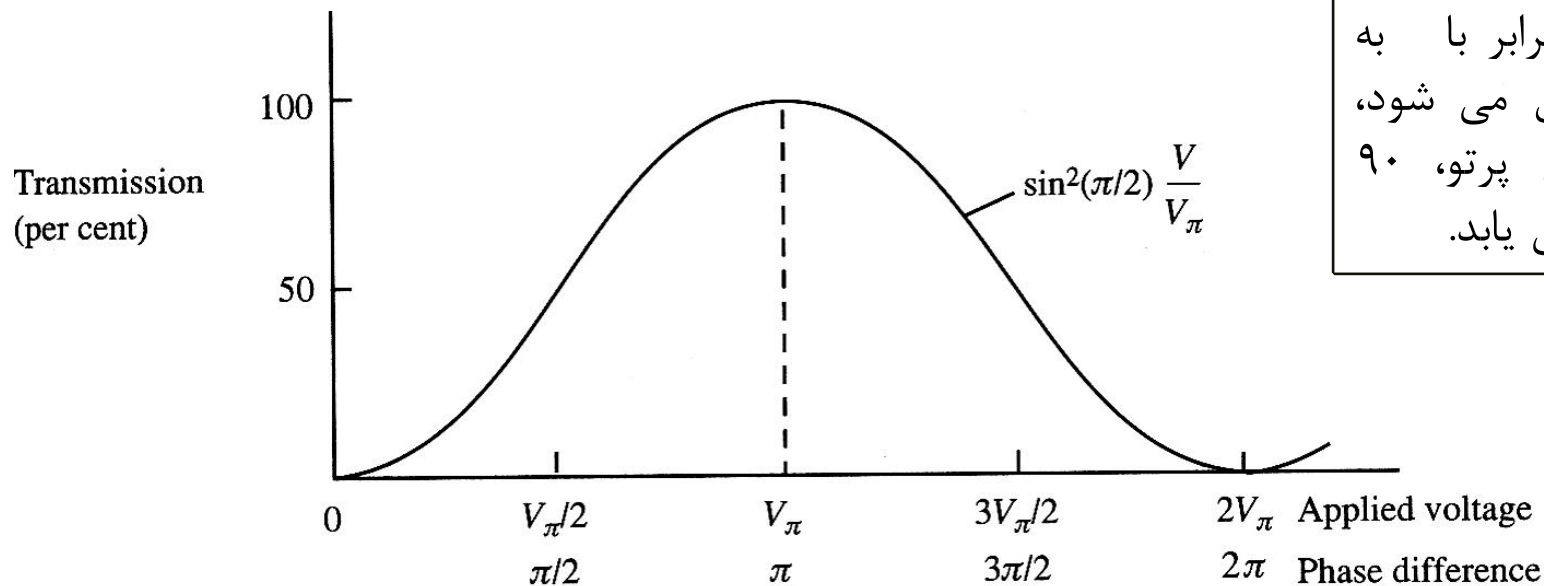
$$I = I_0 \sin^2 \Delta\phi = I_0 \sin^2(\Phi/2)$$

$$\frac{I}{I_0} = \sin^2\left(\frac{\pi}{\lambda_0} r n_o^3 V\right)$$

$$\frac{I}{I_0} = \sin^2\left(\frac{\pi}{2} \frac{V}{V_\pi}\right)$$

half-wave voltage

$$V_\pi (= \lambda_0 / (2 r n_o^3))$$



وقتی ولتاژی برابر با به مدولاتور اعمال می شود، صفحه قطبش پرتو، ۹۰ درجه دوران می یابد.

3.4 Pockels electro-optic modulator

TABLE 3.1 Characteristics of some electro-optic materials used in Pockels cells

Material	Linear electro-optic coefficient, r (pm V ⁻¹)	n_o †	n_e †	Relative permittivity,‡ ϵ_r
KH ₂ PO ₄ (KDP)	10.6	1.51	1.47	42
KD ₂ PO ₄ (KD*P)	26.4	1.51	1.47	50
AH ₂ PO ₄ (ADP)	8.5	1.52	1.48	12
Cadmium telluride (CdTe)	6.8	2.6		7.3
Lithium tantalate (LiTaO ₃)	30.3	2.175	2.180	43
Lithium niobate (LiNbO ₃)	30.8	2.29	2.20	18
Gallium arsenide (GaAs)	1.6	3.6		11.5
Zinc sulfide (ZnS)	2.1	2.32		16
Barium borate (BaB ₂ O ₄)		1.67	1.56	7.4

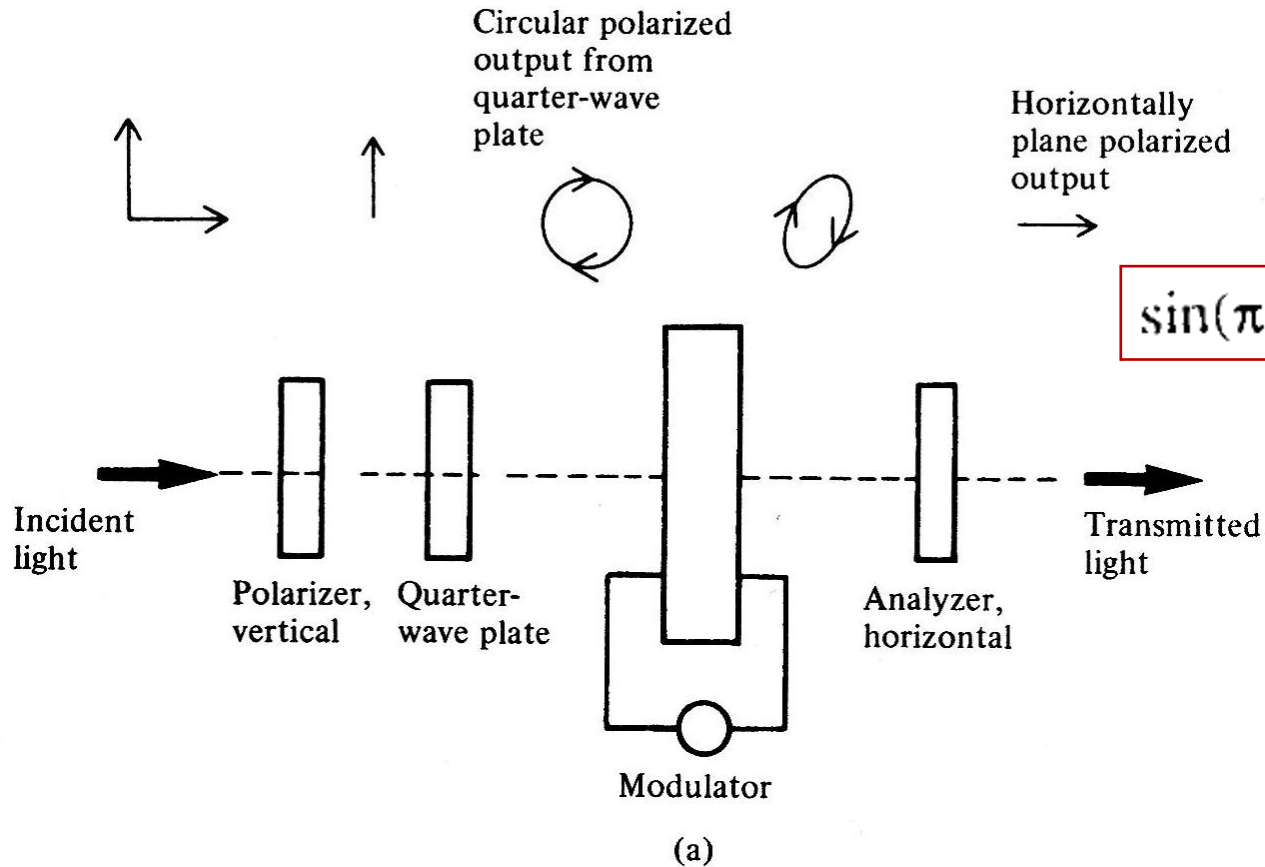
† Values near a wavelength of 550 nm.

‡ Low frequency values.

Pockels electro-optic cell biased with a quarter-wave plate

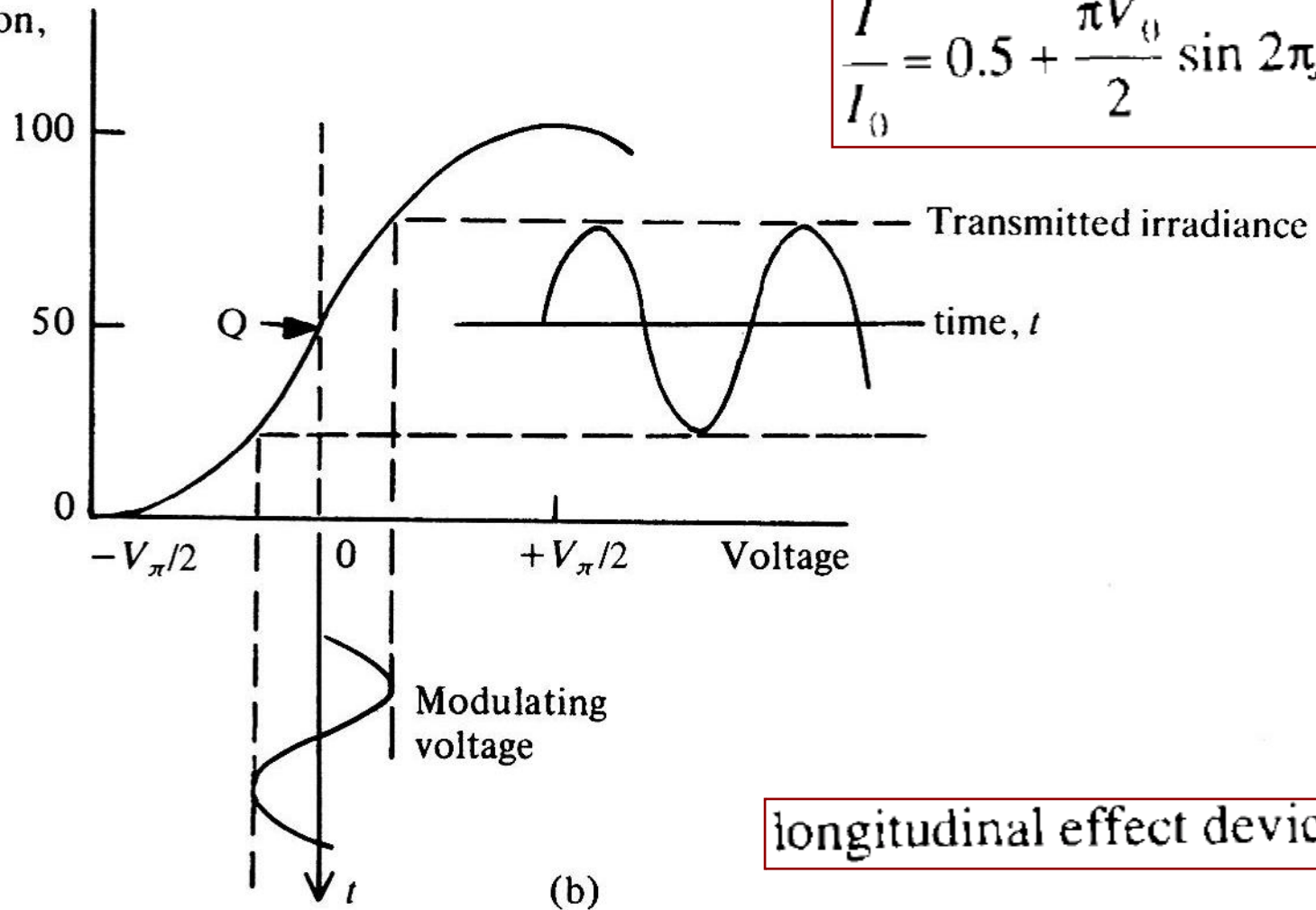
$$\Phi = \frac{\pi}{2} + 2\Delta\phi = \frac{\pi}{2} + \pi \frac{V}{V_{\pi}}$$

$$\frac{I}{I_0} = \sin^2\left(\frac{\pi}{4} + \frac{\pi}{2} \frac{V}{V_{\pi}}\right) = \frac{1}{2} \left(1 + \sin \frac{\pi V}{V_{\pi}}\right)$$



Packels electro-optic cell biased with a quarter-wave plate

Transmission,
per cent



3.5 Kerr modulators

$$\Delta(1/n^2) = r\mathcal{E} + P\mathcal{E}^2$$

$$\Delta n = n_p - n_s = K\lambda_0 E^2$$

❖ تغییر ضریب شکست متناسب با مجذور میدان الکتریکی اعمال شده است.

❖ اختلاف ضریب شکست در مواد برای نورقطبیده موازی و عمود بر محور نوری القایی، از رابطه مقابل بدست می آید. که **K**، ضریب ثابت کر نامیده می شود.

TABLE 3.2 Typical values of the Kerr constant **K** for $\lambda = 589.3$ nm at about 20°C

Material	$K (\times 10^{-14} \text{ m V}^{-2})$
Water	5.2
Nitrobenzene	244
Nitrotoluene	137
Glasses – various	0.03–0.17

❖ بین اعمال میدان الکتریکی و پیدایش اثر کر، تاخیر وجود دارد که این تاخیر در جامدات ممکن است حتی چند ثانیه باشد.

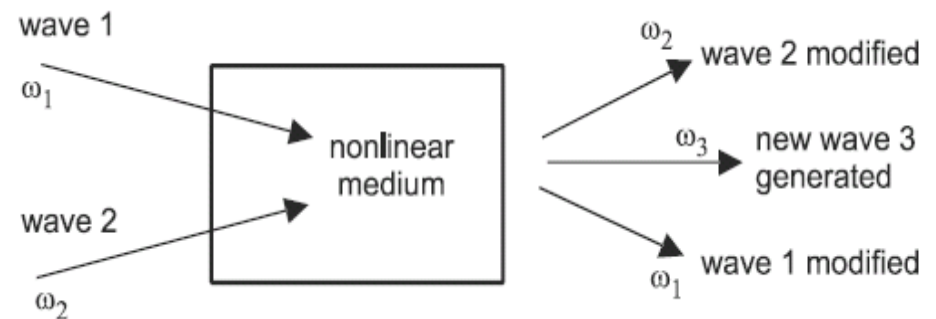
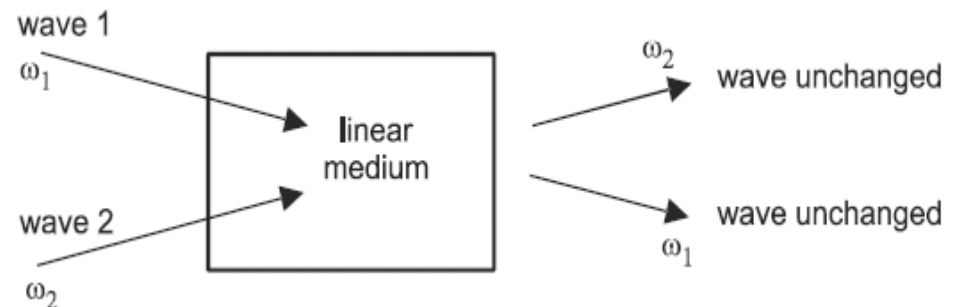
❖ لیکن برای مایعات غیرقطبی، تاخیر بسیار کمتر و گاه حتی کمتر از 10^{-11} s است.

Nonlinear Optics [17]

➤ **A linear interaction:** The presence of wave 2 has no effect on wave 1, and vice versa.

➤ **A nonlinear interaction:** Each wave can **modify** the properties of the other, **changing**, for example, the other wave's **amplitude** or **phase**.

➤ **Self-action:** One of the waves can also modify **its own amplitude** or **phase**.

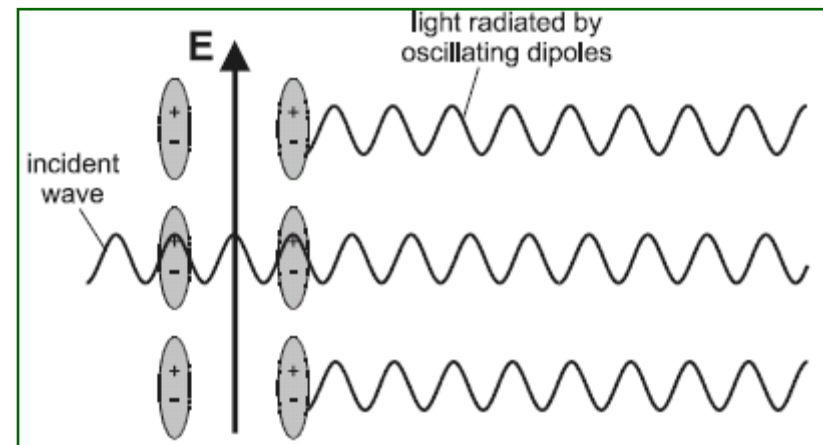
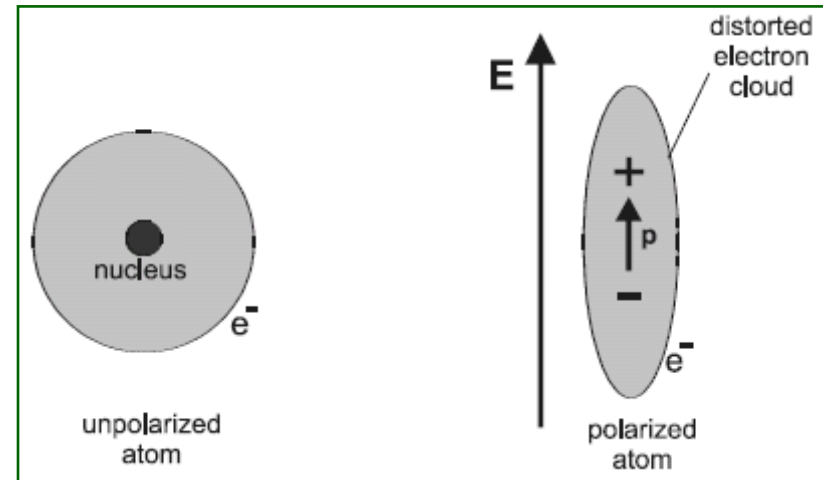


9-1. FUNDAMENTAL MECHANISMS

Electron Cloud Distortion

Figure 9-2

- (a) An electric field distorts the electron cloud in an atom, creating an electric **dipole moment p** .
- (b) The oscillating dipoles driven by the **light wave's E field** radiate additional waves that **interfere** with the original wave, modifying its **propagation speed**.



Electron Cloud Distortion

$$v_p = c/n$$

$$n = \sqrt{\epsilon_r} = \sqrt{\epsilon/\epsilon_0} = \sqrt{1 + \chi}$$

ϵ : permittivity of the medium

ϵ_0 : permittivity of free space

ϵ_r : relative permittivity or *dielectric constant*

(actually not a *constant*, but a *function of frequency*)

χ : *electric susceptibility*

$$\chi \equiv \frac{P_x}{\epsilon_0 E_x} \quad (\text{electric susceptibility})$$

$$P_x = \epsilon_0(\chi_1 E_x + \chi_2 E_x^2 + \chi_3 E_x^3)$$

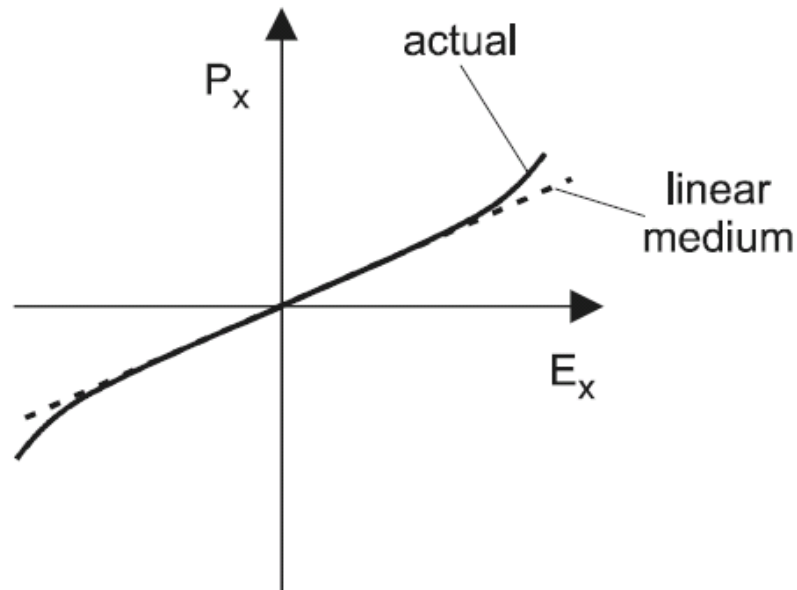
Nonlinear Optics

$$P_x = \varepsilon_0(\chi_1 E_x + \chi_2 E_x^2 + \chi_3 E_x^3)$$

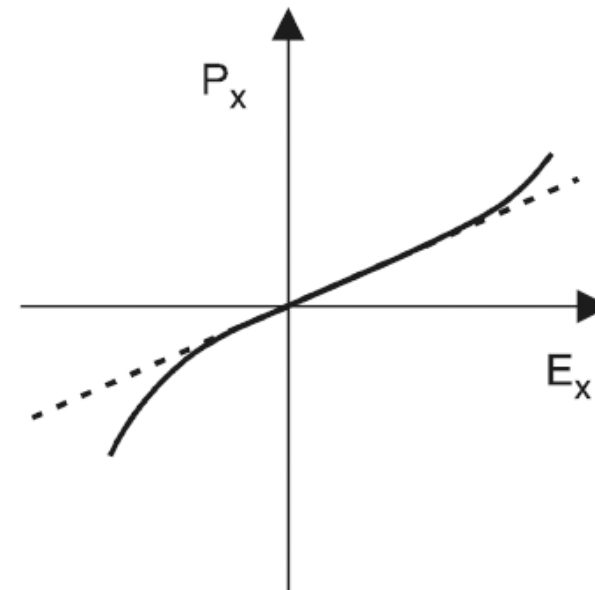
χ_1 : linear susceptibility,

χ_2 (χ_3): second (third)-order susceptibilities

Centrosymmetric materials: (those with inversion symmetry), $\chi_2 = 0$.



centro-symmetric material
symmetric for $x \rightarrow -x$



noncentro-symmetric
not symmetric for $x \rightarrow -x$

Other Nonlinear Mechanisms

1. Molecular Orientation

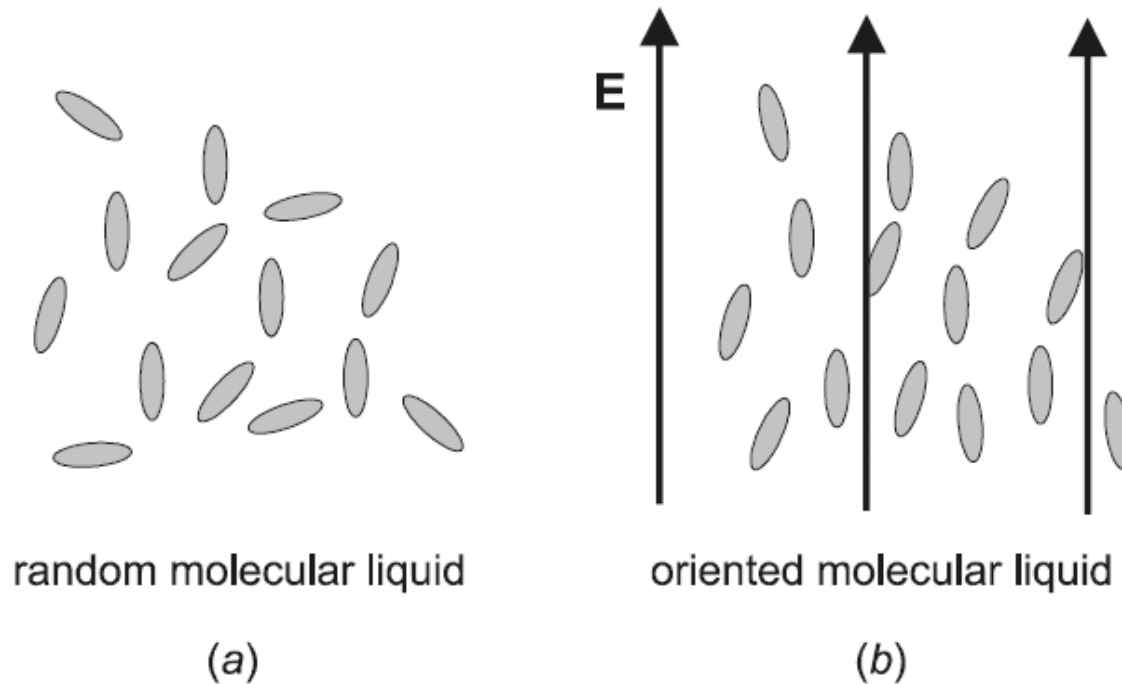


Figure 9-4 A **strong electric field** can orient asymmetrical molecules in a liquid, producing a **nonlinear response**.

Other Nonlinear Mechanisms

2. Electrostriction

The potential energy of a dipole moment \mathbf{p} in a field \mathbf{E} is given by $U = -\mathbf{p} \cdot \mathbf{E}$

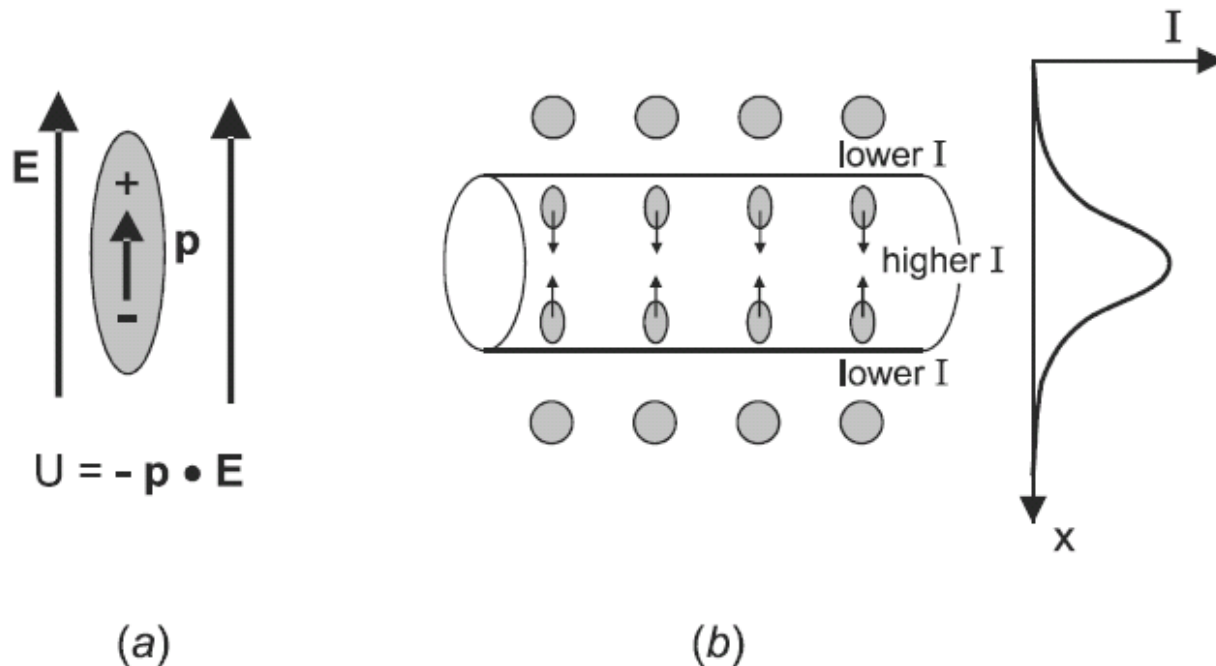


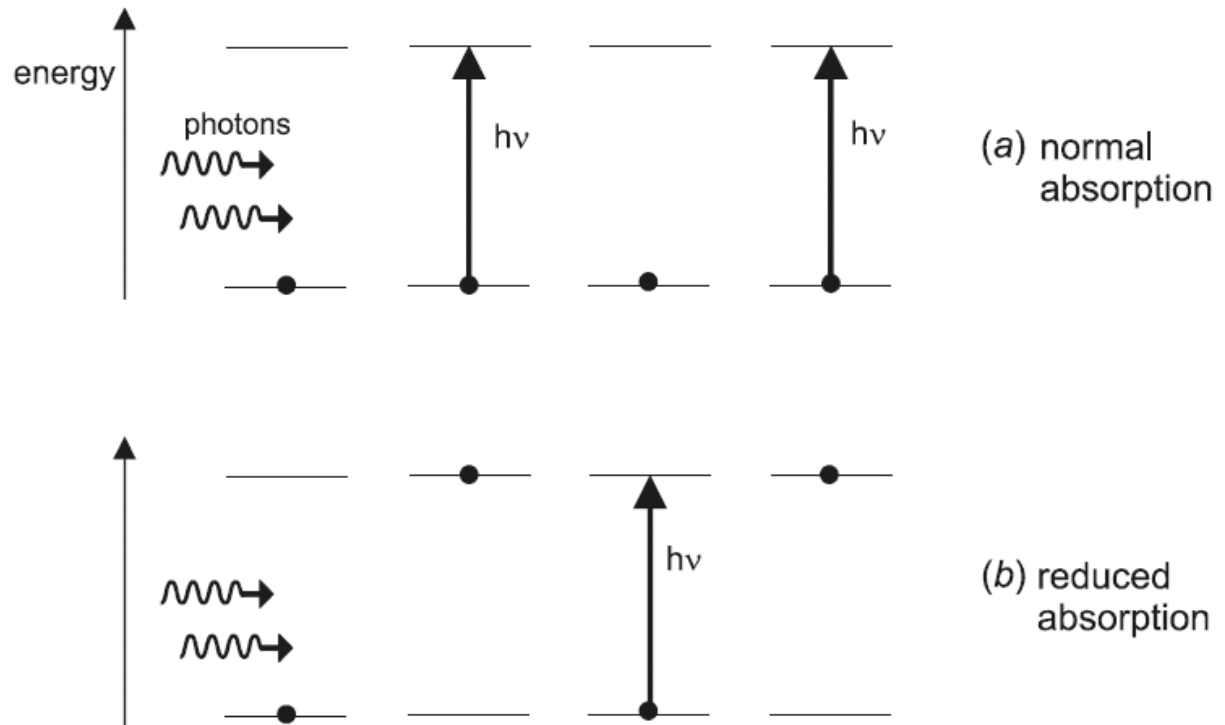
Figure 9-5 (a) A dipole's energy is lower when it is in a larger E field. (b) **Atoms** tend to move to the center of an optical beam, where the E field is highest, a phenomenon known as **electrostriction**.

Other Nonlinear Mechanisms

3. Resonant Absorption

➤ A different type of nonlinearity can occur when there is resonance between $h\nu$ and ΔE .

(a) At **low intensity**, most atoms are in the **ground state 1** and are available for **absorbing a photon**.



optical bleaching

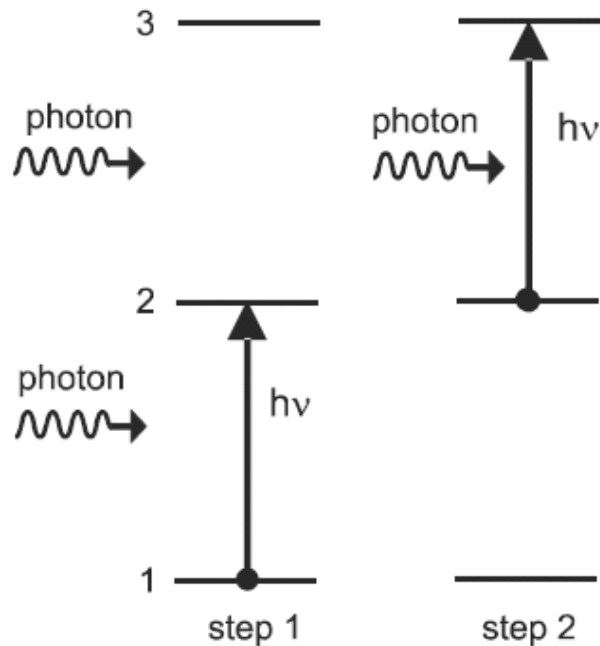
(b) At **high intensity**, a significant fraction of atoms (50% in this example) are in the **excited state 2** and are unavailable for absorbing a photon. This is optical bleaching, an **intensity-dependent absorption**.

Other Nonlinear Mechanisms

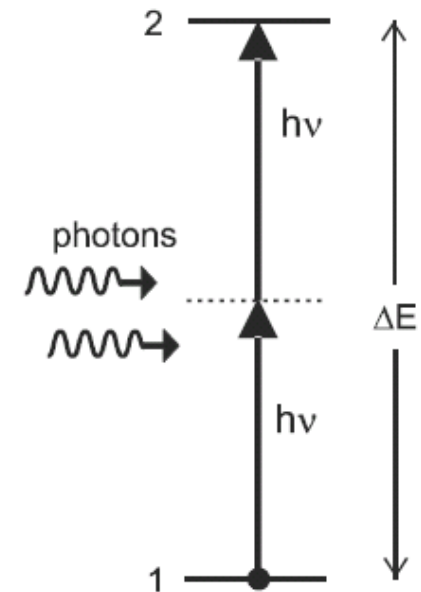
3. Resonant Absorption

Figure 9-7

(a) In a **stepwise absorption process**, an atom is **first** raised to **level 2** by absorbing a photon, and then raised to **level 3** by **absorbing a second photon**.



(a)



(b)

(b) In **two-photon absorption**, the two photons are absorbed **simultaneously**, **without excitation** of any **real intermediate** level. Both of these processes can be used to **increase the absorption** at **high intensity**.

Other Nonlinear Mechanisms

4. Inelastic Scattering

➤ Another type of **nonlinear mechanism** arises from the **Raman** and **Brillouin scattering** processes.

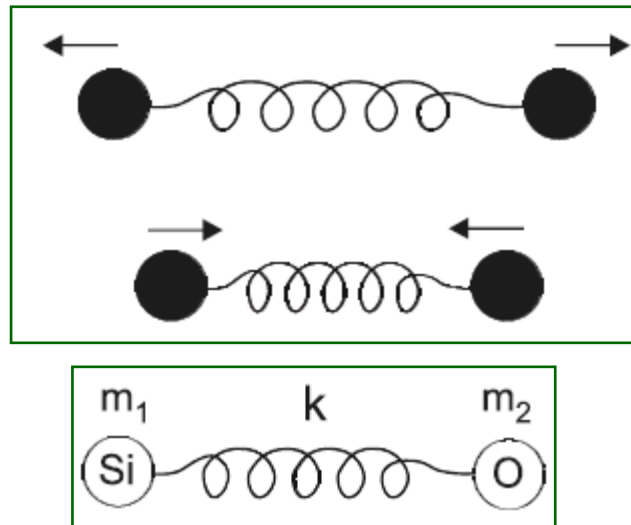


Figure 5-6 **Molecular vibrations** involved in **Raman scattering** can be modelled by masses and springs.

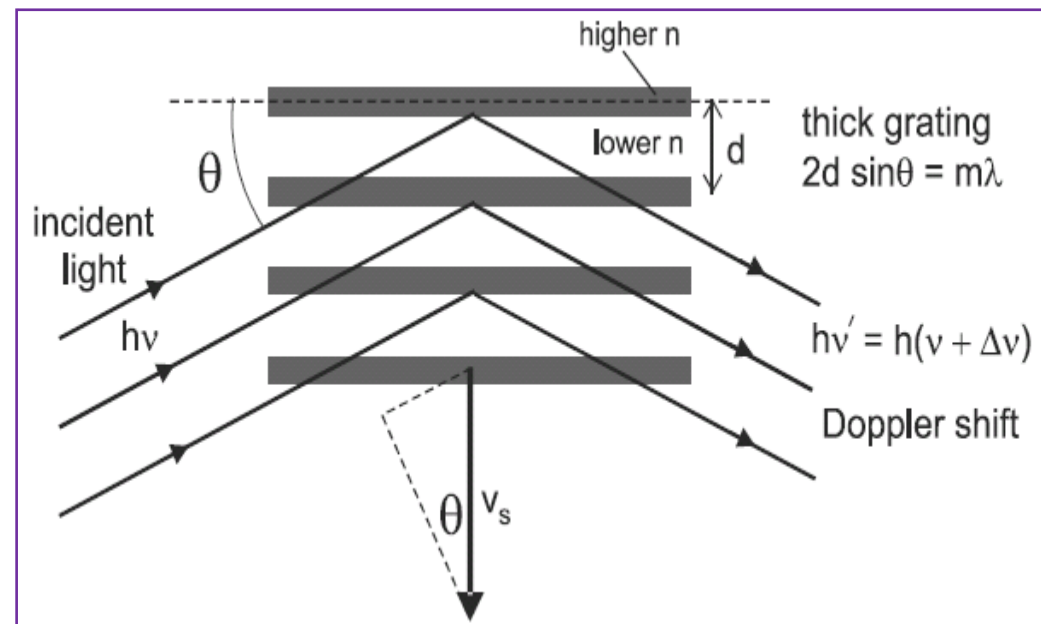


Figure 5-5 **Brillouin scattering** from acoustic waves.

Other Nonlinear Mechanisms

5. Thermal Effects

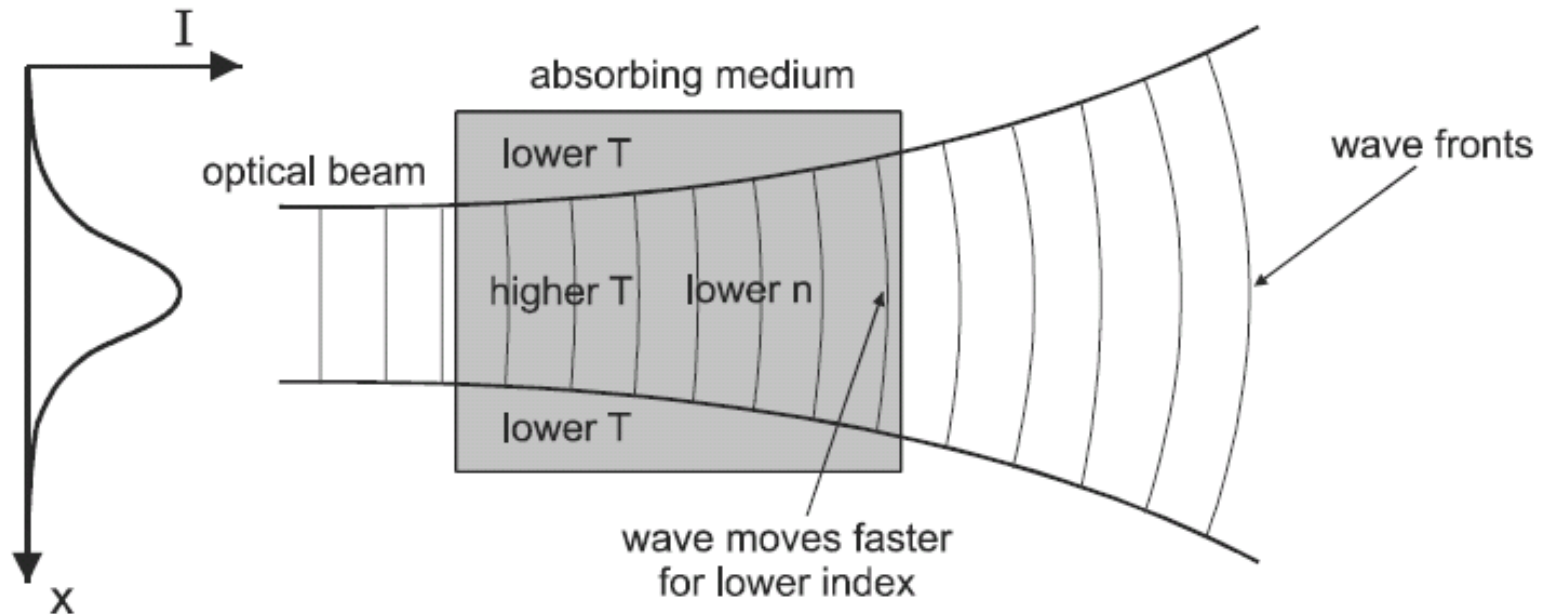


Figure 9-8 Heating of a material by an intense optical beam lowers the refractive index near the beam center. This creates a thermal lens that causes a collimated beam to diverge.

9-2. FREQUENCY CONVERSION

SHG: Second Harmonic Generation or frequency doubling

$$E(t) = A \cos \omega t$$

$$P(t) = \varepsilon_0 \chi_1 A \cos \omega t + \varepsilon_0 \chi_2 A^2 \cos^2 \omega t$$

$$P(t) = P_0 + P_\omega \cos \omega t + P_{2\omega} \cos 2\omega t$$

$$P_0 = \varepsilon_0 \chi_2 A^2 / 2$$

$$P_\omega = \varepsilon_0 \chi_1 A$$

$$P_{2\omega} = \varepsilon_0 \chi_2 A^2 / 2$$

- The **induced polarization** varies with **time** in **three distinct** ways.
- P_0 : *optical rectification*, in which a *static (dc) polarization* is produced in response to the rapidly varying electric field of the light wave.
- $P_\omega \cos \omega t$, causes the atoms to radiate light at frequency ω , and this results in the **linear refractive index**.

SHG: Second Harmonic Generation or frequency doubling

Table 9-1 Second-order nonlinear susceptibility χ_2 for selected crystals

Material	Transparency range (μm)	Linear index ^a at ω n_o/n_e	Linear index ^a at 2ω n_o/n_e	χ_2 (10^{-12} m/V) ^{b,c}
KDP	0.18–1.45	1.495/1.460	1.512/1.471	0.86
LiNbO ₃	0.4–5.5	2.234/2.155	2.325/2.233	12
AgGaS ₂	0.5–13	2.316/2.347	2.383/2.341	40
CdGeAs ₂	2.4–18	3.505/3.591	3.530/3.621	470

^aFundamental at 1.064 μm for KDP and LiNbO₃, 10.6 μm for AgGaS₂ and CdGeAs₂.

^bFor 1.064 \rightarrow 0.532 μm SHG (KDP, LiNbO₃) and 10 \rightarrow 5 μm SHG (AgGaS₂, CdGeAs₂).

^cThe parameter $\mathbf{d} = \chi_2/2$ is an often used alternative definition.

SHG: Second Harmonic Generation or frequency doubling

$$k = n\omega/c$$

$$k_{\omega} = \frac{2\pi}{\lambda_{\omega}} = n_{\omega} \frac{\omega}{c}$$

$$k_{2\omega} = \frac{2\pi}{\lambda_{2\omega}} = n_{2\omega} \frac{2\omega}{c}$$

If $n_{2\omega} = n_{\omega}$, \longrightarrow $k_{2\omega} = 2k_{\omega}$ and $\lambda_{2\omega} = \lambda_{\omega}/2$.

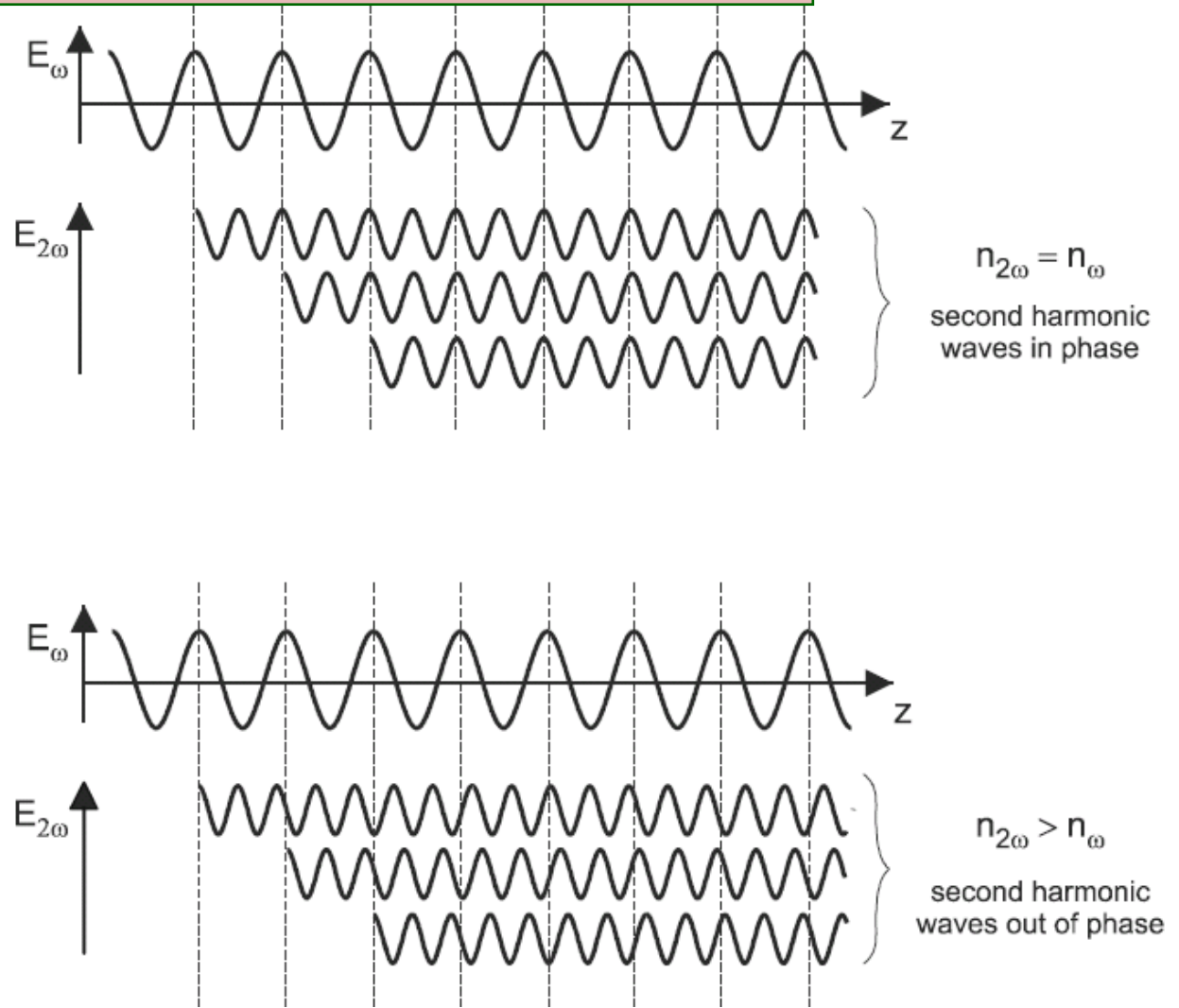
SHG: Second Harmonic Generation or frequency doubling

Figure 9-9

(a) The upper wave is the fundamental and the lower waves are the **second harmonic waves** radiated by different atoms in the material.

Constructive interference occurs when the refractive index is the same at the two frequencies.

(b) When the two indices are different, the waves get out of phase and add by destructive interference.



SHG: Second Harmonic Generation or frequency doubling

The **optimum SHG conversion** is obtained by passing the fundamental wave through a crystal of **length** equal to Lc .

$$L_c(k_{2\omega} - 2k_\omega) \equiv L_c\Delta k = \pi$$

$$\Delta k \equiv k_{2\omega} - 2k_\omega$$

$$\Delta k = (n_{2\omega} - n_\omega) \frac{2\omega}{c} \quad (\text{wavevector mismatch})$$

EXAMPLE 9-1

Light of free-space wavelength 1064 nm from a Nd:YAG laser is passed through a lithium niobate (LiNbO_3) crystal for frequency doubling to 532 nm. The index of refraction at 1064 nm is 2.234, and at 532 nm is 2.325. Determine the optimum crystal length for SHG.

Solution: The optimum length is

$$L_c = \frac{\pi}{\Delta k} = \frac{\pi c}{2\omega\Delta n} = \frac{\lambda_0}{4\Delta n}$$

where λ_0 is the free space wavelength of the fundamental wave and $\Delta n \equiv n_{2\omega} - n_{\omega}$. This evaluates to

$$L_c = \frac{1064 \times 10^{-9} \text{ m}}{4(2.325 - 2.234)} = 2.92 \text{ } \mu\text{m}$$

Three-Wave Mixing

Consider **two light waves** with frequencies ω_1 and ω_2 that **add together** at a given **point** in a **material** to give a **total electric field**:

$$E(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

Assume: $\chi_2 \neq 0$

$$\frac{1}{\epsilon_0} P(t) = \chi_1 [A_1 \cos \omega_1 t + A_2 \cos \omega_2 t] \\ + \chi_2 [A_1^2 \cos^2 \omega_1 t + A_2^2 \cos^2 \omega_2 t + 2A_1 A_2 (\cos \omega_1 t)(\cos \omega_2 t)]$$

$$2(\cos \omega_1 t)(\cos \omega_2 t) = \cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t$$

$$\omega_3 = \omega_1 \pm \omega_2$$

Upconversion:

$$\omega_3 = \omega_1 + \omega_2$$

(the frequency is **shifted upward**)

downconversion

$$\omega_3 = \omega_1 - \omega_2$$

(the frequency is shifted downward)

Three-Wave Mixing

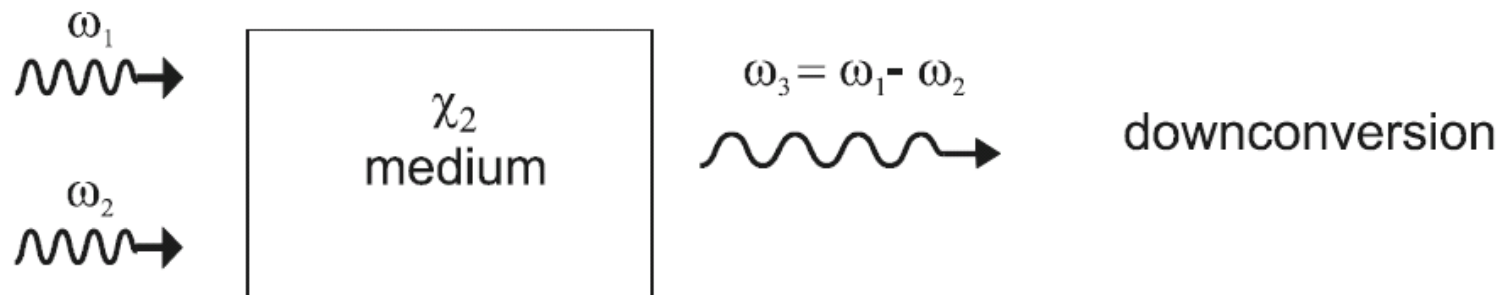
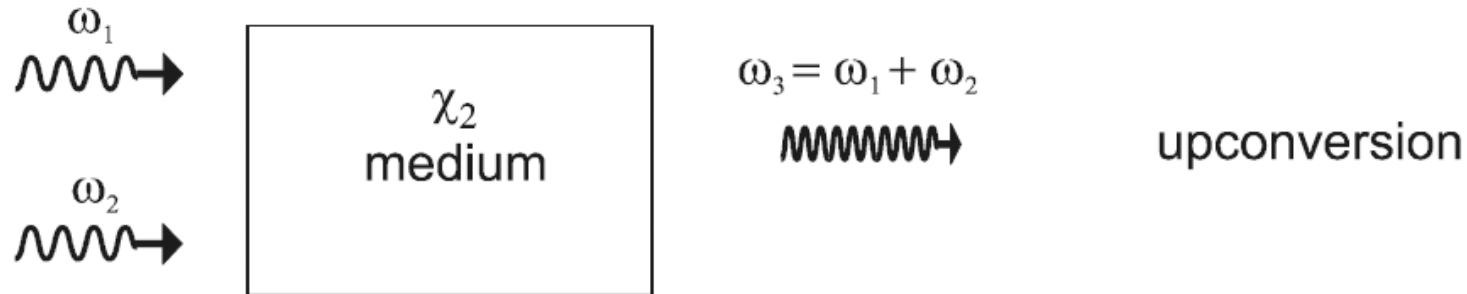


Figure 9-12 Two photons of frequencies ω_1 and ω_2 incident on a χ_2 medium can create a new photon at either frequency $\omega_3 = \omega_1 + \omega_2$ (upconversion) or $\omega_3 = \omega_1 - \omega_2$ (downconversion).

Three-Wave Mixing

$$\hbar\mathbf{k}_1 + \hbar\mathbf{k}_2 = \hbar\mathbf{k}_3 \quad (\text{three-photon phase matching})$$

$$k_i = n_i\omega_i/c \quad n_1\omega_1 + n_2\omega_2 = n_3\omega_3$$

If there is no dispersion ($n_1 = n_2 = n_3$),

$$\hbar\omega_1 + \hbar\omega_2 = \hbar\omega_3 \quad (\text{three-photon energy conservation})$$

where $\omega_1 = \omega_2$. In this case **energy conservation** becomes $2\hbar\omega_1 = \hbar\omega_3$, or simply $2\omega_1 = \omega_3$ (the **frequency is doubled**). Phase matching then reduces to $2n_1\omega_1 = n_3\omega_3$, or simply $n_1 = n_3$.

Three-Wave Mixing

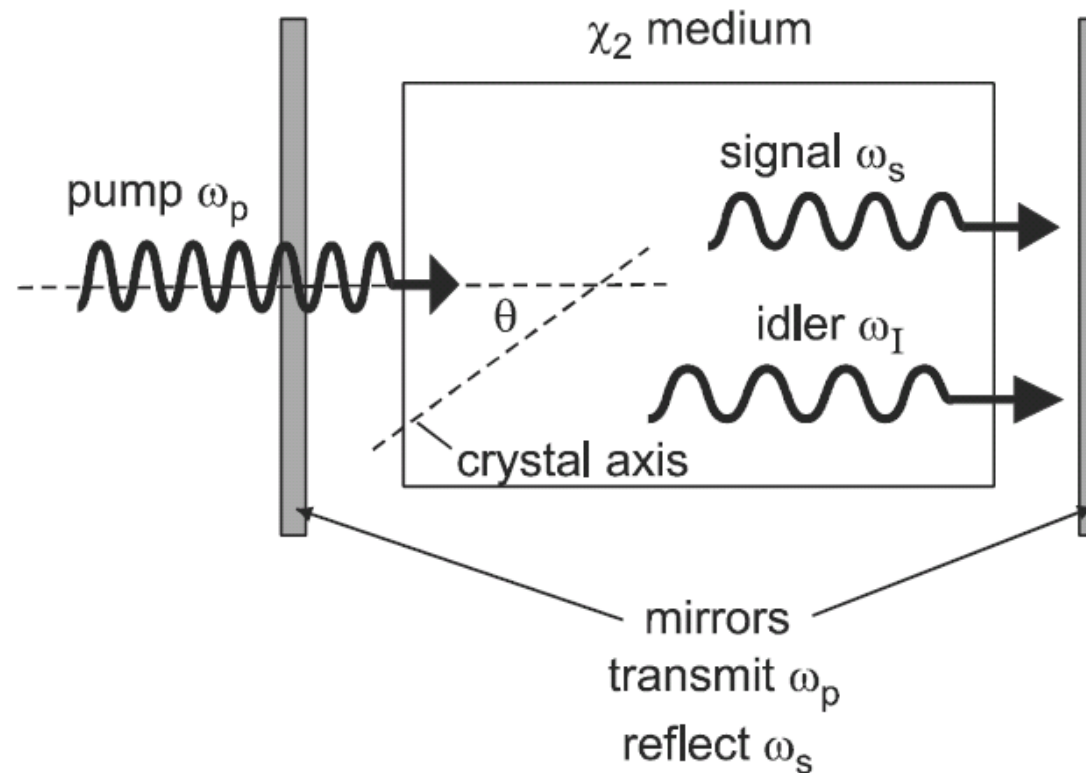


Figure 9-13 In an **optical parametric oscillator (OPO)**, pump photons are converted into lower frequency signal and idler photons by a nonlinear χ_2 medium. Phase matching enables lasing at a frequency ω_s , *which depends on the angle θ .*

Four-Wave Mixing

For **isotropic materials** such as glass, the lowest-order nonlinearity is due to χ_3 , and the time-dependent polarization becomes

$$P(t) = \varepsilon_0[\chi_1 E(t) + \chi_3 E^3(t)] \quad (\text{centrosymmetric medium})$$

$$E(t) = A \cos \omega t$$

$$P(t) = P_0 + P_\omega \cos \omega t + P_{2\omega} \cos 2\omega t + P_{3\omega} \cos 3\omega t$$

$$P_0 = 0$$

$$P_\omega = \varepsilon_0 A [\chi_1 + (3/4)\chi_3 A^2]$$

$$P_{2\omega} = 0$$

$$P_{3\omega} = \frac{1}{4} \varepsilon_0 \chi_3 A^3$$

This is **third harmonic generation** or **frequency tripling**, and can be used to produce even **shorter wavelengths** than SHG.

$$\omega_1, \omega_2, \omega_3, \quad \omega_1 + \omega_2 - \omega_3, \quad \omega_1 - \omega_2 + \omega_3$$

9-3. NONLINEAR REFRACTIVE INDEX

$$\chi'_1 = \chi_1 + \frac{3}{4}\chi_3 A^2 \quad (\text{effective } \chi_1, \text{ optical Kerr effect})$$

$$\Delta\chi_1 = \frac{3}{4}\chi_3 A^2$$

$$n = \sqrt{\epsilon_r} = \sqrt{\epsilon/\epsilon_0} = \sqrt{1 + \chi}$$

$$\Delta n = \frac{1}{2\sqrt{1 + \chi_1}} \Delta\chi_1 = \frac{\Delta\chi_1}{2n} = \frac{3}{8n}\chi_3 A^2$$

(energy density
in E field)

$$\rho = \frac{1}{2} \epsilon_r \epsilon_0 E^2$$

intensity

$$I = \frac{c}{n} \rho = \frac{1}{2} cn \epsilon_0 E^2$$

$$A^2 = \frac{2I}{cn\epsilon_0}$$

$$\Delta n = \frac{3\chi_3 I}{4n^2 c \epsilon_0}$$

$$n \equiv n_0 + n_2 I$$

$$n_2 = \frac{3\chi_3}{4n^2 c \epsilon_0} \quad (\text{nonlinear refractive index})$$

Optical Kerr effect

nonlinear
refractive index:

$$n \equiv n_0 + n_2 I$$

$$n_2 = \frac{3\chi_3}{4n^2 c \epsilon_0}$$

$$I = \frac{c}{n} \rho = \frac{1}{2} c n \epsilon_0 E^2$$

- If I is expressed in MKS units of W/m^2 ,
- then n_2 will be in units of m^2/W , so that $n_2 I$ is **dimensionless**.
- This **variation of refractive index with light intensity** is sometimes **referred to as the *optical Kerr effect***.

Table 9-2 Nonlinear refractive indices for selected crystals

Material	λ (nm)	n_2 (10^{-20} m ² /W)
Pure silica	1300	2.4
Ge-silica	1300	2.6
Water	500–1000	4
Lead silicate glass	1000	30–70
Ta ₂ O ₅	800	72
PPV polymer	880	80
As ₂ S ₃	1320	170
CS ₂	1000	310
GaAs	1000	3000

Optical Switching

Mach-Zehnder interferometer

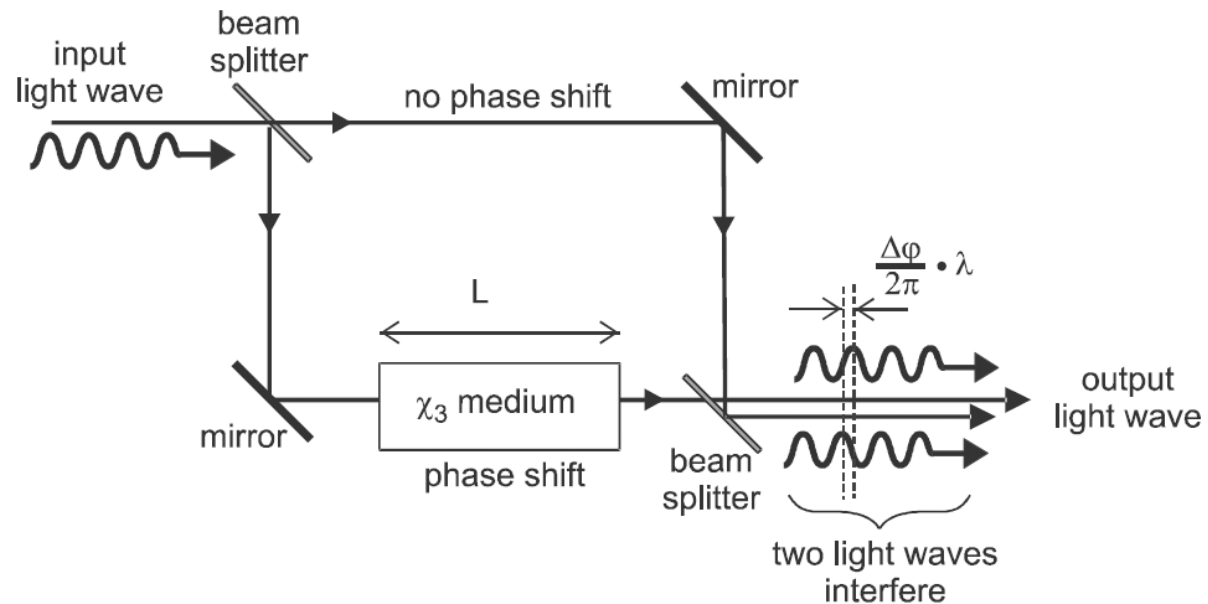
$$n \equiv n_0 + n_2 I$$

$$k = \frac{n\omega}{c} = \frac{n_0\omega}{c} + \frac{n_2 I \omega}{c} = k_0 + \Delta k$$

$$\Delta\phi = \Delta k L = \frac{n_2 I \omega}{c} L = \frac{2\pi n_2 I L}{\lambda_0}$$

where $\lambda_0 = 2\pi c/\omega$ is the free-space wavelength of the light

Self-phase modulation

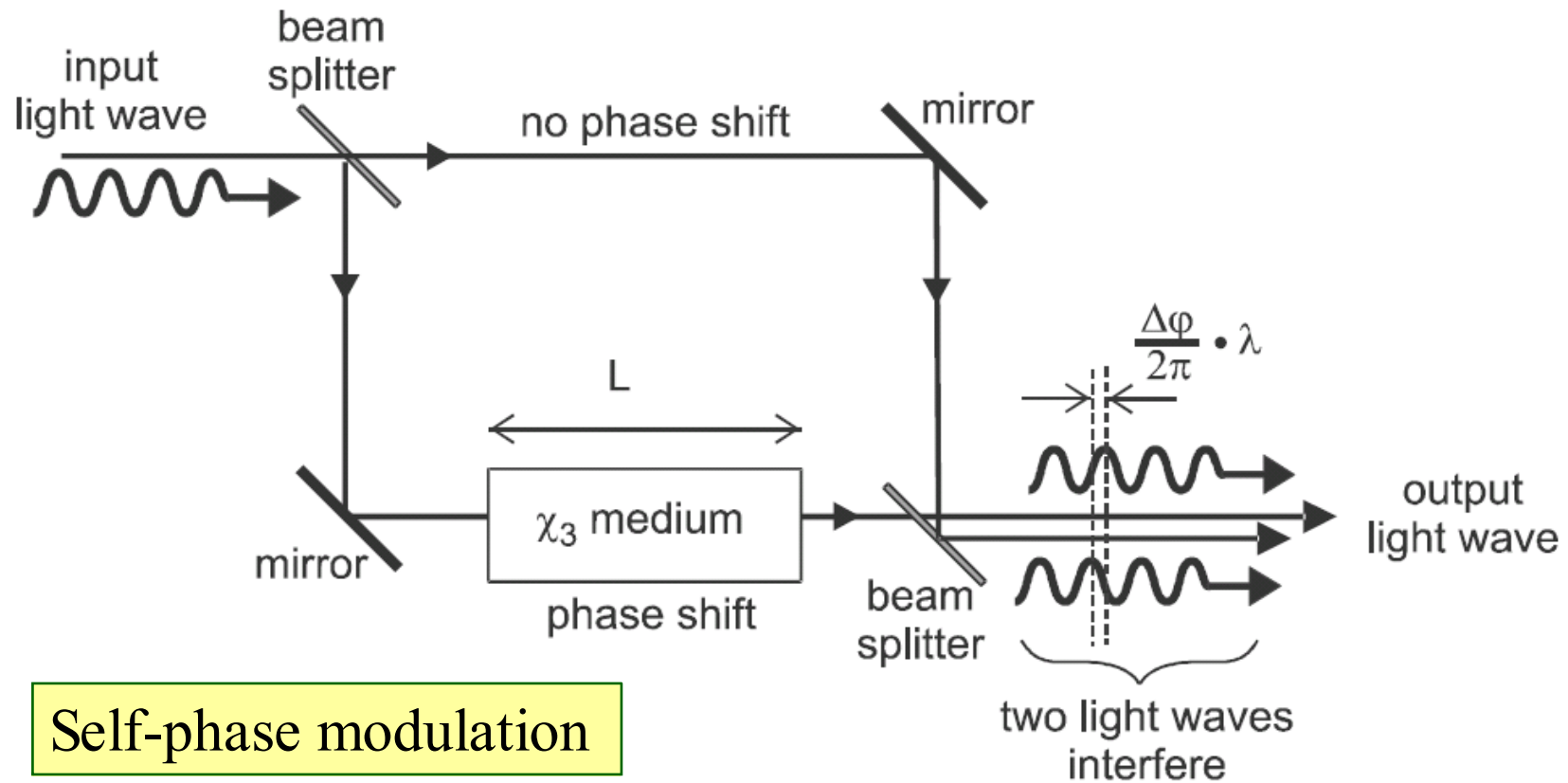


Optical Switching

Mach-Zehnder interferometer

$$I_{\pi} = \frac{\lambda_0}{2n_2L} \quad (\text{switching intensity})$$

$$\pi = \frac{n_2 I_{\pi} 2\pi L}{\lambda_0}$$



Self-phase modulation

Pulse Chirping and Temporal Solitons

$$E(z, t) = A \cos(\omega_0 t - kz) \quad k = (n_0 + n_2 I)k_0$$

$$E_1(t) = A \cos \omega_0 t$$

$$E_2(t) = A \cos [\phi(t)]$$

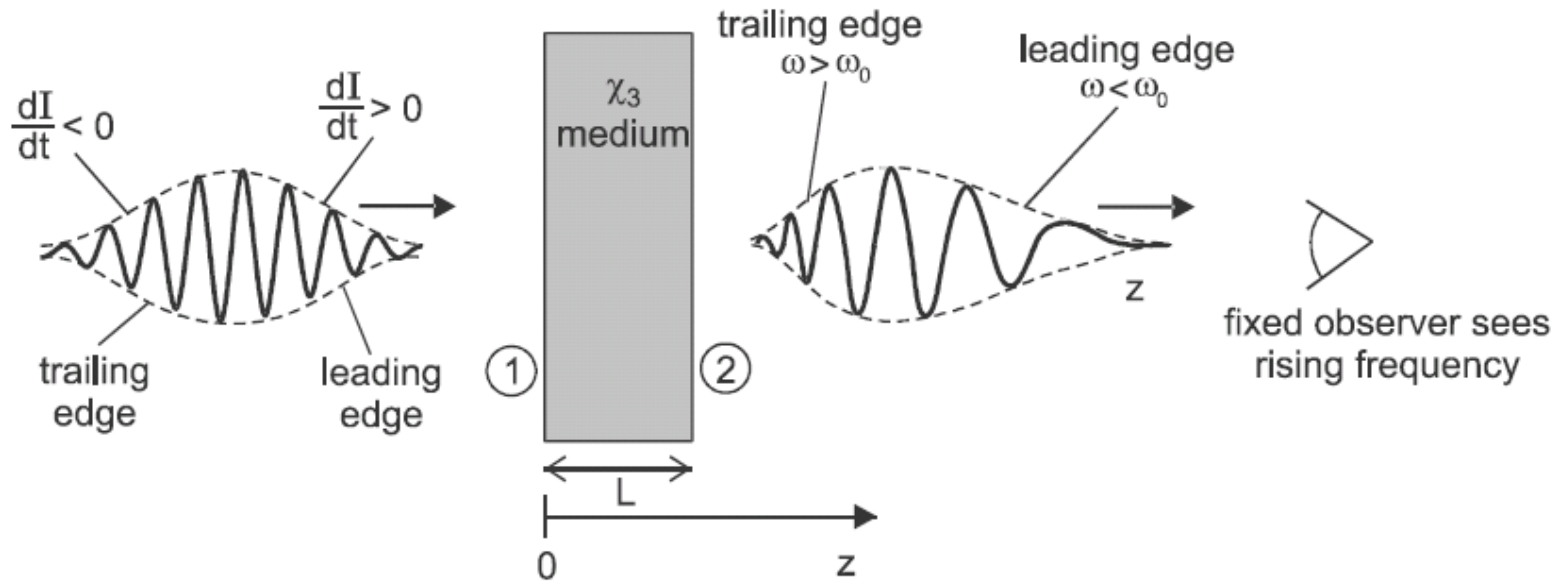
$$\phi(t) = \omega_0 t - [n_0 + n_2 I(t)]k_0 L$$

$$z = 0$$

$$z = L$$

$$\omega \equiv \frac{d\phi}{dt} = \omega_0 - n_2 k_0 L \frac{dI}{dt}$$

(instantaneous frequency)



Pulse Chirping and Temporal Solitons

➤ When the **leading edge** of the pulse enters the medium, $dI/dt > 0$, so that $\omega < \omega_0$:

“**red-shifted**” (shifted to lower frequency, or longer wavelength)

➤ When the **trailing edge** of the pulse is passing through the medium, however, $dI/dt < 0$, and in this case $\omega > \omega_0$.

“**blue-shifted**” (shifted to higher frequency, or shorter wavelength)

➤ The **center** of the pulse is **unshifted** in frequency, since $dI/dt = 0$ there.

Pulse Chirping and Temporal Solitons

- The result would be a measured **frequency** that **changes** in **time**, a phenomenon termed *frequency chirp*.
- *Since the frequency **increases** in time in this example, it is called a **positive chirp**.*
- *Positive chirp results from our assumption that $n_2 > 0$, which is usually the case.*
- ***Negative n_2** can occur at frequencies near a strong material resonance, in which case there would be **negative chirp** (frequency decreasing in time).*
- *The resulting **pulse preserves its shape and frequency distribution as it propagates, and is known as an **optical soliton**.***

9-4. ELECTROOPTIC EFFECTS

- The **refractive index** can vary **linearly** with field (*Pockels effect*)
- or **quadratically** with field (*electrooptic Kerr effect*).
- In general, the **refractive index** varies with applied **field** E_0 as:

$$n = n_0 + aE_0 + bE_0^2$$

$$a = \chi_2/n$$

$$b = 3\chi_3/(2n)$$

Pockels Effect

$$E_0 = V/d$$

$$E(t) = E_0 + A \cos \omega t$$

$$P = \epsilon_0 \chi_1 (E_0 + A \cos \omega t) + \epsilon_0 \chi_2 (E_0 + A \cos \omega t)^2$$

$$P_\omega = \epsilon_0 \chi_1 A + \epsilon_0 2\chi_2 E_0 A = \epsilon_0 \chi_1' A$$

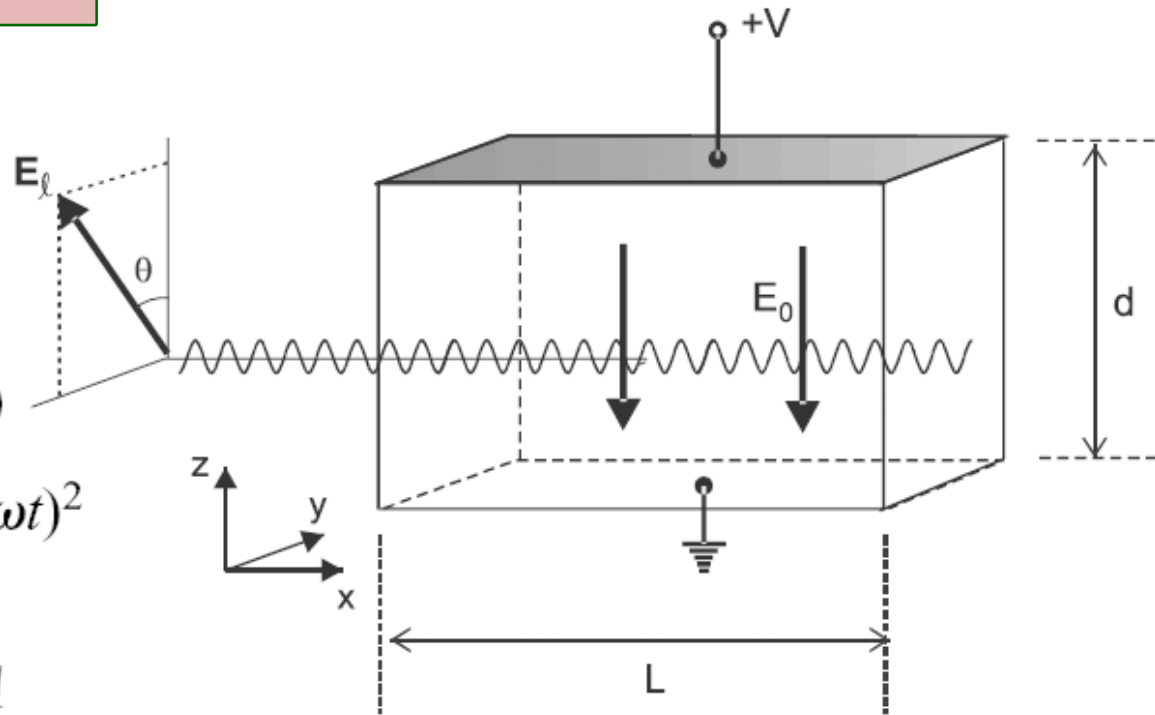


Figure 9-20 Light wave **polarized** in the **y-z** plane, propagating along the **x** axis. The **change** in **refractive index** is **greatest** for $\theta = 0$ and **smallest** for $\theta = 90^\circ$.

Pockels Effect

$$P = \varepsilon_0 \chi_1 (E_0 + A \cos \omega t) + \varepsilon_0 \chi_2 (E_0 + A \cos \omega t)^2$$

$$\begin{aligned} P_\omega &= \varepsilon_0 \chi_1 A + \varepsilon_0 2 \chi_2 E_0 A \\ &= \varepsilon_0 \chi'_1 A \end{aligned}$$

$$\chi'_1 = \chi_1 + 2 \chi_2 E_0 \quad (\text{effective } \chi_1, \text{ Pockels effect})$$

$$\Delta \chi_1 = 2 \chi_2 E_0 \quad \Delta n = \frac{1}{n} \chi_2 E_0 \quad a = \chi_2 / n$$

$$\Delta n \equiv \frac{1}{2} n^3 r E_0 \quad (\text{index change, Pockels effect})$$

r : Pockels coefficient.

$$r = (2/n^4) \chi_2$$

Mach-Zehnder interferometer

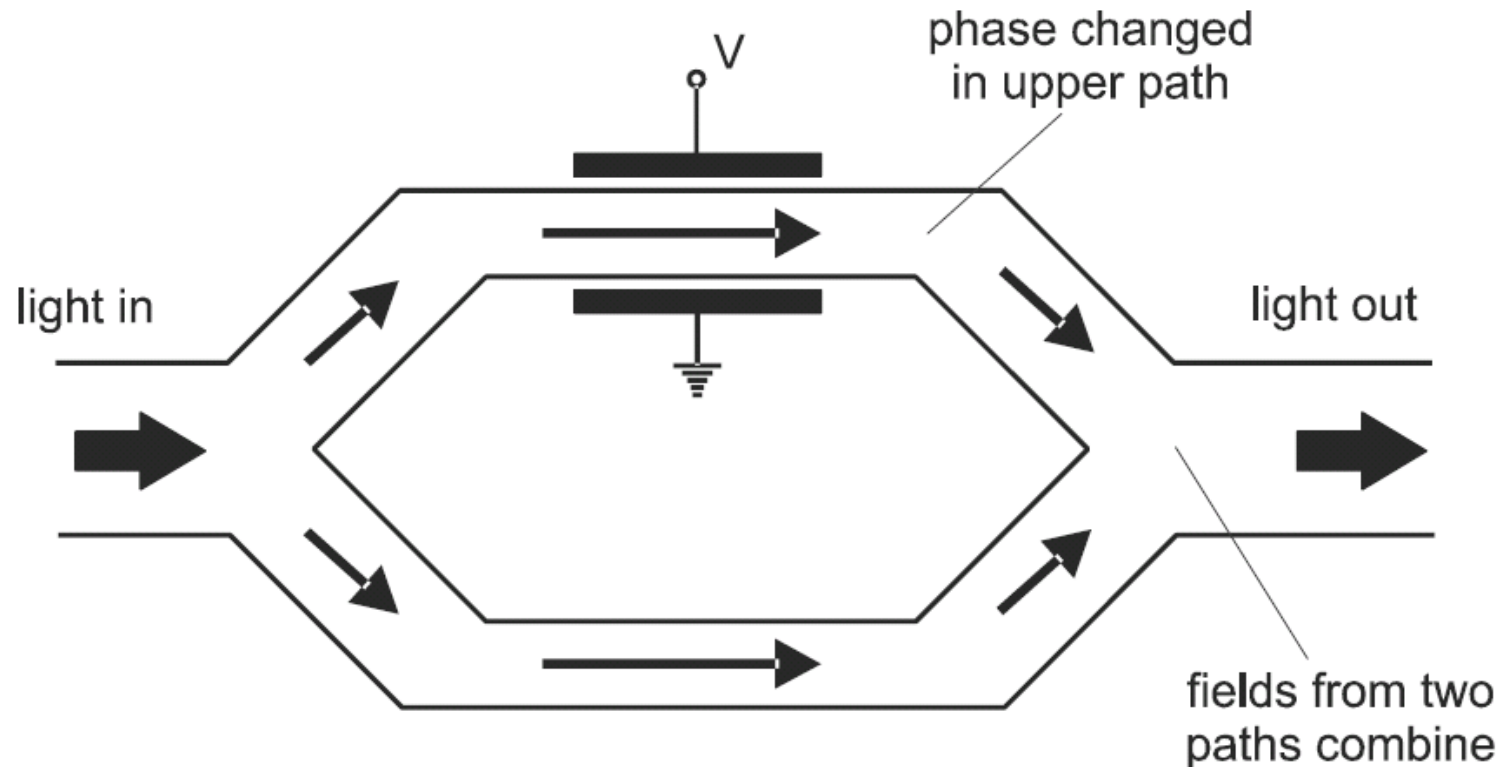


Figure 9-21 A **Mach-Zehnder interferometer** is formed from **lithium niobate** planar waveguides. A **phase shift** in one path **changes** the **amplitude** of the output wave, which enables **electrical modulation** of the light wave.

$$E_{1x} = (A/\sqrt{2}) \cos \omega t$$

$$E_{1y} = (A/\sqrt{2}) \cos \omega t$$

$$\Delta\phi = \frac{2\pi}{\lambda_0} \Delta n L$$

$$= \frac{2\pi}{\lambda_0} \left(n_o^3 r_{22} \frac{V}{d} \right) L$$

$$E_{2x} = (A/\sqrt{2}) \cos \omega t$$

$$E_{2y} = (A/\sqrt{2}) \cos (\omega t + \Delta\phi)$$

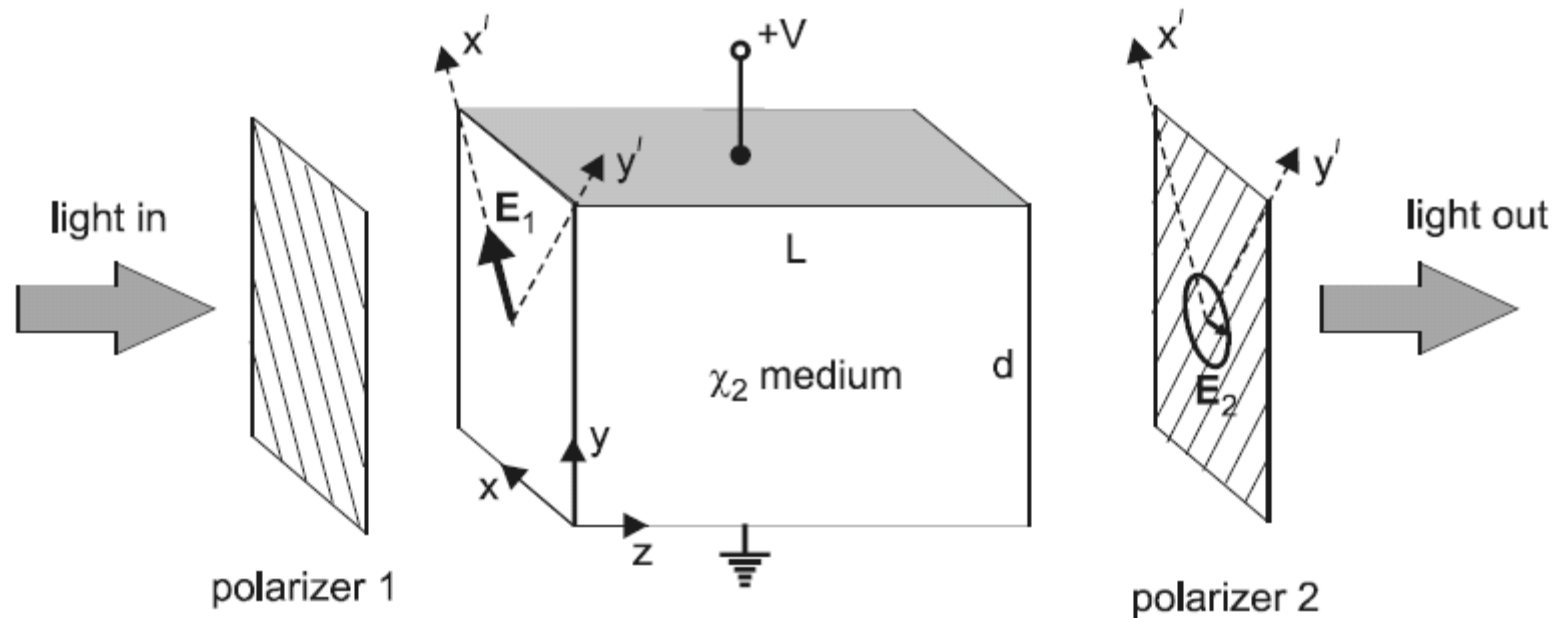


Figure 9-22 In a **Pockels cell**, the **applied voltage** changes the **light's polarization** from **linear** to **elliptical**. When it is placed between **crossed polarizers**, this can be used to **modulate** the transmitted **light intensity**.

Pockels Effect

$$\cos \alpha - \cos \beta = -2 \sin (\alpha + \beta) / 2 \sin (\alpha - \beta) / 2$$

$$\begin{aligned} E_{2y'} &= \frac{1}{\sqrt{2}} E_{2y} - \frac{1}{\sqrt{2}} E_{2x} \\ &= \frac{A}{2} [\cos (\omega t + \Delta \phi) - \cos \omega t] \end{aligned}$$

$$\begin{aligned} \Delta \phi &= \frac{2\pi}{\lambda_0} \Delta n L \\ &= \frac{2\pi}{\lambda_0} \left(n_o^3 r_{22} \frac{V}{d} \right) L \end{aligned}$$

$$= -A \sin \left(\omega t + \frac{\Delta \phi}{2} \right) \sin \left(\frac{\Delta \phi}{2} \right)$$

$$\frac{I_{\text{out}}}{I_{\text{in}}} = \left(\frac{A'}{A} \right)^2 = \sin^2 \left(\frac{\Delta \phi}{2} \right)$$

$$V_{\pi} = \frac{\lambda_0 d}{2 n_o^3 r_{22} L} \quad (\text{switching voltage, Pockels cell}) \quad \Delta \phi = \pi,$$

$$I_{\text{out}} = I_{\text{in}} \sin^2 \left(\frac{\pi}{2} \frac{V}{V_{\pi}} \right) \quad (\text{Pockels cell transmission})$$

Kerr Electro-optic Effect

- The **Pockels effect** is **absent** in **materials** having **inversion symmetry**, such as **liquids** and **glasses**, because $\chi_2 = 0$.
- In these materials, there is still an electrooptic effect, but it is a **higher-order process** that depends on χ_3 .

$$P_{\omega} = \epsilon_0 \chi_1' A$$

$$\chi_1' = \chi_1 + 3\chi_3 E_0^2$$

(effective χ_1 , Kerr electrooptic effect)

$$\Delta\chi_1 = 3\chi_3 E_0^2$$

$$\Delta n = \frac{3\chi_3}{2n} E_0^2$$

Kerr Electro-optic Effect

$$\Delta n = \frac{3\chi_3}{2n} E_0^2 \qquad b = 3\chi_3/(2n) \qquad \Delta n = bE_0^2$$

$$\Delta n = K\lambda_0 E_0^2 \qquad \boxed{K \text{ is the Kerr electrooptic coefficient}}$$

Table 9-3 Kerr electrooptic properties for some materials

Material	$b = K\lambda \ (10^{-20} \text{ m}^2/\text{V}^2)$
Nitrobenzene	206
Water	1.85
Glass	$10^{-2}-1$

