Chapter 3: Modulation of light

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3.1 Elliptical Polarization

\[ \varepsilon_x = i\varepsilon_0 \cos(kz - \omega t) \]
\[ \varepsilon_y = j\varepsilon_0 \sin(kz - \omega t) \]

\[ \varepsilon = \varepsilon_x + \varepsilon_y \]
\[ \varepsilon = \varepsilon_0 [i \cos(kz - \omega t) + j \sin(kz - \omega t)] \]

\[ \phi = \phi_2 - \phi_1 = \pi / 2 \]

A single wave in which the electric vector at a given point in space is constant in amplitude but rotates with angular frequency \( \omega \).

Fig. 3.1 Right circularly polarized light: (a) electric vectors at a given instant in time and (b) rotation of the vector at a given position in space.
Figure 6.1-1  (a) Rotation of the endpoint of the electric-field vector in the x-y plane at a fixed position z. (b) Snapshot of the trajectory of the endpoint of the electric-field vector at a fixed time t.

Figure 6.1-2  Linearly polarized light. (a) Time course at a fixed position z. (b) A snapshot (fixed time t).
Figure 6.1-3  Trajectories of the endpoint of the electric-field vector of a circularly polarized plane wave. (a) Time course at a fixed position $z$. (b) A snapshot (fixed time $t$). The sense of rotation in (a) is opposite that in (b) because the traveling wave depends on $t - z/c$. 
3.1 Elliptical Polarization

The resultant wave is said to be **elliptically polarized** and in fact **plane** and **circular polarization** are **special cases** of elliptical polarization.

\[
\begin{align*}
\varepsilon_x &= i\varepsilon_0 \cos(kz - \omega t) \\
\varepsilon_y &= j\varepsilon_0' \sin(kz - \omega t)
\end{align*}
\]

\[\varepsilon_0 \neq \varepsilon_0'\]

Fig. 3.1 **Right elliptically polarized light**: electric vectors at (a) a given instant in time and (b) a given position in space.
3.2 Birefringence (doubly refracting)

Many important crystalline optical materials such as calcite (CaCO₃), quartz (SiO₂) and KDP (Potassium dihydrogen phosphate, KH₂PO₄) are anisotropic.

\[ P = \varepsilon_0 \chi E \]

\[ P = a_1 E + \frac{1}{2} a_2 E^2 + \frac{1}{6} a_3 E^3 + \ldots \]

\[ a_1 = \varepsilon_0 \chi^{(1)} \]

\[ \chi = \frac{P_x}{\varepsilon_0 E_x} \quad \text{(electric susceptibility)} \]

\[ P = \varepsilon_0 \left( \chi^{(1)} \cdot E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots \right) = P^{(1)} + P^{(2)} + P^{(3)} + \ldots \]
3.2 Birefringence (doubly refracting)

\[ D = \varepsilon_0 E + P = \varepsilon_0 E + \varepsilon_0 \chi E = \varepsilon_0 (1 + \chi) E \]

\[
\begin{bmatrix}
D_x \\
D_y \\
D_z
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_0 (1 + \chi_x) & 0 & 0 \\
0 & \varepsilon_0 (1 + \chi_y) & 0 \\
0 & 0 & \varepsilon_0 (1 + \chi_z)
\end{bmatrix} \cdot 
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}
\]

\[ n_x = \sqrt{1 + \chi_x} \quad n_y = \sqrt{1 + \chi_y} \quad n_z = \sqrt{1 + \chi_z} \]
3.2 Birefringence

 временی نور غیر قطعیسه (یا با قطعیش وابسته به ساختار بلور)، بر یک بلور دویشکستی میتابد، نور منتشر شده از میان بلور را می توان ترکیبی از دو موج مستقل در نظر گرفت.

 پرتوهای معمولی و غیرمعمولی دارای جهات قطعیش دارای جهات قطعیش عمود بریکدیگر است.
3.2 Birefringence

The reflective index ellipsoid for a uniaxial crystal.

\[ X, y: n_0 \]
\[ Z: n_e \]
\[ OP: n_0 \]
\[ OQ: n_e(\theta) \]
3.2 Birefringence

Normal (index) surfaces for a **positive uniaxial** crystal projected on the xz
3.2.1 Phase plates

Quarter-wave, half-wave and whole-wave plate

- Phase plates
  - Tilt angle to blurs a mirror of a mirror path to create a path for light to pass through.
  - Tilt angle, which is a special case, is proportional to the axis of the mirror.
  - Phase plate is a systematic phase change with respect to the axis of the mirror.

- Phase plates
  - A phase change of a mirror path is created between E and O waves.

- As the phase change increases, the phase change between O and E waves increases.
- If the medium is a polarizer, the path of the mirror is perpendicular to the axis of the mirror.
- Only the phase change between O and E waves is observed.

- Quarter-wave, half-wave and whole-wave plate:
  - \( n_0 d - n_e d = \lambda_0 / 4 \)
3.3 Optical activity

- بلورهایی خاص (و نیز برخی از مایعات) میتوانند صفحه قطبش نور عبوری از داخل خود را دوران دهند. این گونه مواد دارای کنش نوری هستند.

- بلور کوارتز یکی از این نوع است.

- چرخش ایجاد شده، به ضخامت بلور و طول موج نور بستگی دارد.

- این بلورها دارای ضراپ قطبیت مختلف هستند.

- این بلورها دارای ضراپ شکست متفاوت برای نورهای مختلف قطبیده مختلط هستند.
3.4 Electro-optic effect

- when an electric field is applied across an optical medium, the distribution of electrons within it is distorted so that the polarizability and hence the refractive index of medium changes anisotropically.

- The result of this electro-optic effect may be to introduce new optic axes into naturally doubly refracting crystals, (KDP).

- Or to make naturally isotropic crystals doubly refracting (GaAS).

- The change in refractive index as a function of the applied field:

\[ \Delta(1/n^2) = r E + P E^2 \]

- \( r \): linear electro-optic coefficient
- \( P \): quadratic electro-optic coefficient

\( r.E: \) Pockels effect
\( P.E^2: \) Kerr effect
3.4.1 Pockels electro-optic modulator

If the electric field in KDP, the electric field is applied along the z-axis, the principal axes x, y, and z rotate 45 degrees and new waveplates form at 45 degrees. The electric field induces a phase shift:

\[
\Delta \left( \frac{1}{n^2} \right) = -2 \frac{\Delta n}{n^3} = r_{63} E_z
\]

\[
n'_x = n_0 + \frac{n_0^3}{2} r_{63} E_z
\]

\[
n'_y = n_0 - \frac{n_0^3}{2} r_{63} E_z
\]
3.4 Pockels electro-optic modulator

\[ \mathcal{E} = \mathcal{E}_0 \cos(\omega t - k z) \]

\[ \mathcal{E}_x' = \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t - k z) \]

\[ \mathcal{E}_y' = \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t - k z) \]

\[ \phi_x' = \frac{2\pi}{\lambda_0} n_x L \]

\[ \phi_y' = \frac{2\pi}{\lambda_0} n_y L \]

\[ \phi_x = \phi_0 + \Delta \phi \]

\[ \phi_y = \phi_0 - \Delta \phi \]

\[ \Delta \phi = \frac{\pi}{\lambda_0} L r n_0^3 \varepsilon_z = \frac{\pi}{\lambda_0} r n_0^3 V \]

\[ \Phi = \phi_x' - \phi_y' = 2\Delta \phi = \frac{2\pi}{\lambda_0} r n_0^3 V \]
3.4 Pockels electro-optic modulator

\[
\begin{align*}
\mathcal{E}_y &= \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t + \Delta \phi) \\
\mathcal{E}_{y'} &= \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t - \Delta \phi) \\
\mathcal{E} &= -\frac{\mathcal{E}_0}{2} [\cos(\omega t + \Delta \phi) - \cos(\omega t - \Delta \phi)]
\end{align*}
\]

\[\mathcal{E} = -\mathcal{E}_0 \sin \Delta \phi \sin \omega t\]
3.4 Pockels electro-optic modulator

\[ E = -E_0 \sin \Delta \phi \sin \omega t \]

\[ I = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} E^2 \, dt \]

\[ I = I_0 \sin^2 \Phi = I_0 \sin^2(\Phi/2) \]

\[ \frac{I}{I_0} = \sin^2 \left( \frac{\pi}{\lambda_0} \frac{r n_0^3 V}{2} \right) \]

\[ \frac{I}{I_0} = \sin^2 \left( \frac{\pi}{2} \frac{V}{V_\pi} \right) \]

\[ V_\pi = \frac{\lambda_0}{2r n_0^3} \]

Transmission (per cent)

<table>
<thead>
<tr>
<th>Applied voltage</th>
<th>Phase difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2\pi</td>
</tr>
<tr>
<td>0.5\pi</td>
<td>\pi</td>
</tr>
<tr>
<td>V_\pi/2</td>
<td>\pi</td>
</tr>
<tr>
<td>V_\pi/2</td>
<td>3\pi/2</td>
</tr>
<tr>
<td>2V_\pi/2</td>
<td>2\pi</td>
</tr>
</tbody>
</table>

\[ \sin^2(\pi/2) \frac{V}{V_\pi} \]
### 3.4 Pockels electro-optic modulator

**TABLE 3.1 Characteristics of some electro-optic materials used in Pockels cells**

<table>
<thead>
<tr>
<th>Material</th>
<th>Linear electro-optic coefficient, ( r ) (pm V(^{-1}))</th>
<th>( n_o )†</th>
<th>( n_e )†</th>
<th>Relative permittivity, ( \varepsilon_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{KH}_2\text{PO}_4 ) (KDP)</td>
<td>10.6</td>
<td>1.51</td>
<td>1.47</td>
<td>42</td>
</tr>
<tr>
<td>( \text{KD}_2\text{PO}_4 ) (KD*P)</td>
<td>26.4</td>
<td>1.51</td>
<td>1.47</td>
<td>50</td>
</tr>
<tr>
<td>( \text{AH}_2\text{PO}_4 ) (ADP)</td>
<td>8.5</td>
<td>1.52</td>
<td>1.48</td>
<td>12</td>
</tr>
<tr>
<td>Cadmium telluride (CdTe)</td>
<td>6.8</td>
<td>2.6</td>
<td></td>
<td>7.3</td>
</tr>
<tr>
<td>Lithium tantalate (LiTaO(_3))</td>
<td>30.3</td>
<td>2.175</td>
<td>2.180</td>
<td>43</td>
</tr>
<tr>
<td>Lithium niobate (LiNbO(_3))</td>
<td>30.8</td>
<td>2.29</td>
<td>2.20</td>
<td>18</td>
</tr>
<tr>
<td>Gallium arsenide (GaAs)</td>
<td>1.6</td>
<td>3.6</td>
<td></td>
<td>11.5</td>
</tr>
<tr>
<td>Zinc sulfide (ZnS)</td>
<td>2.1</td>
<td>2.32</td>
<td>1.56</td>
<td>7.4</td>
</tr>
<tr>
<td>Barium borate (BaB(_2)O(_4))</td>
<td>1.67</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† Values near a wavelength of 550 nm.
‡ Low frequency values.
Pockels electro-optic cell biased with a quarter-wave plate

\[
\Phi = \frac{\pi}{2} + 2\Delta \phi = \frac{\pi}{2} + \pi \frac{V}{V_\pi}
\]

\[
\frac{I}{I_0} = \sin^2 \left( \frac{\pi}{4} + \frac{\pi}{2} \frac{V}{V_\pi} \right) = \frac{1}{2} \left( 1 + \sin \frac{\pi V}{V_\pi} \right)
\]

\[
\sin \left( \frac{\pi V}{V_\pi} \right) = \frac{\pi V}{V_\pi}
\]
Packels electro-optic cell biased with a quarter-wave plate

\[
\frac{I}{I_0} = 0.5 + \frac{\pi V(t)}{2} \sin 2\pi f t
\]

(b) Longitudinal effect device
3.5 Kerr modulators

\[ \Delta(1/n^2) = r^2 \varepsilon + P^2 \varepsilon \]

\[ \Delta n = n_p - n_s = K\lambda_0 E^2 \]

**TABLE 3.2 Typical values of the Kerr constant**

<table>
<thead>
<tr>
<th>Material</th>
<th>( K \times 10^{-14} \text{ m V}^{-2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>5.2</td>
</tr>
<tr>
<td>Nitrobenzene</td>
<td>244</td>
</tr>
<tr>
<td>Nitrotoluene</td>
<td>137</td>
</tr>
<tr>
<td>Glasses - various</td>
<td>0.03 - 0.17</td>
</tr>
</tbody>
</table>

**Notes:**
- تغيير ضريب شکست متناسب با مجذور میدان الکتریکی
- اعمال شده است.
- اختلاف ضريب شکست در مواد برای نور نقطیه موایی و عمود بر محور نوری القایی، از رابطه مقابل بدست می‌آید. 
- که، ضريب ثابت کر نامیده می‌شود.
- بين اعمال میدان الکتریکی و پیدايش اثر کر، تأخیر وجود دارد که این تأخیر در جامدات ممکن است حتي چند ثانيه باشد.
- ليكن برای مایعات غير نقطي، تأخير بسيار کمتر و گاه حتي کمتر از 10^{-11} است.
Nonlinear Optics

- **A linear interaction:** The presence of wave 2 has no effect on wave 1, and vice versa.

- **A nonlinear interaction:** Each wave can modify the properties of the other, changing, for example, the other wave’s amplitude or phase.

- **Self-action:** One of the waves can also modify its own amplitude or phase.
Electron Cloud Distortion

Figure 9-2
(a) An electric field distorts the electron cloud in an atom, creating an electric dipole moment $p$.

(b) The oscillating dipoles driven by the light wave’s $E$ field radiate additional waves that interfere with the original wave, modifying its propagation speed.
Electron Cloud Distortion

\[ v_p = \frac{c}{n} \]

\[ n = \sqrt{\varepsilon_r} = \sqrt{\varepsilon/\varepsilon_0} = \sqrt{1 + \chi} \]

\[ \varepsilon: \text{permittivity of the medium} \]

\[ \varepsilon_0: \text{permittivity of free space} \]

\[ \varepsilon_r: \text{relative permittivity or dielectric constant} \]

\( \text{(actually not a constant, but a function of frequency)} \)

\[ \chi: \text{electric susceptibility} \]

\[ \chi \equiv \frac{P_x}{\varepsilon_0 E_x} \quad \text{(electric susceptibility)} \]

\[ P_x = \varepsilon_0 \left( \chi_1 E_x + \chi_2 E_x^2 + \chi_3 E_x^3 \right) \]
Nonlinear Optics

\[ P_x = \varepsilon_0 (\chi_1 E_x + \chi_2 E_x^2 + \chi_3 E_x^3) \]

\( \chi_1 \): linear susceptibility, \( \chi_2 (\chi_3) \): second (third)-order susceptibilities

**Centrosymmetric materials**: (those with inversion symmetry), \( \chi_2 = 0 \).
Other Nonlinear Mechanisms

1. Molecular Orientation

Figure 9-4 A **strong electric field** can orient asymmetrical molecules in a liquid, producing a **nonlinear response**.
Other Nonlinear Mechanisms

2. Electrostriction

The potential energy of a dipole moment $p$ in a field $E$ is given by $U = -p \cdot E$

Figure 9-5 (a) A dipole’s energy is lower when it is in a larger $E$ field. (b) Atoms tend to move to the center of an optical beam, where the $E$ field is highest, a phenomenon known as electrostriction.
A different type of nonlinearity can occur when there is resonance between $hv$ and $\Delta E$.

(a) At low intensity, most atoms are in the ground state $1$ and are available for absorbing a photon.

(b) At high intensity, a significant fraction of atoms (50% in this example) are in the excited state $2$ and are unavailable for absorbing a photon. This is optical bleaching, an intensity-dependent absorption.
Other Nonlinear Mechanisms

3. Resonant Absorption

Figure 9-7
(a) In a stepwise absorption process, an atom is first raised to level 2 by absorbing a photon, and then raised to level 3 by absorbing a second photon.

(b) In two-photon absorption, the two photons are absorbed simultaneously, without excitation of any real intermediate level. Both of these processes can be used to increase the absorption at high intensity.
Another type of **nonlinear mechanism** arises from the Raman and Brillouin scattering processes.

Figure 5-5 *Brillouin scattering* from acoustic waves.

Figure 5-6 *Molecular vibrations* involved in Raman scattering can be modelled by masses and springs.
Figure 9-8 Heating of a material by an intense optical beam lowers the refractive index near the beam center. This creates a thermal lens that causes a collimated beam to diverge.
9-2. FREQUENCY CONVERSION

SHG: Second Harmonic Generation or frequency doubling

\[ E(t) = A \cos \omega t \]

\[ P(t) = \varepsilon_0 \chi_1 A \cos \omega t + \varepsilon_0 \chi_2 A^2 \cos^2 \omega t \]

\[ P(t) = P_0 + P_\omega \cos \omega t + P_{2\omega} \cos 2\omega t \]

- The induced polarization varies with time in three distinct ways.
- \( P_0 \): optical rectification, in which a static (dc) polarization is produced in response to the rapidly varying electric field of the light wave.
- \( P_\omega \cos \omega t \), causes the atoms to radiate light at frequency \( \omega \), and this results in the linear refractive index.
Table 9-1 Second-order nonlinear susceptibility $\chi_2$ for selected crystals

<table>
<thead>
<tr>
<th>Material</th>
<th>Transparency range (μm)</th>
<th>Linear index$^a$ at $\omega$ $n_o/n_e$</th>
<th>Linear index$^a$ at $2\omega$ $n_o/n_e$</th>
<th>$\chi_2$ ($10^{-12}$ m/V)$^{b,c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KDP</td>
<td>0.18–1.45</td>
<td>1.495/1.460</td>
<td>1.512/1.471</td>
<td>0.86</td>
</tr>
<tr>
<td>LiNbO$_3$</td>
<td>0.4–5.5</td>
<td>2.234/2.155</td>
<td>2.325/2.233</td>
<td>12</td>
</tr>
<tr>
<td>AgGaS$_2$</td>
<td>0.5–13</td>
<td>2.316/2.347</td>
<td>2.383/2.341</td>
<td>40</td>
</tr>
<tr>
<td>CdGeAs$_2$</td>
<td>2.4–18</td>
<td>3.505/3.591</td>
<td>3.530/3.621</td>
<td>470</td>
</tr>
</tbody>
</table>

$^a$ Fundamental at 1.064 μm for KDP and LiNbO$_3$, 10.6 μm for AgGaS$_2$ and CdGeAs$_2$.

$^b$ For 1.064 →0.532 μm SHG (KDP, LiNbO$_3$) and 10 →5 μm SHG (AgGaS$_2$, CdGeAs$_2$).

$^c$ The parameter $d = \chi_2/2$ is an often used alternative definition.
If \( n_{2\omega} = n_{\omega} \), \( k_{2\omega} = 2k_{\omega} \) and \( \lambda_{2\omega} = \lambda_{\omega}/2 \).
**Figure 9-9**

(a) The upper wave is the fundamental and the lower waves are the second harmonic waves radiated by different atoms in the material. Constructive interference occurs when the refractive index is the same at the two frequencies.

(b) When the two indices are different, the waves get out of phase and add by destructive interference.
The optimum SHG conversion is obtained by passing the fundamental wave through a crystal of length equal to \( L_c \).

\[
L_c(k_{2\omega} - 2k_\omega) \equiv L_c\Delta k = \pi
\]

\[
\Delta k \equiv k_{2\omega} - 2k_\omega
\]

\[
\Delta k = (n_{2\omega} - n_\omega) \frac{2\omega}{c} \quad \text{(wavevector mismatch)}
\]
EXAMPLE 9-1

Light of free-space wavelength 1064 nm from a Nd:YAG laser is passed through a lithium niobate (LiNbO₃) crystal for frequency doubling to 532 nm. The index of refraction at 1064 nm is 2.234, and at 532 nm is 2.325. Determine the optimum crystal length for SHG.

Solution: The optimum length is

\[ L_c = \frac{\pi}{\Delta k} = \frac{\pi c}{2\omega \Delta n} = \frac{\lambda_0}{4\Delta n} \]

where \( \lambda_0 \) is the free space wavelength of the fundamental wave and \( \Delta n \equiv n_{2\omega} - n_\omega \). This evaluates to

\[ L_c = \frac{1064 \times 10^{-9} \text{ m}}{4(2.325 - 2.234)} = 2.92 \mu \text{m} \]
Three-Wave Mixing

Consider two light waves with frequencies $\omega_1$ and $\omega_2$ that add together at a given point in a material to give a total electric field:

$$E(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

Assume: $\chi_2 \neq 0$

$$\frac{1}{\varepsilon_0} P(t) = \chi_1[A_1 \cos \omega_1 t + A_2 \cos \omega_2 t] + \chi_2[A_1^2 \cos^2 \omega_1 t + A_2^2 \cos^2 \omega_2 t + 2A_1A_2(\cos \omega_1 t)(\cos \omega_2 t)]$$

$$2(\cos \omega_1 t)(\cos \omega_2 t) = \cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t$$

$$\omega_3 = \omega_1 \pm \omega_2$$

**Upconversion:** $\omega_3 = \omega_1 + \omega_2$ (the frequency is **shifted upward**)

**Downconversion:** $\omega_3 = \omega_1 - \omega_2$ (the frequency is shifted downward)
Three-Wave Mixing

Two photons of frequencies $\omega_1$ and $\omega_2$ incident on a $\chi_2$ medium can create a new photon at either frequency $\omega_3 = \omega_1 + \omega_2$ (upconversion) or $\omega_3 = \omega_1 - \omega_2$ (downconversion).

Figure 9-12 Two photons of frequencies $\omega_1$ and $\omega_2$ incident on a $\chi_2$ medium can create a new photon at either frequency $\omega_3 = \omega_1 + \omega_2$ (upconversion) or $\omega_3 = \omega_1 - \omega_2$ (downconversion).
Three-Wave Mixing

\[ \hbar k_1 + \hbar k_2 = \hbar k_3 \] (three-photon phase matching)

\[ k_i = n_i \omega_i / c \quad n_1 \omega_1 + n_2 \omega_2 = n_3 \omega_3 \]

If there is no dispersion \((n_1 = n_2 = n_3)\),

\[ \hbar \omega_1 + \hbar \omega_2 = \hbar \omega_3 \] (three-photon energy conservation)

where \(\omega_1 = \omega_2\). In this case energy conservation becomes \(2\hbar \omega_1 = \hbar \omega_3\), or simply \(2\omega_1 = \omega_3\) (the frequency is doubled). Phase matching then reduces to \(2n_1 \omega_1 = n_3 \omega_3\), or simply \(n_1 = n_3\).
Three-Wave Mixing

Figure 9-13 In an optical parametric oscillator (OPO), pump photons are converted into lower frequency signal and idler photons by a nonlinear $\chi_2$ medium. Phase matching enables lasing at a frequency $\omega_S$, which depends on the angle $\theta$. 
Four-Wave Mixing

For isotropic materials such as glass, the lowest-order nonlinearity is due to $\chi_3$, and the time-dependent polarization becomes

$$P(t) = \varepsilon_0[\chi_1 E(t) + \chi_3 E^3(t)]$$

(centrosymmetric medium)

$$E(t) = A \cos \omega t$$

$$P(t) = P_0 + P_\omega \cos \omega t + P_{2\omega} \cos 2\omega t + P_{3\omega} \cos 3\omega t$$

$$P_0 = 0$$

$$P_\omega = \varepsilon_0 A [\chi_1 + (3/4)\chi_3 A^2]$$

$$P_{2\omega} = 0$$

$$P_{3\omega} = \frac{1}{4} \varepsilon_0 \chi_3 A^3$$

This is third harmonic generation or frequency tripling, and can be used to produce even shorter wavelengths than SHG.

$$\omega_1, \omega_2, \omega_3, \omega_1 + \omega_2 - \omega_3, \omega_1 - \omega_2 + \omega_3$$
9-3. NONLINEAR REFRACTIVE INDEX

\[ \chi_1' = \chi_1 + \frac{3}{4} \chi_3 A^2 \quad \text{(effective } \chi_1, \text{ optical Kerr effect)} \]

\[ \Delta n = \frac{1}{2 \sqrt{1 + \chi_1}} \quad \Delta \chi_1 = \frac{\Delta \chi_1}{2n} = \frac{3}{8n} \chi_3 A^2 \]

\[ n = \sqrt{\varepsilon_r} = \sqrt{\varepsilon/\varepsilon_0} = \sqrt{1 + \chi} \]

\[ \rho = \frac{1}{2} \varepsilon_r \varepsilon_0 E^2 \]

\[ I = \frac{c}{n} \rho = \frac{1}{2 c n \varepsilon_0 E^2} \]

\[ A^2 = \frac{2I}{cn \varepsilon_0} \]

\[ \Delta n = \frac{3 \chi_3 I}{4n^2 c \varepsilon_0} \]

\[ n_0 + n_2 I \]

\[ n_2 = \frac{3 \chi_3}{4n^2 c \varepsilon_0} \quad \text{(nonlinear refractive index)} \]
nonlinear refractive index:

\[ n = n_0 + n_2 I \]

\[ n_2 = \frac{3 \chi_3}{4 n^2 c \varepsilon_0} \]

If \( I \) is expressed in MKS units of \( \text{W/m}^2 \),

then \( n_2 \) will be in units of \( \text{m}^2/\text{W} \), so that \( n_2 I \) is dimensionless.

This variation of refractive index with light intensity is sometimes referred to as the **optical Kerr effect**.
### Table 9-2  Nonlinear refractive indices for selected crystals

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda$ (nm)</th>
<th>$n_2$ (10$^{-20}$ m$^2$/W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure silica</td>
<td>1300</td>
<td>2.4</td>
</tr>
<tr>
<td>Ge–silica</td>
<td>1300</td>
<td>2.6</td>
</tr>
<tr>
<td>Water</td>
<td>500–1000</td>
<td>4</td>
</tr>
<tr>
<td>Lead silicate glass</td>
<td>1000</td>
<td>30–70</td>
</tr>
<tr>
<td>Ta$_2$O$_5$</td>
<td>800</td>
<td>72</td>
</tr>
<tr>
<td>PPV polymer</td>
<td>880</td>
<td>80</td>
</tr>
<tr>
<td>As$_2$S$_3$</td>
<td>1320</td>
<td>170</td>
</tr>
<tr>
<td>CS$_2$</td>
<td>1000</td>
<td>310</td>
</tr>
<tr>
<td>GaAs</td>
<td>1000</td>
<td>3000</td>
</tr>
</tbody>
</table>
Optical Switching

\[ n \equiv n_0 + n_2 I \]

Mach–Zehnder interferometer

\[ k = \frac{n \omega}{c} = \frac{n_0 \omega}{c} + \frac{n_2 I \omega}{c} = k_0 + \Delta k \]

Self-phase modulation

\[ \Delta \phi = \Delta k L = \frac{n_2 I \omega}{c} L = \frac{2\pi n_2 I L}{\lambda_0} \]

where \( \lambda_0 = 2\pi c/\omega \) is the free-space wavelength of the light
Optical Switching

$$I_\pi = \frac{\lambda_0}{2n_2L}$$  (switching intensity)

$$\pi = \frac{n_2I_\pi^2 \pi L}{\lambda_0}$$

Mach–Zehnder interferometer

Self-phase modulation

Input light wave

Beam splitter

No phase shift

Mirror

Mirror

$\chi_3$ medium

Phase shift

Beam splitter

$\frac{\Delta \phi}{2\pi} \cdot \lambda$

L

two light waves interfere

Output light wave
Pulse Chirping and Temporal Solitons

\[ E(z, t) = A \cos(\omega_0 t - kz) \quad k = (n_0 + n_2 I)k_0 \]

\[ E_1(t) = A \cos \omega_0 t \quad z = 0 \]

\[ E_2(t) = A \cos [\phi(t)] \quad \phi(t) = \omega_0 t - [n_0 + n_2 I(t)]k_0 L \]

\[ \omega \equiv \frac{d\phi}{dt} = \omega_0 - n_2 k_0 L \frac{dI}{dt} \]

\[ \chi_3 \text{ medium} \]

fixed observer sees rising frequency

leading edge \( \omega < \omega_0 \)

trailing edge \( \omega > \omega_0 \)
When the **leading edge** of the pulse enters the medium, $\frac{dI}{dt} > 0$, so that $\omega < \omega_0$.

“**red-shifted**” (shifted to lower frequency, or longer wavelength)

When the **trailing edge** of the pulse is passing through the medium, however, $\frac{dI}{dt} < 0$, and in this case $\omega > \omega_0$.

“**blue-shifted**” (shifted to higher frequency, or shorter wavelength)

The **center** of the pulse is **unshifted** in frequency, since $\frac{dI}{dt} = 0$ there.
Pulse Chirping and Temporal Solitons

- The result would be a measured frequency that changes in time, a phenomenon termed frequency chirp.

- Since the frequency increases in time in this example, it is called a positive chirp.

- Positive chirp results from our assumption that \( n_2 > 0 \), which is usually the case.

- Negative \( n_2 \) can occur at frequencies near a strong material resonance, in which case there would be negative chirp (frequency decreasing in time).

- The resulting pulse preserves its shape and frequency distribution as it propagates, and is known as an optical soliton.
9-4. ELECTROOPTIC EFFECTS

➢ The refractive index can vary linearly with field (Pockels effect).

➢ or quadratically with field (electrooptic Kerr effect).

➢ In general, the refractive index varies with applied field $E_0$ as:

$$n = n_0 + aE_0 + bE_0^2$$

$$a = \chi_2/n \quad b = 3\chi_3/(2n)$$
Pockels Effect

\[ E_0 = \frac{V}{d} \]

\[ E(t) = E_0 + A \cos \omega t \]

\[ P = \varepsilon_0 \chi_1(E_0 + A \cos \omega t) + \varepsilon_0 \chi_2(E_0 + A \cos \omega t)^2 \]

\[ P_\omega = \varepsilon_0 \chi_1 A + \varepsilon_0 2 \chi_2 E_0 A = \varepsilon_0 \chi'_1 A \]

**Figure 9-20** Light wave polarized in the y–z plane, propagating along the x axis. The change in refractive index is greatest for \( \theta = 0 \) and smallest for \( \theta = 90^\circ \).
Pockels Effect

\[ P = \varepsilon_0 \chi_1 (E_0 + A \cos \omega t) + \varepsilon_0 \chi_2 (E_0 + A \cos \omega t)^2 \]

\[ P_\omega = \varepsilon_0 \chi_1 A + \varepsilon_0 2 \chi_2 E_0 A \]

\[ = \varepsilon_0 \chi'_1 A \]

\[ \chi'_1 = \chi_1 + 2 \chi_2 E_0 \quad \text{(effective } \chi_1, \text{ Pockels effect)} \]

\[ \Delta \chi_1 = 2 \chi_2 E_0 \quad \Delta n = \frac{1}{n} \chi_2 E_0 \quad a = \chi_2 / n \]

\[ \Delta n \equiv \frac{1}{2} n^3 r E_0 \quad \text{(index change, Pockels effect)} \]

\[ r \text{ : Pockels coefficient.} \quad r = (2/n^4) \chi_2 \]
A Mach–Zehnder interferometer is formed from lithium niobate planar waveguides. A phase shift in one path changes the amplitude of the output wave, which enables electrical modulation of the light wave.
In a Pockels cell, the applied voltage changes the light’s polarization from linear to elliptical. When it is placed between crossed polarizers, this can be used to modulate the transmitted light intensity.

\[ E_{1x} = \left( \frac{A}{\sqrt{2}} \right) \cos \omega t \]
\[ E_{1y} = \left( \frac{A}{\sqrt{2}} \right) \cos \omega t \]

\[ \Delta \phi = \frac{2\pi}{\lambda_0} \frac{\Delta n L}{n_0^2 r_{22} \frac{V}{d}} \]

\[ E_{2x} = \left( \frac{A}{\sqrt{2}} \right) \cos \omega t \]
\[ E_{2y} = \left( \frac{A}{\sqrt{2}} \right) \cos (\omega t + \Delta \phi) \]
Pockels Effect

\[ E_{2y'} = \frac{1}{\sqrt{2}} E_{2y} - \frac{1}{\sqrt{2}} E_{2x} \]

\[ = \frac{A}{2} \left[ \cos (\omega t + \Delta \phi) - \cos \omega t \right] \]

\[ = -A \sin \left( \omega t + \frac{\Delta \phi}{2} \right) \sin \left( \frac{\Delta \phi}{2} \right) \]

\[ \Delta \phi = \frac{2 \pi}{\lambda_0} \Delta n L \]

\[ = \frac{2 \pi}{\lambda_0} \left( n_o^3 r_{22} \frac{V}{d} \right) L \]

\[ \frac{I_{\text{out}}}{I_{\text{in}}} = \left( \frac{A'}{A} \right)^2 = \sin^2 \left( \frac{\Delta \phi}{2} \right) \]

\[ V_\pi = \frac{\lambda_0 d}{2 n_o^3 r_{22} L} \] (switching voltage, Pockels cell)

\[ \Delta \phi = \pi \]

\[ I_{\text{out}} = I_{\text{in}} \sin^2 \left( \frac{\pi}{2} \frac{V}{V_\pi} \right) \] (Pockels cell transmission)
The **Pockels effect** is **absent** in **materials** having **inversion symmetry**, such as **liquids** and **glasses**, because $\chi_2 = 0$.

In these materials, there is still an electrooptic effect, but it is a **higher-order process** that depends on $\chi_3$.

\[ P_\omega = \varepsilon_0 \chi'_1 A \]

\[ \chi'_1 = \chi_1 + 3\chi_3 E_0^2 \]

(Effective $\chi_1$, Kerr electrooptic effect)

\[ \Delta \chi_1 = 3\chi_3 E_0^2 \]

\[ \Delta n = \frac{3\chi_3}{2n} E_0^2 \]
Kerr Electro-optic Effect

$$\Delta n = \frac{3\chi_3}{2n} E_0^2$$

$$b = \frac{3\chi_3}{(2n)}$$

$$\Delta n = bE_0^2$$

$$\Delta n = K\lambda_0 E_0^2$$

*K is the Kerr electrooptic coefficient*

**Table 9-3**  Kerr electrooptic properties for some materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$b = K\lambda$  ($10^{-20}$ m$^2$/V$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrobenzene</td>
<td>206</td>
</tr>
<tr>
<td>Water</td>
<td>1.85</td>
</tr>
<tr>
<td>Glass</td>
<td>$10^{-2} - 1$</td>
</tr>
</tbody>
</table>
Lecture 3: Modulation of light