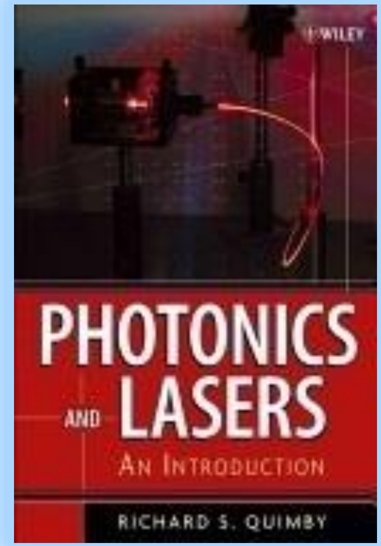
Photonic Crystal Optics

➢Reference:

Photonics and Lasers An Introduction - Chapter 8

Author: **Richard S. Quimby**



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Reference: **Chapter 8**- **Photonics and Lasers An Introduction**

Author: **Richard S. Quimby**

1-D

Step-Index Grating
Sinusoidal Index Grating
Photonic Band Gap

2-D

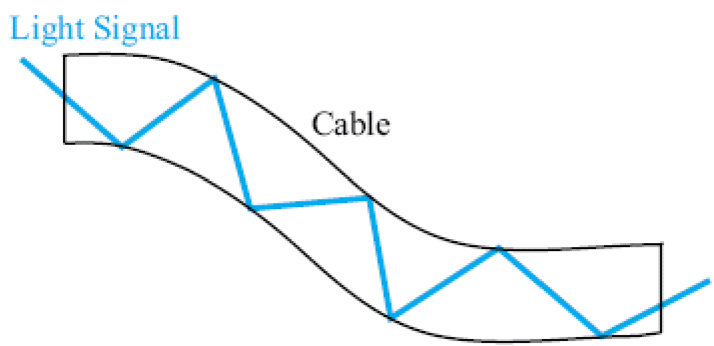
Planar Geometry
Fiber Geometry

3-D

3-D Photonic Crystals

Introduction (1)

- The conventional method **Utilize** → Total Internal Reflection (TIR)



- **TIR** fundamental Principle Operation
  - Nearly all Fiber Optic Devices
  - Planar Waveguide Devices

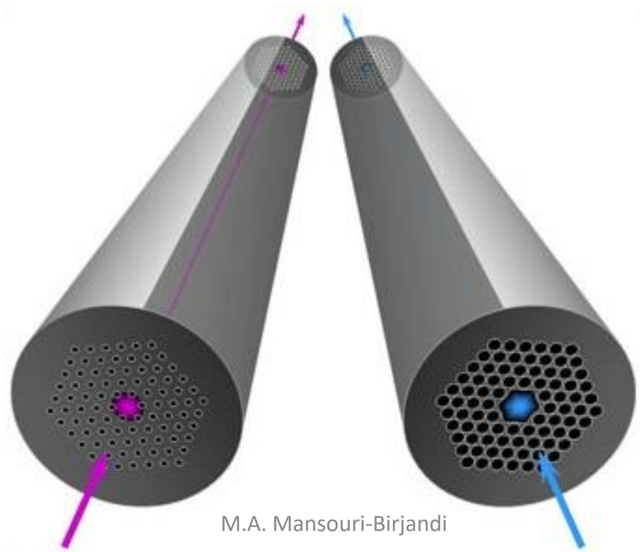
Introduction (2)

- Recently, for controlling the flow of light

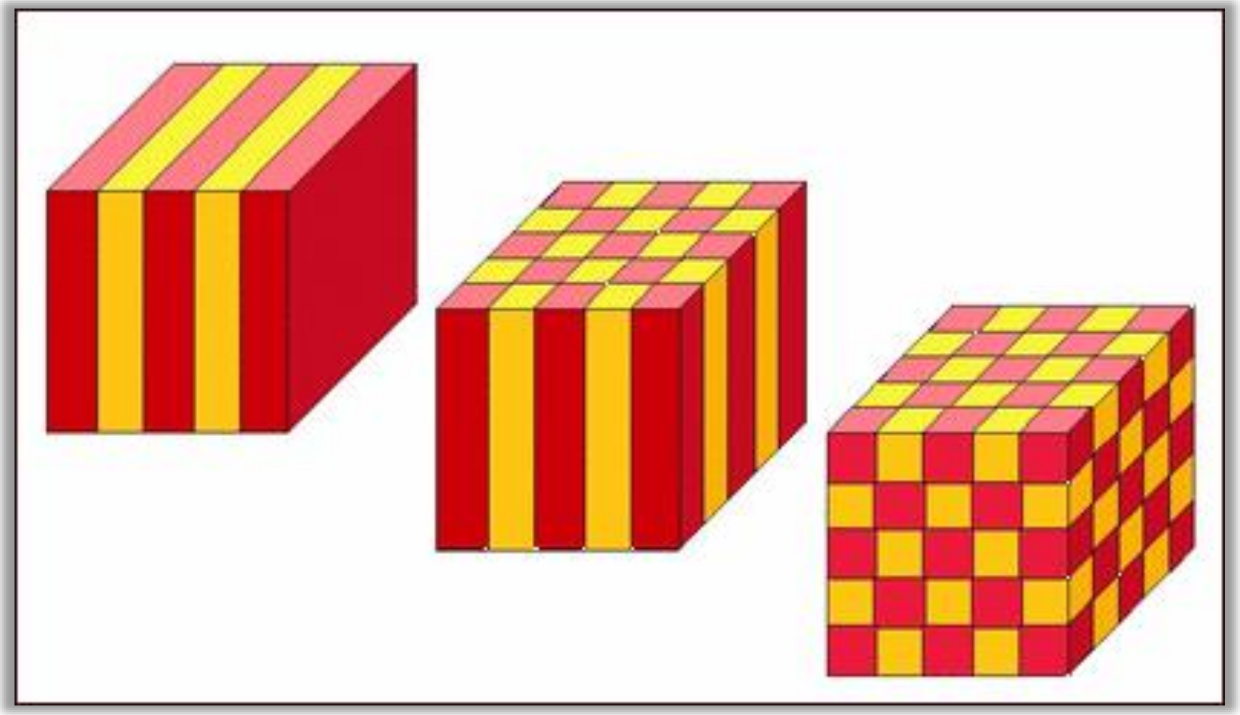
Utilize some modifications on microstructure of the cladding region

➡ Light cannot propagate in there

The **refractive index (n)** varies periodically in space
with a repetition distance on the order of the wavelength of light



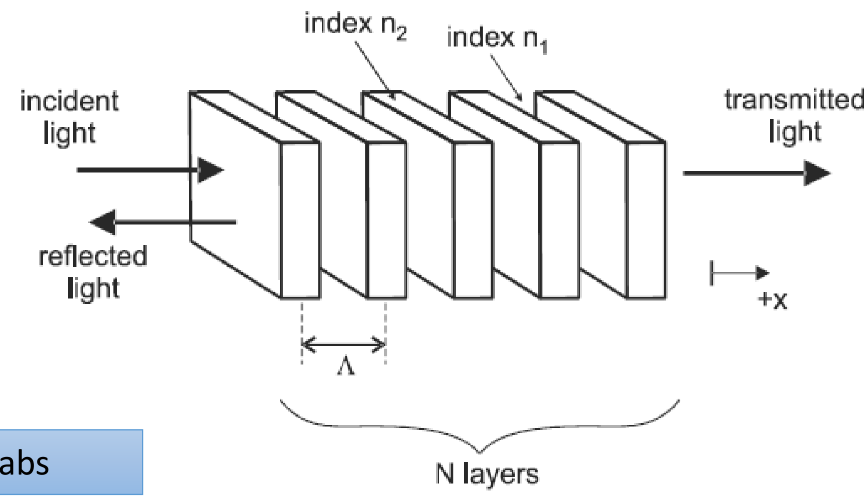
Introduction (3)



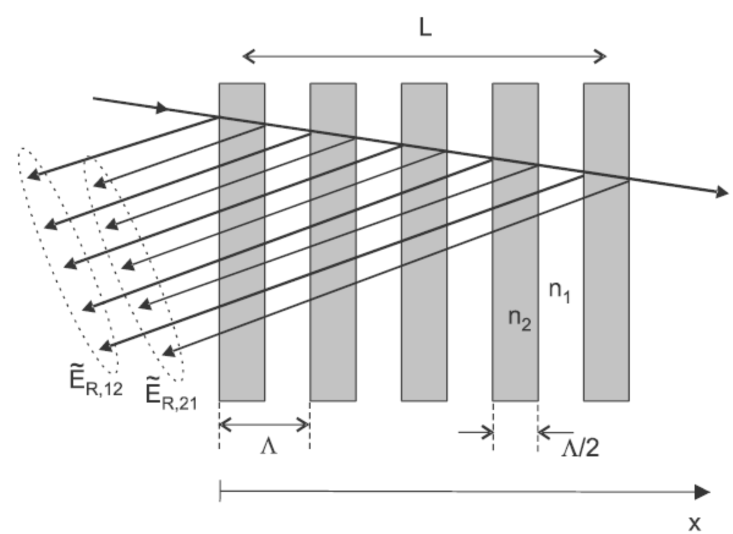
1D, 2D, 3D Photonic crystals

1D Photonic Crystals { Step-Index Grating (1) }

❖ Bragg grating the simplest 1D PhC



n_2	The refractive index of the slabs
n_1	The refractive index of the medium between slabs
$\Delta n \equiv n_2 - n_1$	The index difference
Λ	The center-to-center spacing of the slabs
$\Lambda/2$	The thickness of each slab
$L = N\Lambda$	The total length of the photonic crystal
$E_i(x,t)$	The **incident light** wave's electric field



1D Photonic Crystals { Step-Index Grating (2) }

$$E_i(x,t) = A\cos(\omega t - kx) = A\Re e^{i(\omega t - kx)} \quad \text{(1)}$$

$\Re$	The "**Real Part**" of equation
ω	The "**angular frequency**" of the light
$k = 2\pi\, n/\lambda 0$	The **wave vector** magnitude
$\lambda 0$	Free-space wavelength

❖ Assume $\Delta n \ll 1 \longrightarrow n_1 \approx n_2 \approx n$

E field of light reflected from the jth interface at $x = 0$

$$E_{rj}(0,t) \equiv \Re \tilde{E}_{rj} e^{i\omega t} \quad \text{(2)}$$

$\tilde{E}_{rj}$	A complex amplitude → relative phase, magnitude
At $x = 0$	$\tilde{E}_{rj} = \mathrm{A}$ and a real number

The total electric field of the reflected light ➡ by adding the contribution from each interface

1D Photonic Crystals { Step-Index Grating (3) }

- The *Fresnel equations for the reflected and transmitted E fields*

- *For **p** (parallel) polarization (TM)*

$$\left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$

$$\left(\frac{E_t}{E_i}\right)_{\parallel} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$

(chap 2.11)

- For **s** polarization (TE)

(*senkrecht* German for perpendicular**)**

$$\left(\frac{E_r}{E_i}\right)_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$\left(\frac{E_t}{E_i}\right)_{\perp} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

(chap 2.12)

- Reflections

Incident from index *n1*

Incident from index *n2*

The *Fresnel equations* at normal incidence $\theta = 0$

$$r = \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2} \cong -\frac{\Delta n}{2n} \qquad (3)$$

Ei and Er are evaluated just before and after reflection

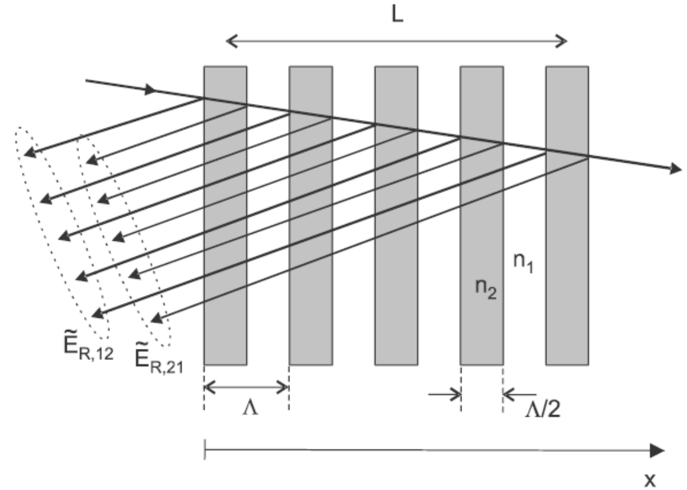
1D Photonic Crystals { Step-Index Grating (4) }

- Reflections
  - Incident from index *n1*
  - Incident from index *n2*

obey the same relation, except that

n1* and *n2* are *interchanged

so $r \approx \Delta n / 2n$



- Assuming $|r| \ll 1 \longrightarrow$ The total reflected field at *x = 0 due to reflections of the **first type***

$$\tilde{E}_{r,12} = -\frac{\Delta n}{2n} A[1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad (4)$$

$$\delta = k(2\Lambda) = \frac{2\pi\, n}{\lambda_0}(2\Lambda) = \frac{4\pi\, n\Lambda}{\lambda_0} \quad (5)$$

phase delay due to propagation over the round-trip distance 2Λ between slabs

$$\tilde{E}_{r,21} = \frac{\Delta n}{2n} A e^{-i\delta/2}\, [1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad (6)$$

total reflected field of the **second type**

1D Photonic Crystals { Step-Index Grating (5) }

$$\tilde{E}_{r,21} = \frac{\Delta n}{2n} A e^{-i\delta/2}\left[1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}\right] \quad (6)$$

total reflected field of the **second type**

❖ Additional round-trip propagation distance $2\left(\Lambda/2\right)$ for each **type 2** reflection

✓ Adding the terms inside the **square brackets**

❖ Visualize them as vectors

❖ Each vector has the same magnitude

❖ $0, -\delta, -2\delta,$ are the **angles from** the real axis

constructive interference

Bragg resonance, all vectors in phase

when $\delta = m2\pi$

m *is an integer* giving the order of diffraction

➢ First-Order Bragg diffraction (***m = 1***)

$$\lambda_B = 2n\Lambda \quad (7)$$

Bragg wavelength in free-space

1D Photonic Crystals { Step-Index Grating (6) }

- $\delta = 2\pi \xrightarrow{\text{Satisfies}}$ Bragg condition $\longrightarrow \exp(-i\delta/2) = -1 \longrightarrow \tilde{E}_{r,12} = \tilde{E}_{r,21}$

✓ **Total reflected** complex amplitude from all interfaces

$$\tilde{E}_r = \tilde{E}_{r,12} + \tilde{E}_{r,21}$$

$$= -\frac{\Delta n}{n} A[1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \quad \text{(8)}$$

✓ *N **unity** terms*

$$\xrightarrow{\delta = 2\pi} \quad \tilde{E}_r = -\frac{\Delta n}{n} AN$$

✓ Power reflectivity $\xrightarrow[\tilde{E}_{rj} = A]{x = 0}$

$$R_{\max} = \left|\frac{\tilde{E}_r}{\tilde{E}_i}\right|^2 = \left|-\frac{\Delta n}{n} N\right|^2 = N^2\left(\frac{\Delta n}{n}\right)^2 \quad \text{(9)}$$

$$\xrightarrow[\lambda_B = 2n\Lambda]{L = N\Lambda} \quad R_{\max} = \left(\frac{L}{\Lambda}\frac{\Delta n}{n}\right)^2 = \left(\frac{2\Delta n L}{\lambda_B}\right)^2 \quad \text{(10)} \longrightarrow R_{\max} = (\kappa L)2 \quad \text{(11)} \quad \xrightarrow[\frac{\Delta n}{n} \ll 1]{\kappa L \ll 1} \quad \boxed{R_{\max} \ll 1}$$

peak reflectivity, **weak grating**

$\underrightarrow{\kappa \equiv 2\Delta n/\lambda B}$ **Attenuation factor** of light as it propagates through the **Bragg grating**

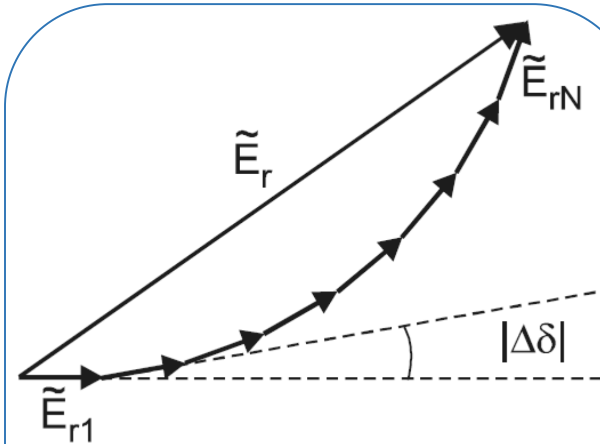
1D Photonic Crystals { Step-Index Grating (7) }

- The incident wavelength slightly detuned from λ_B

  ⇒ **phases** progressively further apart

$$\tilde{E}_r = \tilde{E}_{r,12} + \tilde{E}_{r,21} = -\frac{\Delta n}{n} A[1 + e^{-i\delta} + e^{-i2\delta} + \ldots + e^{-i(N-1)\delta}] \qquad (8)$$

Phases difference $\phi = -\delta$

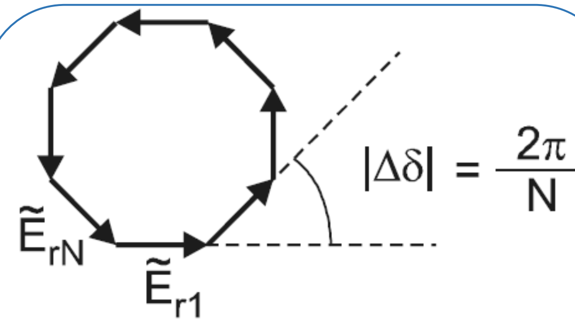


Slightly off resonance
small difference in phase
between vectors

- If the wavelength changes by $\Delta\lambda \longrightarrow \Delta\phi = -\Delta\delta$

$N\,\Delta\phi = 2\pi \longrightarrow$ The grating reflectivity goes from a **maximum** to **zero** over a range of wavelengths $\Delta\lambda$

$\Delta\lambda$ spectral half-width of the Bragg resonance



vectors add to zero
destructive interference

1D Photonic Crystals { Step-Index Grating (8) }

- Evaluation of $\Delta\lambda$

$$\delta = k(2\Lambda) = \frac{2\pi n}{\lambda_0}(2\Lambda) = \frac{4\pi n\Lambda}{\lambda_0} \quad \textbf{(5)}$$

$$\Delta\phi = -\Delta\delta = -\frac{d\delta}{d\lambda}\Delta\lambda = \frac{4\pi n\Lambda}{\lambda_0^2}\Delta\lambda \quad \textbf{(12)}$$

- Near the Bragg resonance $\lambda_0 \approx \lambda_B$

$$\lambda_B = 2n\Lambda$$

$$\Delta\phi = 2\pi/N$$

$$\frac{2\pi}{N} = \frac{2\pi\lambda_B}{\lambda_0^2}\Delta\lambda$$

$$\frac{\Delta\lambda}{\lambda_0} = \frac{1}{N}$$

- *Quality factor* $\rightarrow \quad Q = \dfrac{\lambda}{\Delta\lambda} \quad \rightarrow \quad Q \approx N$

spectral half-width

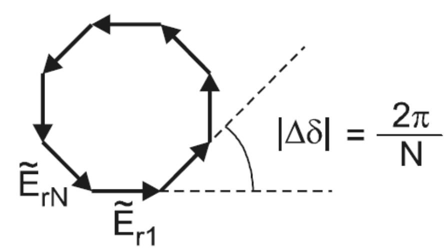
weak grating

- If the incident wavelength is detuned from λ_B *more than* $\Delta\lambda$

→ continue to curl around in the complex plane

a secondary maximum $N\Delta\phi \simeq 3\pi$

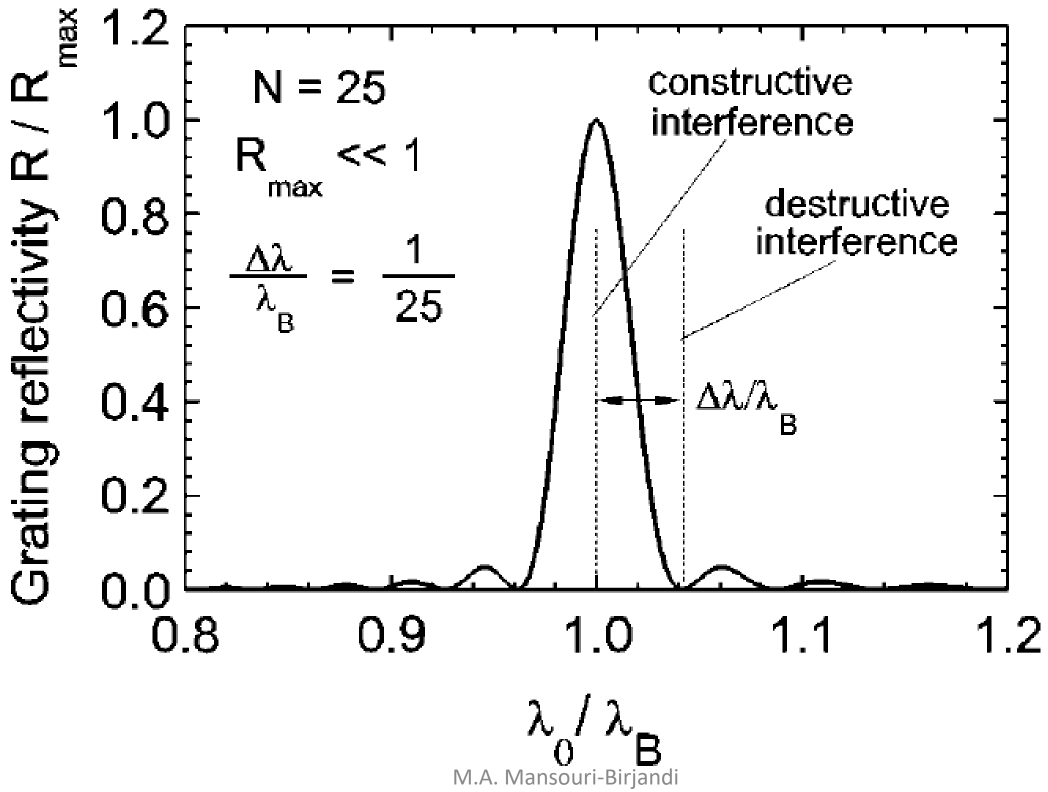
another zero $N\Delta\phi = 4\pi$



1D Photonic Crystals { Step-Index Grating (9) }

This pattern **continues** with increasing detuning

Oscillatory dependence of reflectivity on wavelength

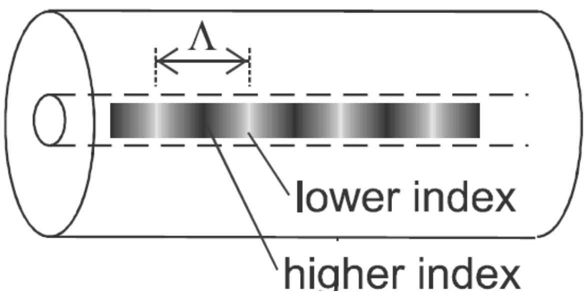


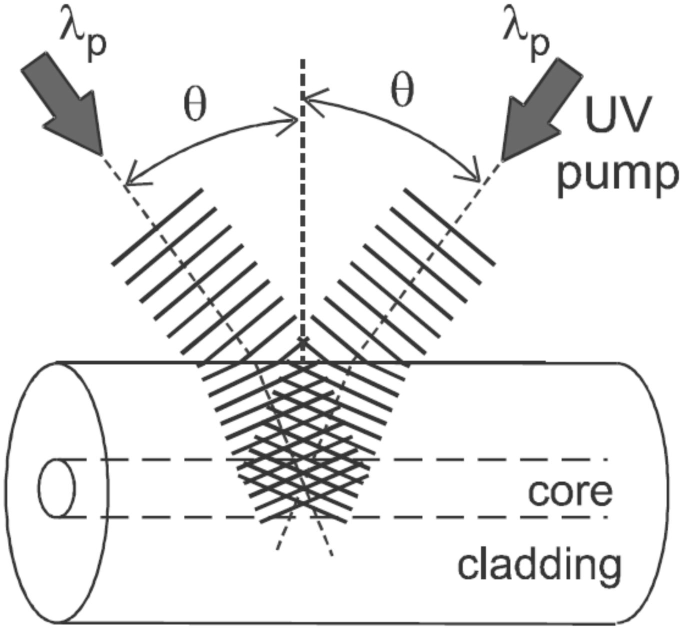
1D Photonic Crystals { **Sinusoidal Index Grating (1)** }

➢ The refractive index (**n**) in a Bragg grating often **varies** sinusoidally with position

fiber Bragg grating

$$\Lambda = \frac{\lambda_p}{2 \sin \theta}$$ Separation between planes of **maximum intensity**





1D Photonic Crystals { Sinusoidal Index Grating (2) }

- If **pump intensity** and **exposure time** is short enough
  - Resulting index variation may be expressed by the sinusoidal form

$$\boxed{n(x) = \bar{n} + \Delta n \cos(\frac{2\pi x}{\Lambda})}$$

average index

spacing between index maxima

$$\left\{ \begin{array}{ll} \lambda_B = 2n\Lambda & \textit{Bragg reflection} \text{ wavelength} \\ \Delta\lambda = \dfrac{\lambda_0}{N} & \text{Resonance half-width} \\ R_{\max} \approx (\kappa L)^2 & \text{Peak reflectivity} \end{array} \right.$$

One small difference

attenuation constant

$$\left\{ \begin{array}{lll} \text{sinusoidal grating} & \longrightarrow & \kappa \equiv \dfrac{\pi \Delta n}{\lambda_B} \\ \text{step-index grating} & \longrightarrow & \kappa \equiv 2\Delta n/\lambda_B \end{array} \right.$$

1D Photonic Crystals { Sinusoidal Index Grating (2) }

Arbitrary κL's *of interest* $\xrightarrow{\text{coupled mode theory}}$ $R_{max} = \tanh^2(\kappa L)$

(peak reflectivity, arbitrary *L*)

In the limit $\kappa L \ll 1 \longrightarrow R_{max} \approx (\kappa L)^2$

In the limit $\kappa L \gg 1 \longrightarrow R_{max} \to 1.$

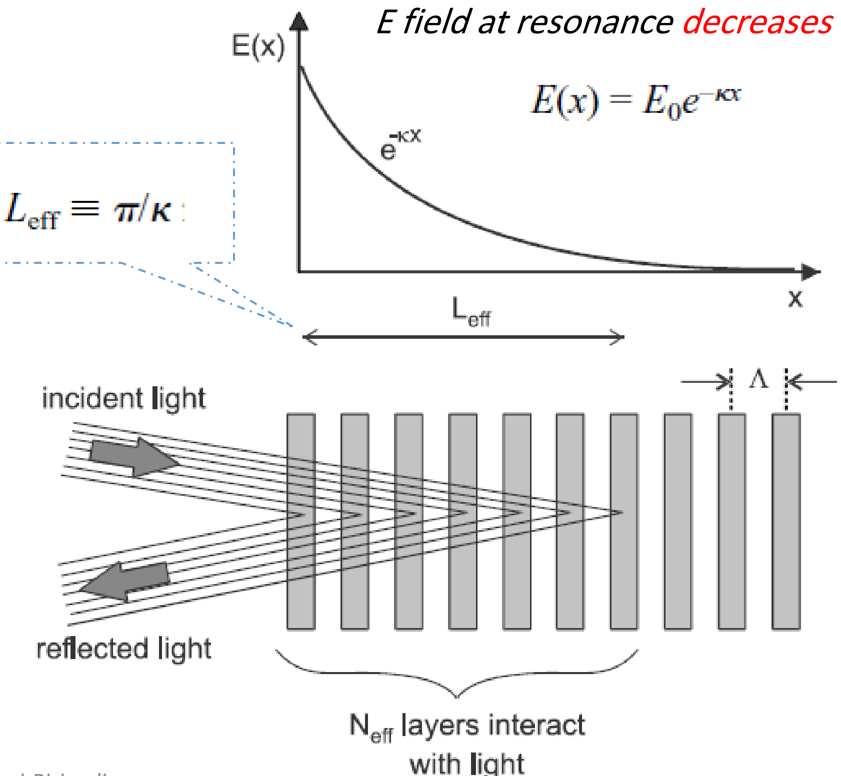
$$N_{eff} = \frac{L_{eff}}{\Lambda} = \left(\frac{\lambda_B}{\Delta n}\right)\left(\frac{2n}{\lambda_B}\right) = \frac{2n}{\Delta n}$$

$$\downarrow$$

$$\frac{\Delta\lambda}{\lambda_0} = \frac{1}{N_{eff}} = \frac{\Delta n}{2n}$$

$$\boxed{\frac{\Delta\lambda}{\lambda_0} = \begin{cases} \lambda_0/(2nL) & \text{for } \kappa L \ll 1 \\ \Delta n/(2n) & \text{for } \kappa L \gg 1 \end{cases}}$$

$L_{eff} \equiv \pi/\kappa$



1D Photonic Crystals { Sinusoidal Index Grating (2) }

✓ Most important application of Bragg gratings — **mirrors in a laser cavity**

✓ High reflectivity that is obtained for $\kappa L \gg 1 \longrightarrow$ Selectivity for the laser output

✓ Fiber Bragg grating as a sensor $\rightarrow$ physical parameters
- Strain
- Temperature change
- Pressure change

1D Photonic Crystals { **Photonic Band Gap (1)** }

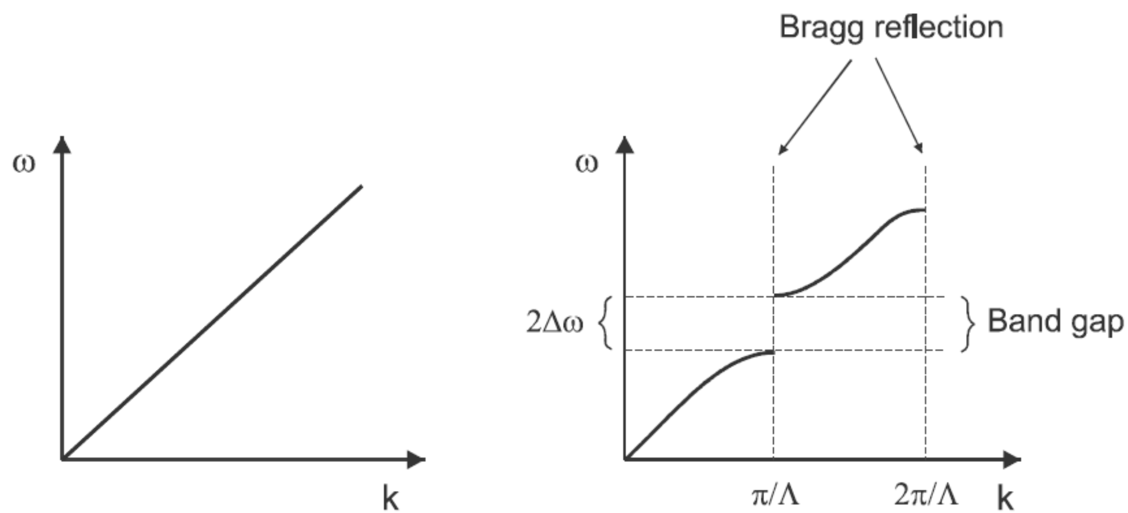
- One of the important characteristics of a Bragg grating

→ Exponential attenuation of light for wavelengths close to λ_B

→ For a sufficiently **long grating** → 100 % reflection

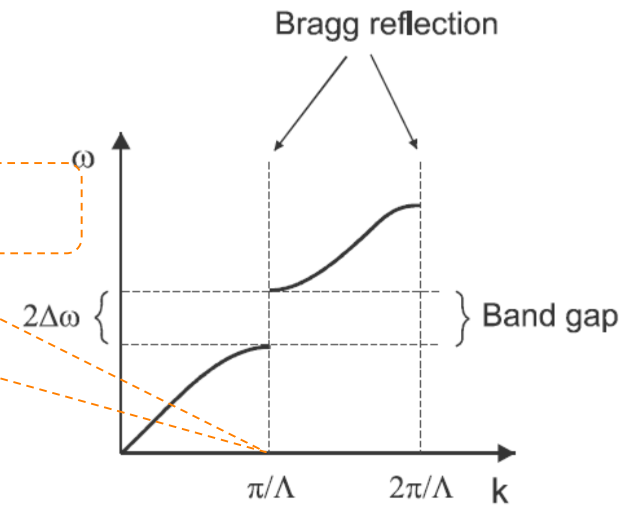
The range of wavelengths for which light is attenuated is referred to as the ***stop band***

A **frequency gap** in the **photon spectrum** is known as a ***photonic band gap***



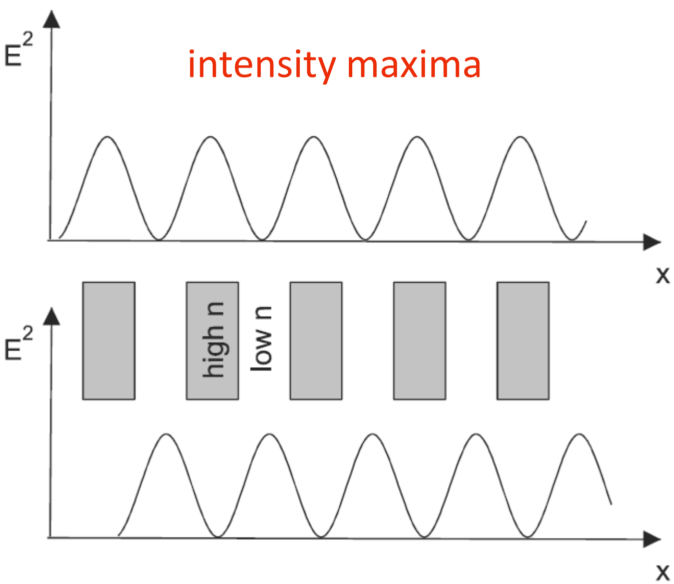
1D Photonic Crystals { **Photonic Band Gap (1)** }

- There are two values of the light frequency $k = \dfrac{\pi}{\Lambda}$?



❖ The wave is not propagating, but is actually a **standing wave**

❖ Physical origin of two frequency ? intensity profiles



higher n_{eff}

lower ω

lower n_{eff}

higher ω

effective index relates → frequency, wave vector

$$\omega = \left(\frac{c}{n_{eff}}\right) k = \left(\frac{c}{n_{eff}}\right)\frac{\pi}{\Lambda} \qquad (23)$$

$\Delta n \uparrow$ ➡ gap $\uparrow$

1D Photonic Crystals { **Photonic Band Gap (2)** }

✓ Calculating $\Delta\omega$

$$\omega = \frac{2\pi c}{\lambda_0} \longrightarrow \Delta\omega = -\frac{2\pi c}{\lambda_0^2}\Delta\lambda = -\frac{2\pi c}{\lambda_0}.\frac{\Delta\lambda}{\lambda_0}$$

$$\frac{\Delta\lambda}{\lambda_0} = \begin{cases} \lambda_0/(2nL) & \text{for } \kappa L \ll 1 \\ \Delta n/(2n) & \text{for } \kappa L \gg 1 \end{cases}$$

$$\frac{\Delta\omega}{\omega} = \frac{\Delta\lambda}{\lambda_0} = \frac{\Delta n}{2n}$$

$$\frac{\text{frequency gap}}{\text{center frequency}} = \frac{2\Delta\omega}{\omega} = \frac{\Delta n}{n}$$

1D Photonic Crystals { **Localized Modes (1)** }

✓ If we have some frequencies within the photonic band gap

➡ Light wave's *E field are exponentially attenuated*

localized modes

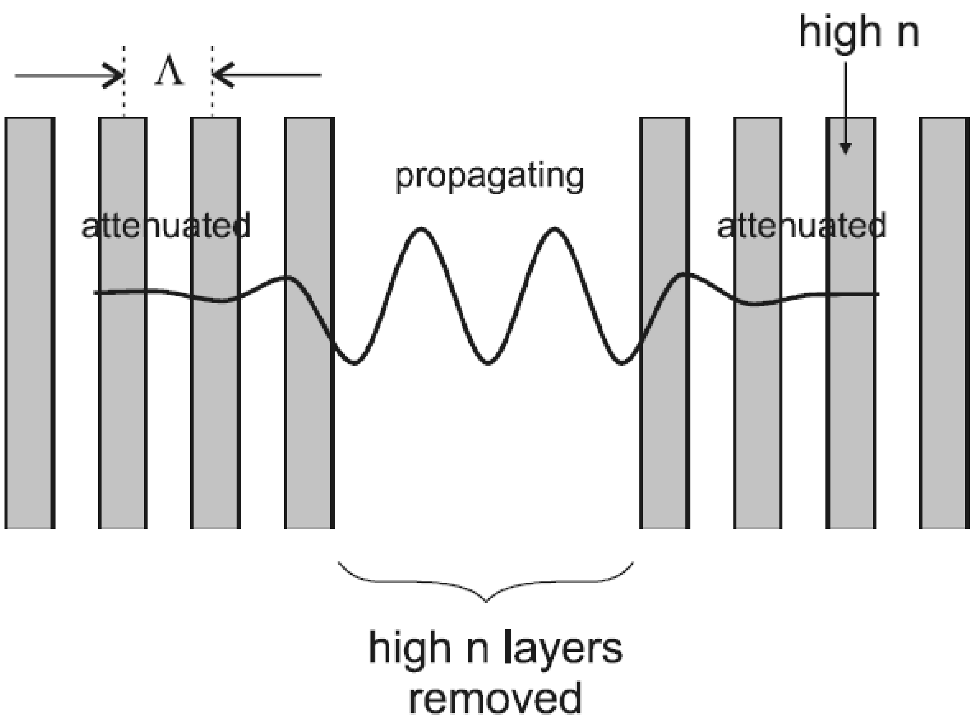
✓ If we create **discontinuity** or **defect** in the structure

➡ Light energy residing in **optical mode** confined to vicinity of the defect

❑ Simplest type of defect → edge / surface of a photonic crystal

❑ 1D Bragg grating defects → **begin point** of grating / **removing** one or more of the high-index layers

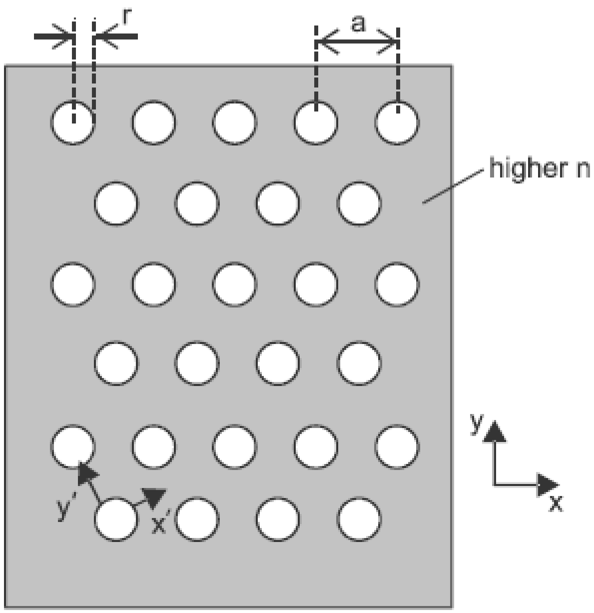
1D Photonic Crystals { **Localized Modes** (2) }



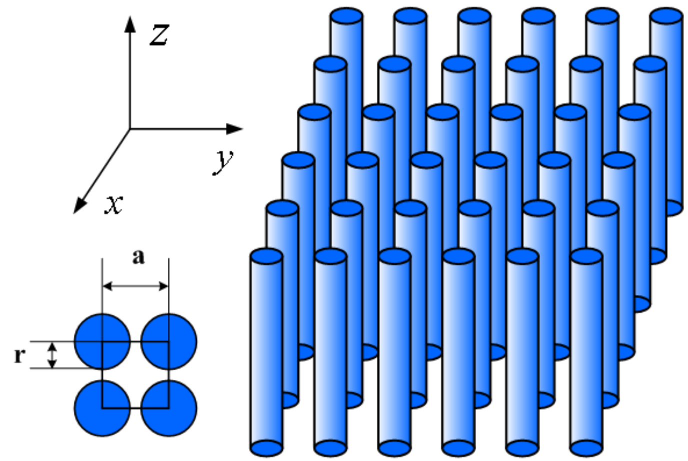
Layers are removed → The result is a localized mode

an **optical cavity** formed by two semiinfinite Bragg gratings

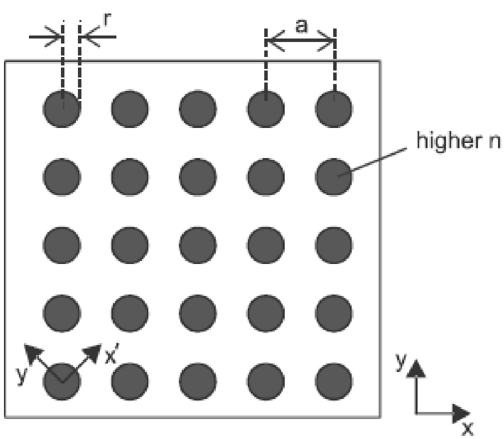
2D Photonic Crystals



2-D Triangular Lattice - Holes in a Substrate with higher n



2-D Square lattice - Rods in Air



Two symmetry directions *x and x' are shown.*

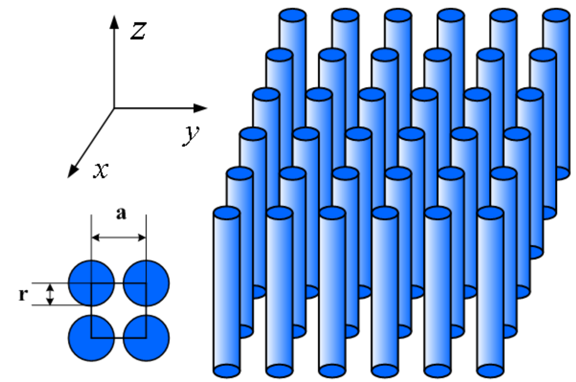
Planar Geometry (1)

❑ For 2D photonic crystals

1- Propagation mostly **in the *x–y plane***

2- Propagate mostly **along the *z* axis**

1) Propagation in the *x–y plane* and **Square Lattice**



➢ Propagating along ***+X* axis** → refractive index spatially **vary**

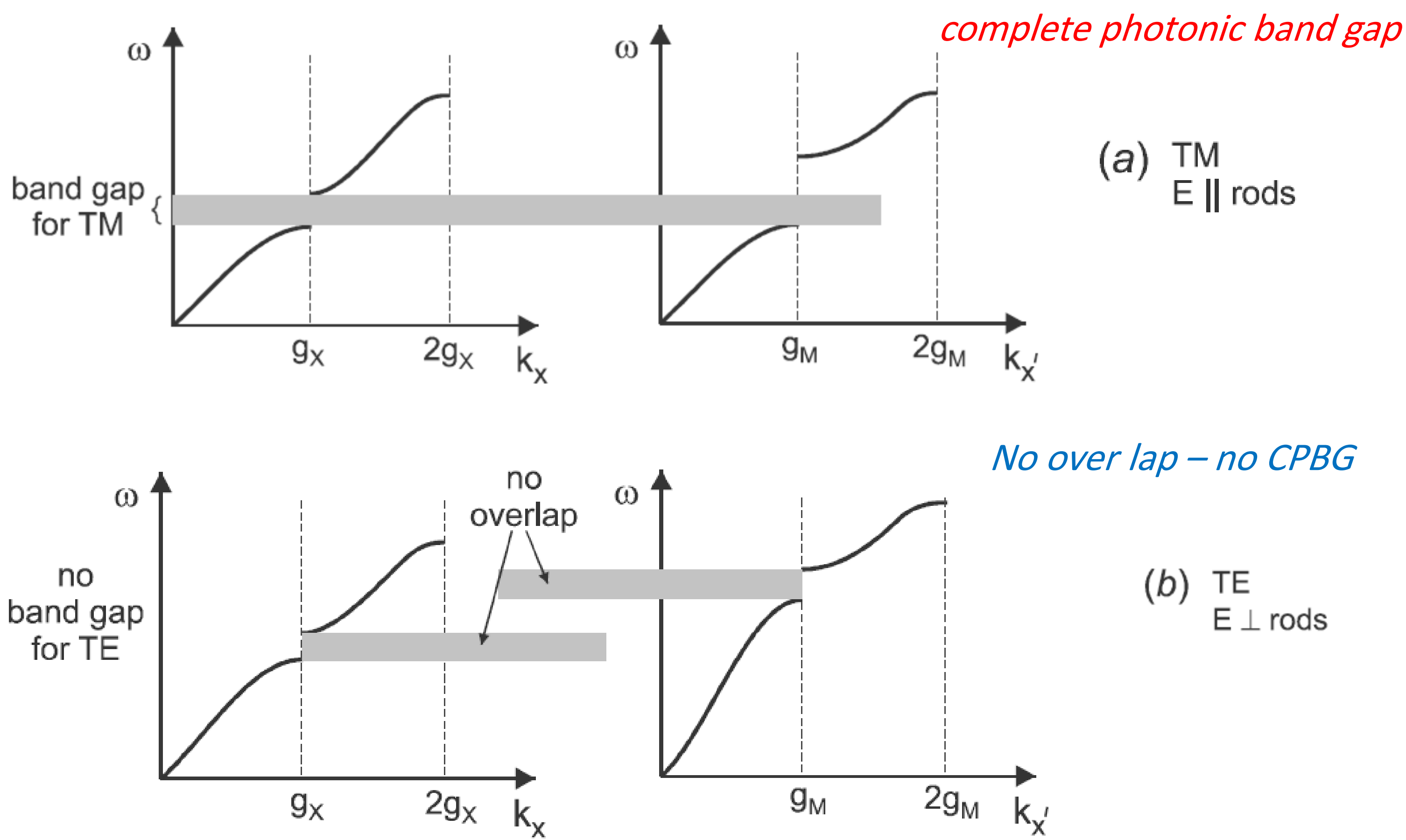
Like **1D Bragg grating**

❑ Bragg condition → light will be strongly reflected, and gap opens in dispersion relation

❖ 2D Difference with **1D Bragg grating** → 2D is **periodic** in 2 direction **x , x'**

Planar Geometry (2)

2 directions → 2 Band gaps → Different position and width



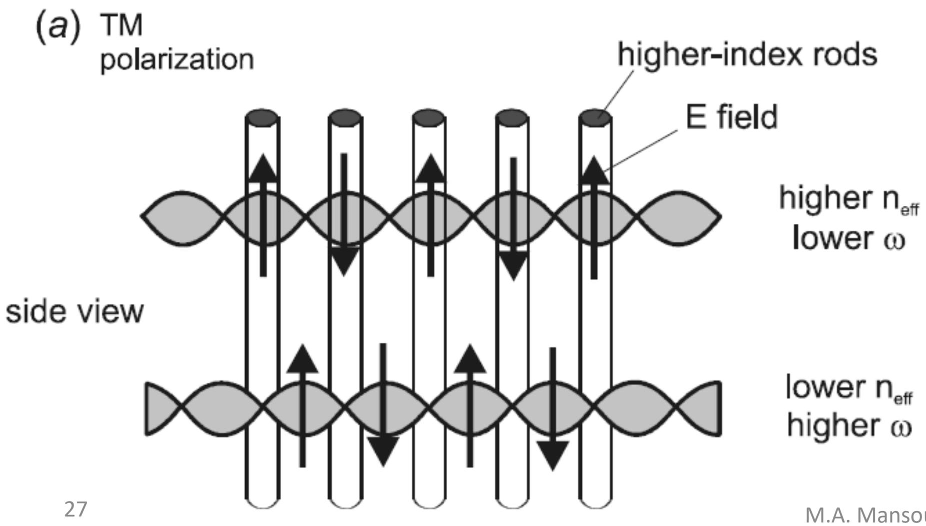
Planar Geometry (3)

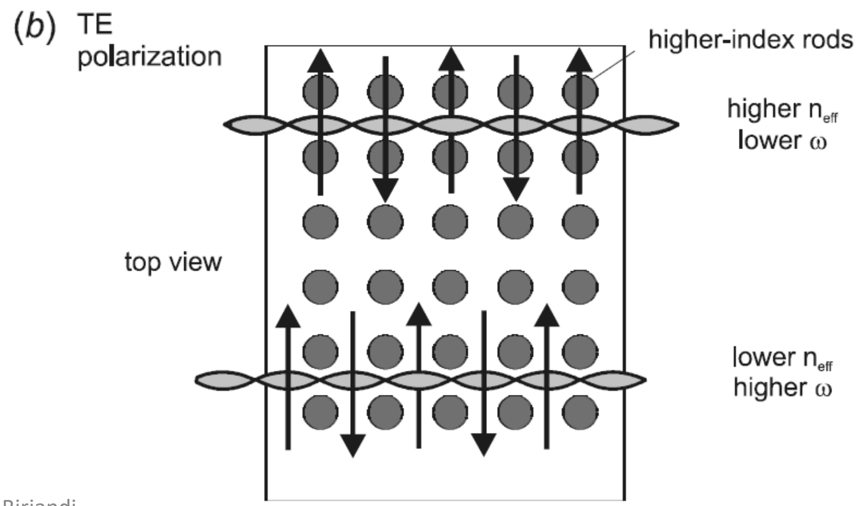
TE polarization	*E field **along y** and **perpendicular** to rods (with B along z)*

TM polarization	*E* field **along *z*** *(with B along –y and **perpendicular** to the rods)*

- **Physical origin** of difference between TM and TE polarization

→ Intensity peaks in the standing waves





Planar Geometry (3)

confine light in two dimensions

band gaps for different directions and polarizations all **overlap** in some frequency range

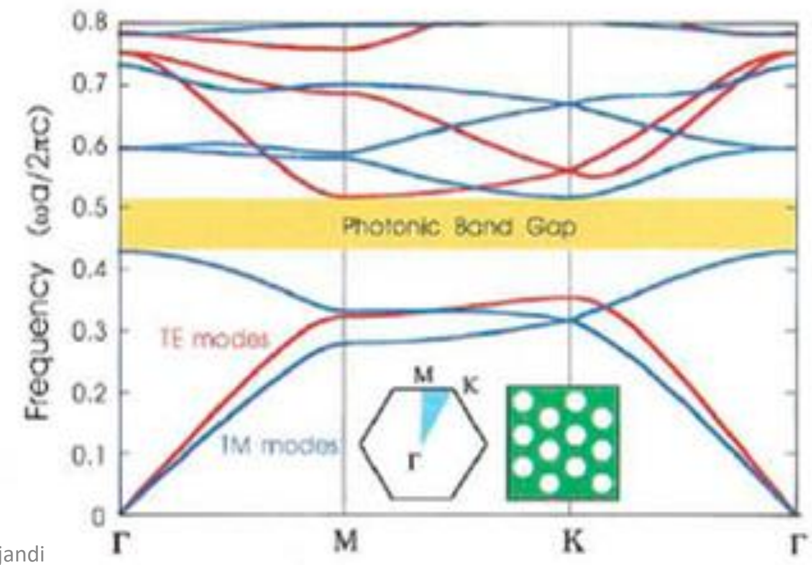
1. A square lattice of dielectric rods

There is **PBG** for **both** TE and TM polarization

Does not overlap → No **CPBG**

2. A triangular lattice of air holes

Overlap ✓



M.A. Mansouri-Birjandi

2D Photonic Crystals - Fiber Geometry

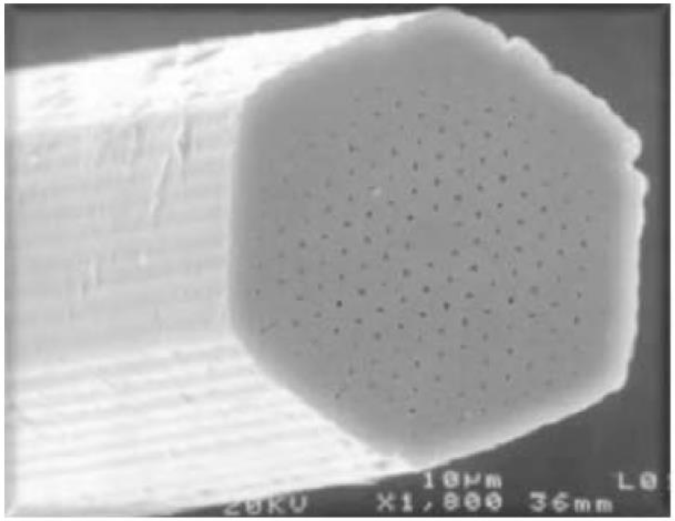
2. Light propagating mostly in the ***z direction***

component perpendicular to the **rods** or **air holes** are **Negligible**

Usage: **optical fiber**

- Guiding types in optical fibers
  - Total internal reflection (TIR)
  - Photonic band gap (PBG)

Guiding by Effective Index (1)



Early ***photonic crystal fiber*** called holey fiber

Single missing air hole, allows confinement of light by TIR

- Advantages
  - only one glass type
  - no doping — Cladding region being "doped" with air holes

Guiding by Effective Index (2)

❖ Light guidance of Photonic crystal fibers → Single mode / Multi mode

❖ Single-mode guidance of light in conventional step-index fiber →

$$V \equiv \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2}$$

$$V < 2.405$$

❖ Single-mode guidance for holey fiber →

$$V_{PCF} \equiv \frac{2\pi \Lambda}{\lambda}\sqrt{n_c^2 - n_{cl}^2(\lambda)}$$

$$V_{PCF} < \pi$$

❖ Effective cladding index is not a **constant** → varies strongly **with wavelength**

❖ Shorter wavelengths, light is **better confined** and **index is higher**

❖ Longer wavelengths, **diffraction** prevents the light from being well confined

❖ glass region between the **air holes** should be sufficiently wide → ✓ **diffraction**

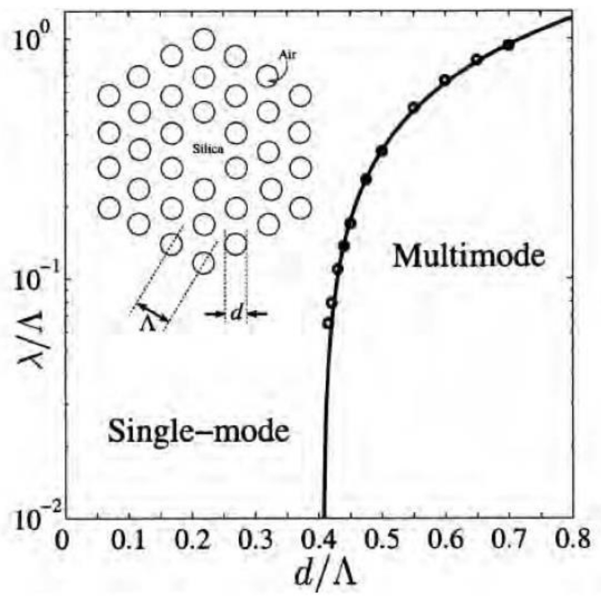
holey fiber *is also called* ***Endlessly single-mode***

Guiding by Effective Index (3)

❖ Boundaries between **single-mode** and **multimode**

$\frac{d}{\Lambda}$ increases

λ decreases



This fibers can be scaled up

Larger

high-power fiber lasers

Smaller

Frequency conversion

Guiding by Photonic Band Gap (1)

- Condition of Photonic band-gap in Crystal type (triangular lattice of air holes)

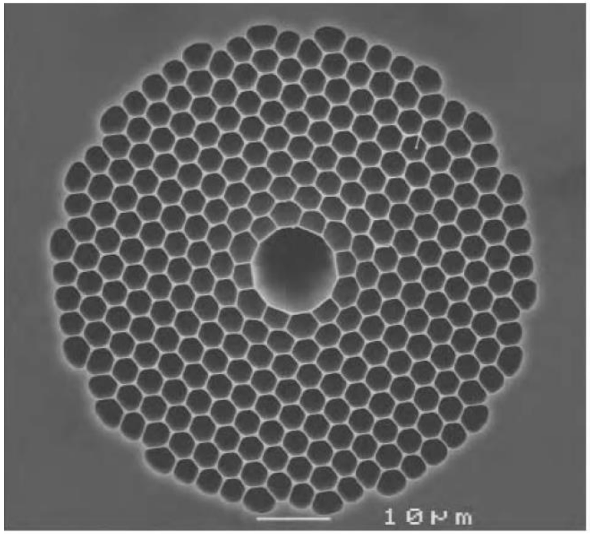
→ substrate material → $\varepsilon_r > 7.2$, $n > 2.7$

- (n) Silica glass <1.5 → **Seems** no Photonic band-gap guiding for holey fiber

- ❑ $\varepsilon_r > 7.2$ is only for light propagating perpendicular to the air holes (TM)

- ❑ For propagation parallel to holes (TE) if one use proper geometry of air holes

Hollow-core fiber



→ **Complete PBG**

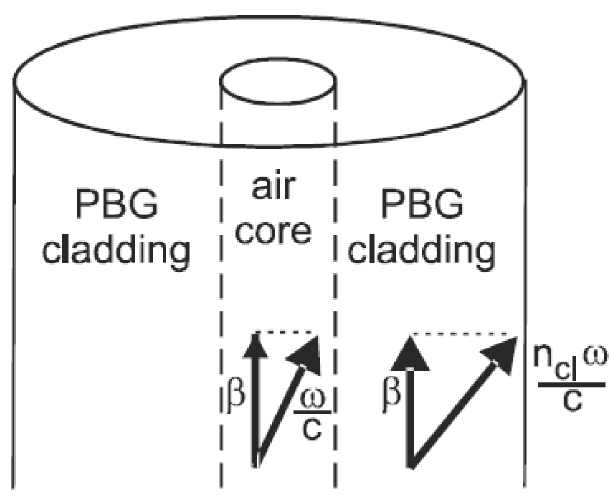
Guiding by Photonic Band Gap (2)

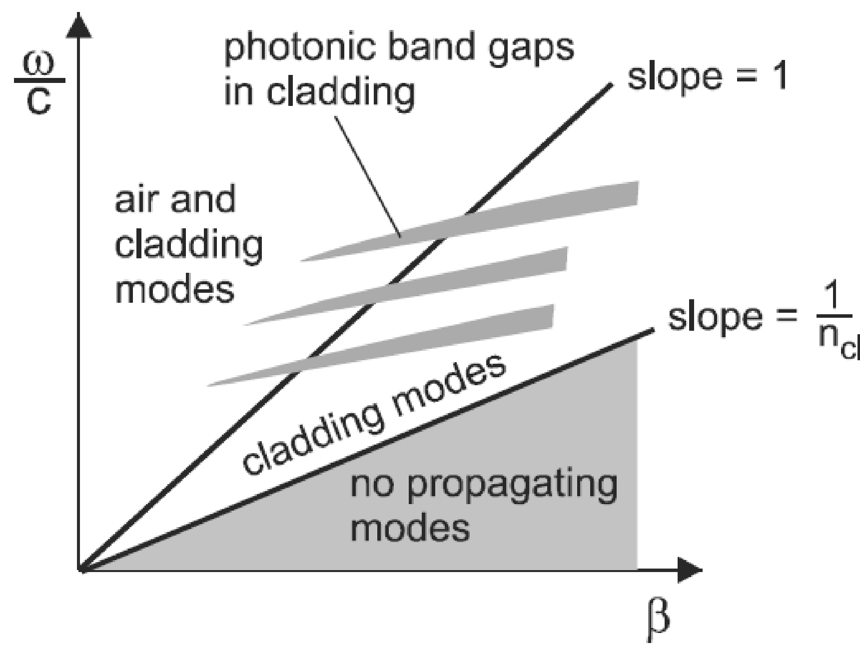
❖ PBG → Δn

→ Photonic crystal structure

→ Propagating direction of light ⟶ β axial wave vector

❑ Modes with larger β ⟶ directed more nearly down the fiber axis ⇐ same ω





Guiding by Photonic Band Gap (3)

- **Absorption** and **Rayleigh scattering** in the fiber core → much lower in hollow-core fiber

  → Less amplification for further distance

- Much higher optical powers

- Less dispersion → Very short optical pulses propagate without significant spreading in time

All because light is propagating mostly in **air**, rather than in the glass

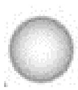
3D Photonic Crystals (1)

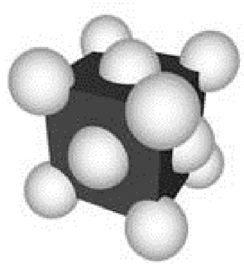
➢ In 1987 The concept of the 3-D photonic crystal introduced independently

- **E. Yablonovitch** — Improving the efficiency of semiconductor **laser** devices
- **S. John** — “**Localizing**” or confining light to a small region of space

a material with a **complete photonic band gap**

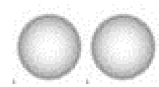
diamond-type
Need n > 1.87

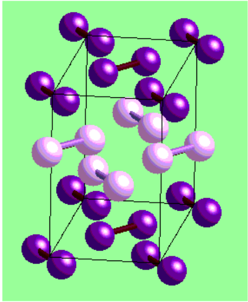


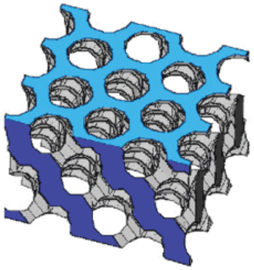


FCC cubic

Opal







inverse opal *Need n > 2.8*
Narrow band gap

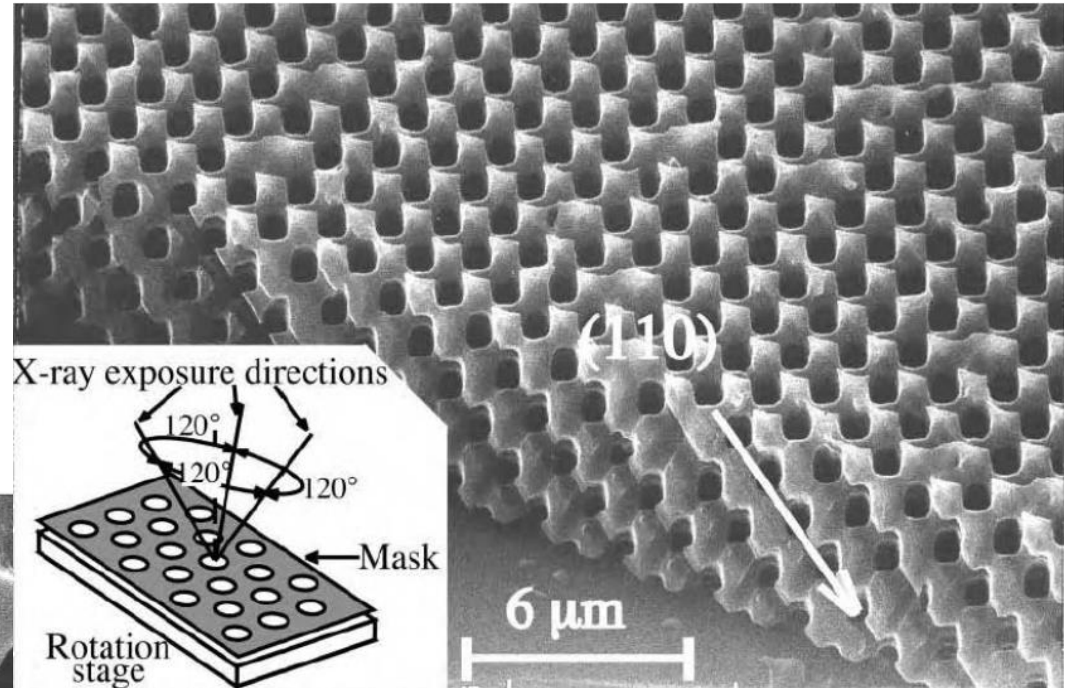
3D Photonic Crystals (2)

- Creation Methods

  ✓ "**Top-down**" approach

Yablonovite

Lincoln log



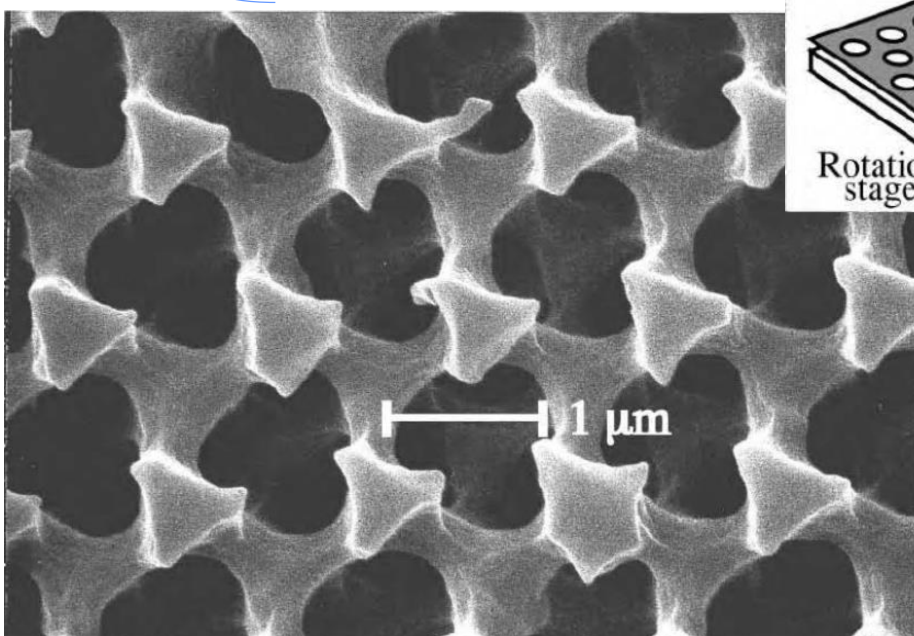
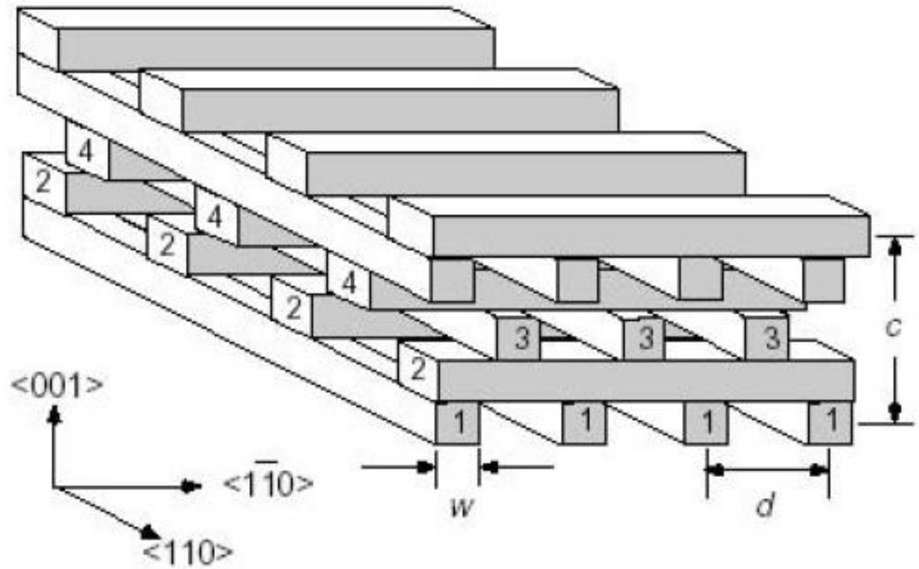


Figure 8-21 Yablonovite is formed by drilling holes into a dielectric at three precise angles, and results in a diamond -like structure. The holes can be created by exposure to X-rays through a mask

3D Photonic Crystals (3)

- Creation Methods
    - ✓ “**Top-down**” approach



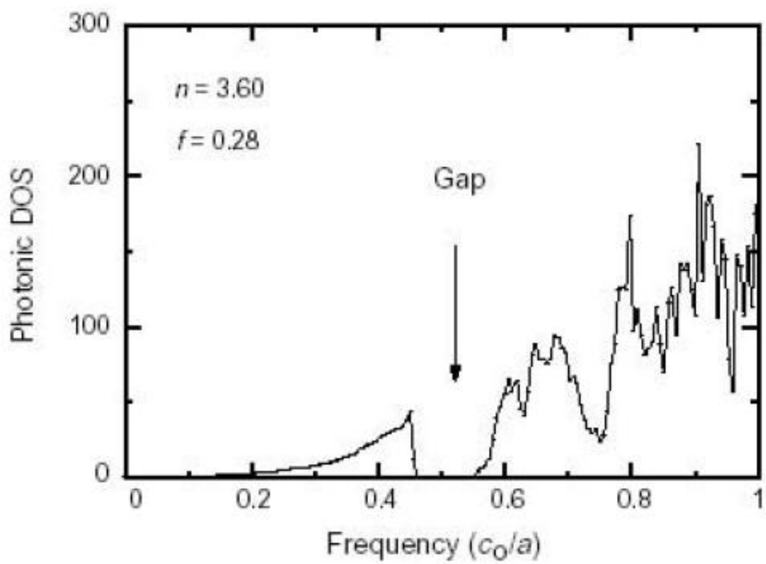


Figure 8-22 (a) The “Lincoln log” or “woodpile” structure has a diamond-type lattice symmetry, and exhibits a 3-D photonic band gap.
(b) Calculated density of states when *n* bars *= 3.6 and fill 28% of the space.*

density of states (number of propagating modes per unit frequency interval)

3D Photonic Crystals (4)

- Creation Methods

  - ✓ "**bottom-up**" or "**self-assembly**" approaches

fossilization

self-assembling of SiO_2 Spheres with diameter of 100 to 1000 nm in the fcc structure →

Injecting a high-index material between the spheres

→

selective etching of the original SiO_2 material

In "**Top-down**" approach for a given refractive index **wider band gap** can be achieved

In "**bottom-up**" approach better for mass-production levels