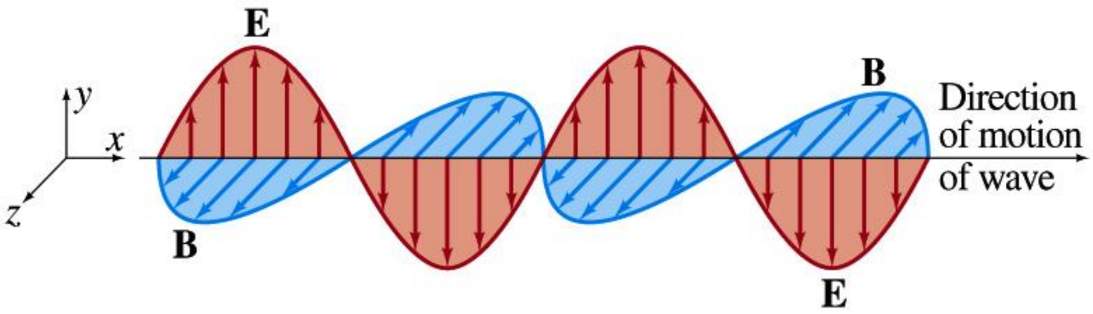
Physical Optics

دانشگاه سیستان و بلوچستان
دانشکده مهندسی برق و کامپیوتر



The wave nature of light

Interference, Diffraction, Polarization, …

زاهدان- دانشگاه سیستان و بلوچستان- دانشکده مهندسی برق و کامپیوتر-

گروه مهندسی برق و الکترونیک- محمدعلی منصوری بیرجندی

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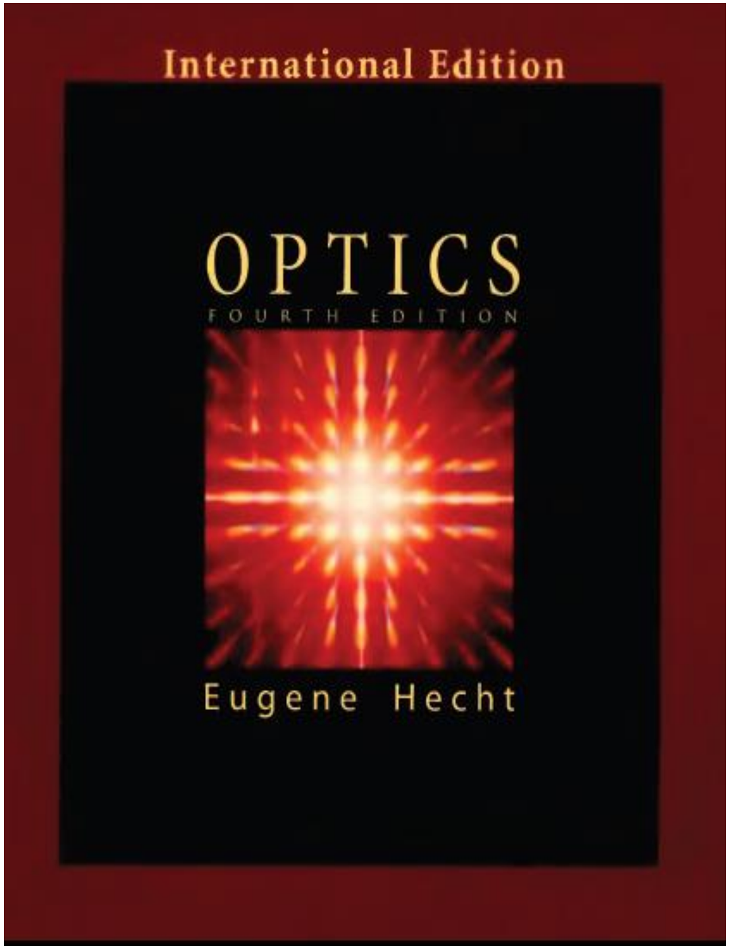
References

- ☐ **Main references**
  - **Textbook:**



- **Eugenem Hecht,**

Optics- 4th Edition

Addison Wesly, 2002

- ترجمه کتاب فوق:

اپتیک، ویراست ۴

یوجن هشت، [مترجم] محمود دیانی،

موسسه علمی فرهنگی نص، ۱۳۸۵

فهرست مطالب

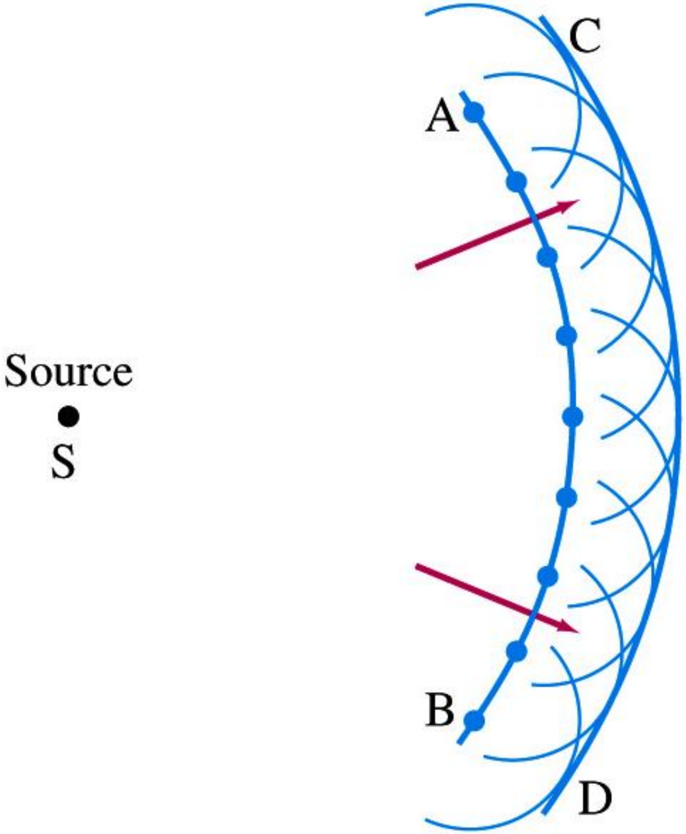
واژه نامه

Birefringence	دوشکستی	Interference	تداخل
Brewster’s angle	زاویه بروستر	Isotropic	همسانگرد
Constructive	سازنده	Linear	خطی
Critical angle	زاویه بحرانی	Polarization	قطبش
Diffraction	پراش	Reflection	بازتاب
Dispersion	پاشندگی	Refraction	شکست
Dispersive	پاشنده	Refractive Index	ضریب شکست
Destructive	مخرب	Thin Film	لایه نازک
Homogeneous	همگن	Total internal reflection	بازتاب کلی داخلی
		Superposition	جمع آثار

Huygens’ Principle

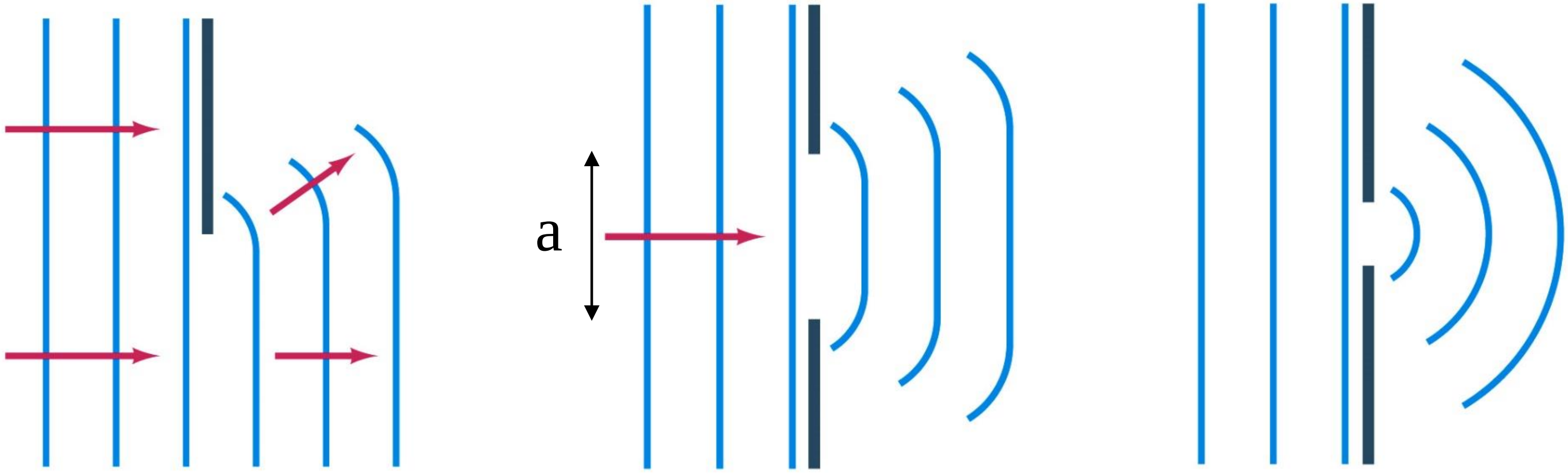
- هر نقطه ای از جبهه موج انتشاری به عنوان منبع موجک های کروی عمل میکند، به طوری که موجک در بعضی اوقات عقبتر از پوش این موج است.

- اگر موج انتشاری دارای یک فرکانس و سرعت خاص باشد، موجک های ثانویه دارای همان فرکانس و سرعت هستند.



“Isotropic”

Diffraction (پراش)



- Diffraction – Bending of light into the shadow region
- Grimaldi - 17th Century observation of diffraction
- Diffraction vs. Refraction (بازتاب)?

$$\lambda \approx a$$

Diffraction (پراش)

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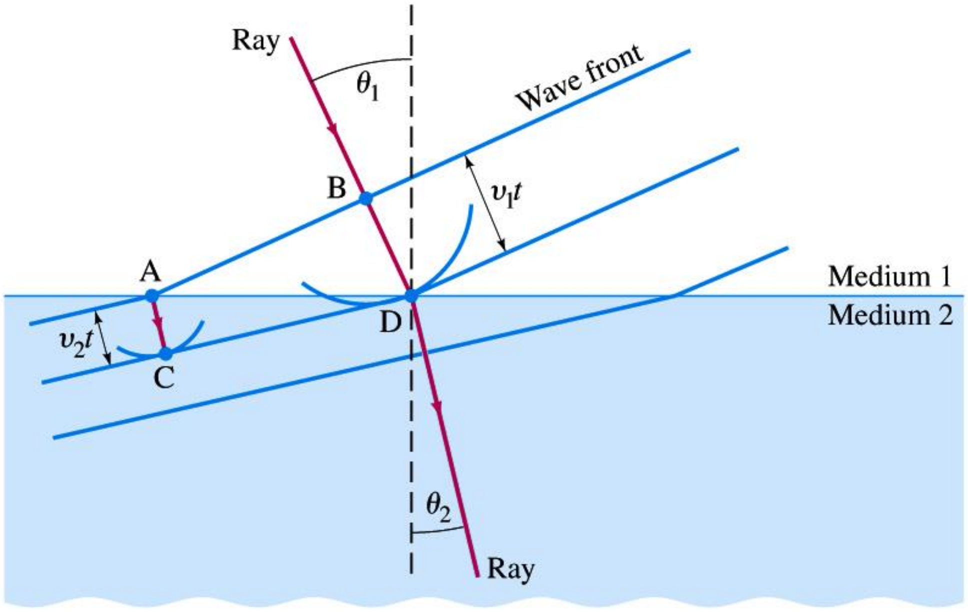
Explanation of Snell’s Law



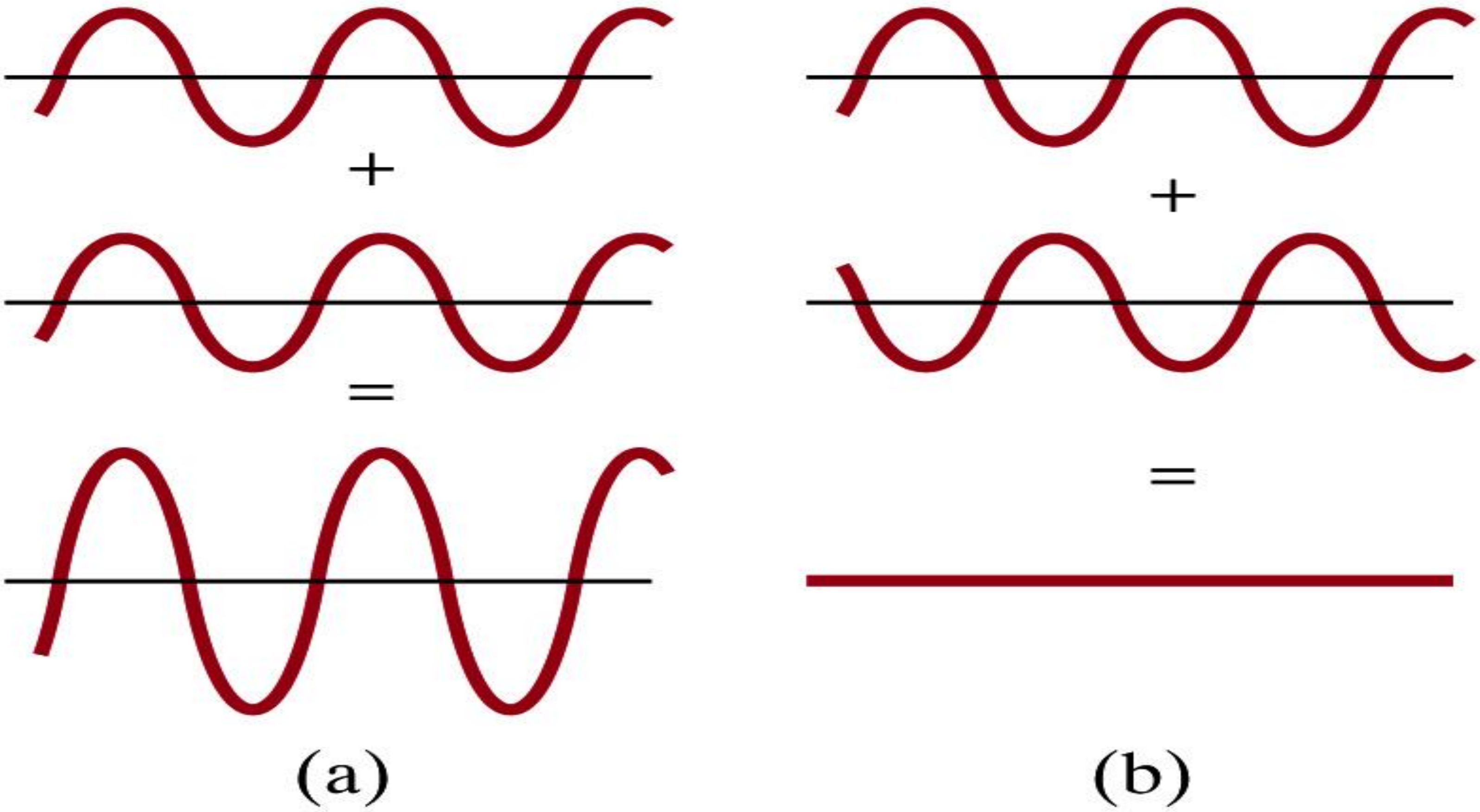
$$\sin\theta_1 = \frac{BD}{AD} = \frac{v_1\Delta t}{AD}$$

$$\sin\theta_2 = \frac{AC}{AD} = \frac{v_2\Delta t}{AD}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1\Delta t}{v_2\Delta t} = \frac{c/n_1}{c/n_2}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

Superposition of waves



Constructive Interference

Destructive Interference

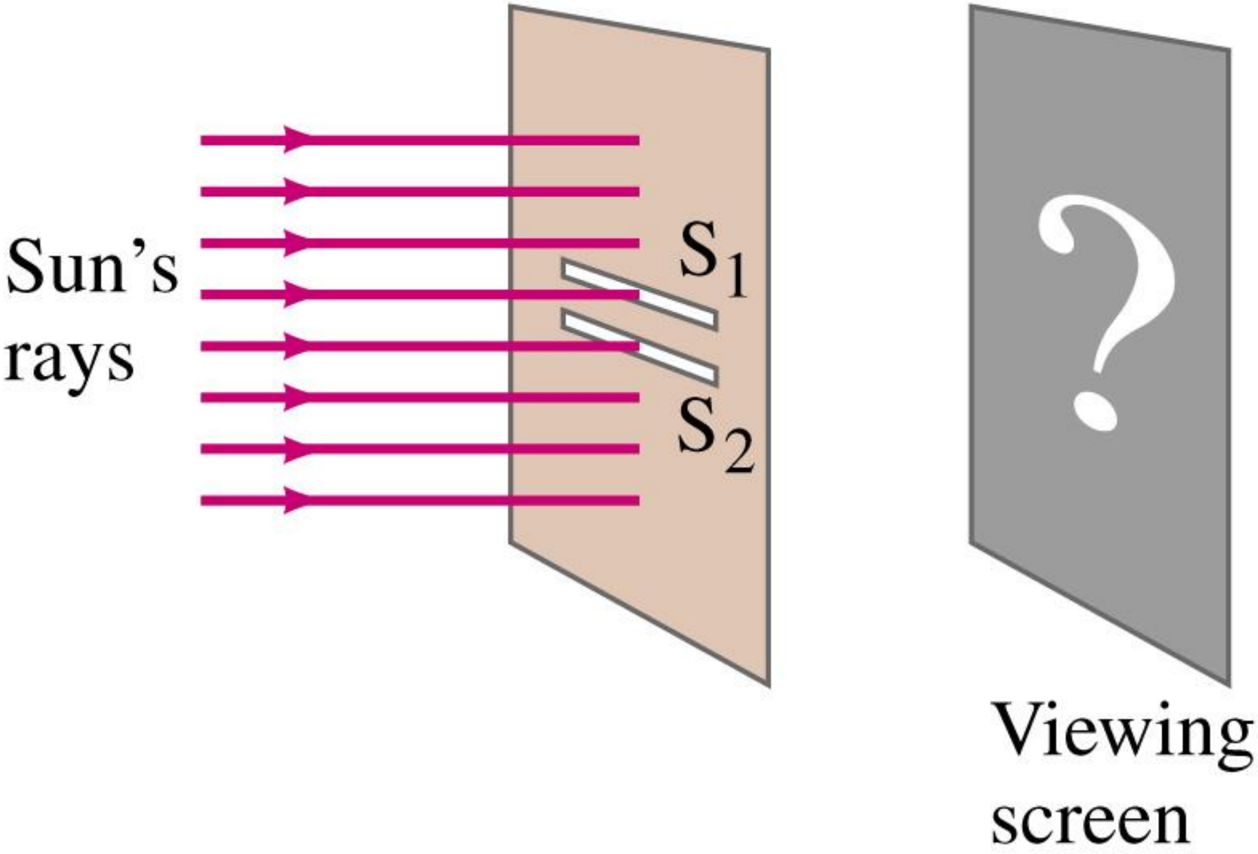
Conditions for Interference

- برای مشاهده تداخل در امواج نور، باید دو شرایط زیر را داشته باشیم:

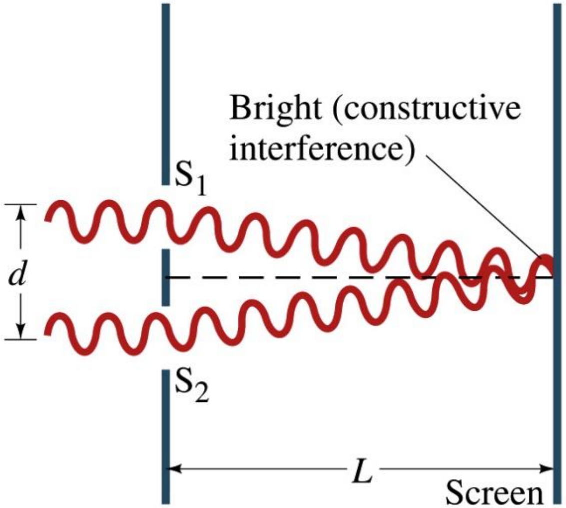
۱) منابع باید همدوس باشند. (**Coherent**)
(یعنی امواج یک فاز ثابت را نسبت به یکدیگر حفظ کنند)

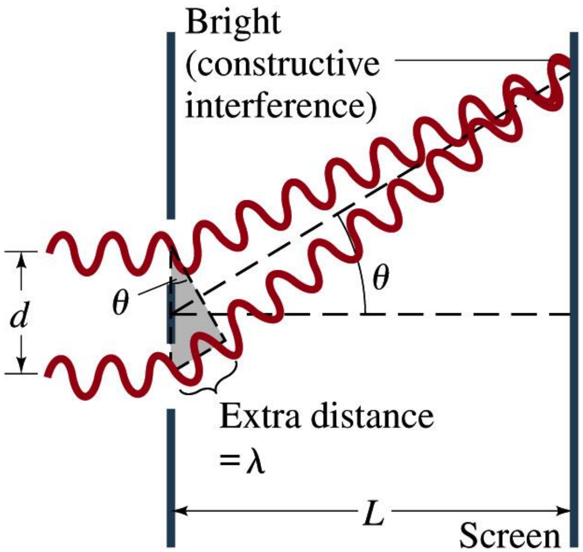
۲) منابع باید تک رنگ باشند. (**Monochromatic**)
(یعنی دارای یک طول موج باشند)

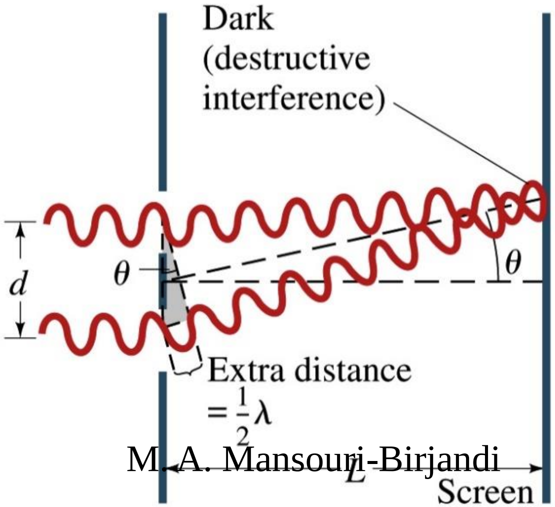
Young's Experiment (1)



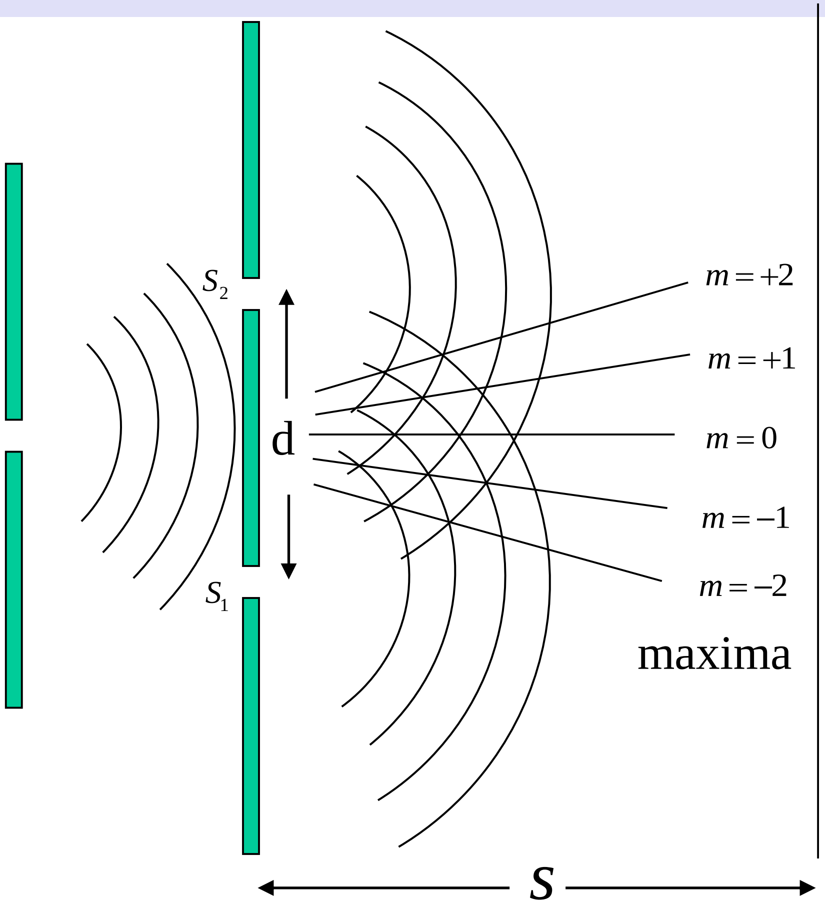
Young's Experiment (2)

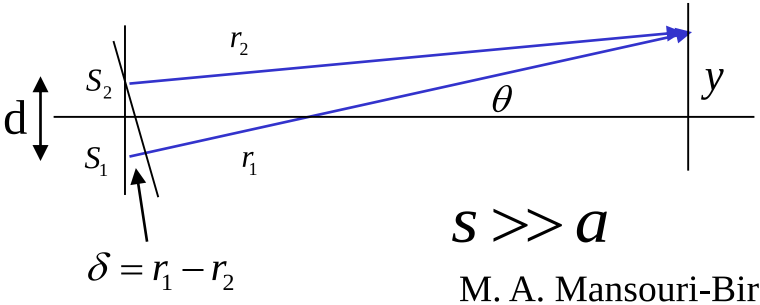






Young's Experiment (3)





$$s >> a$$

Maxima occur when:

$$\delta = r_1 - r_2 = m\lambda$$

$$\sin\theta \approx \frac{\delta}{d}$$

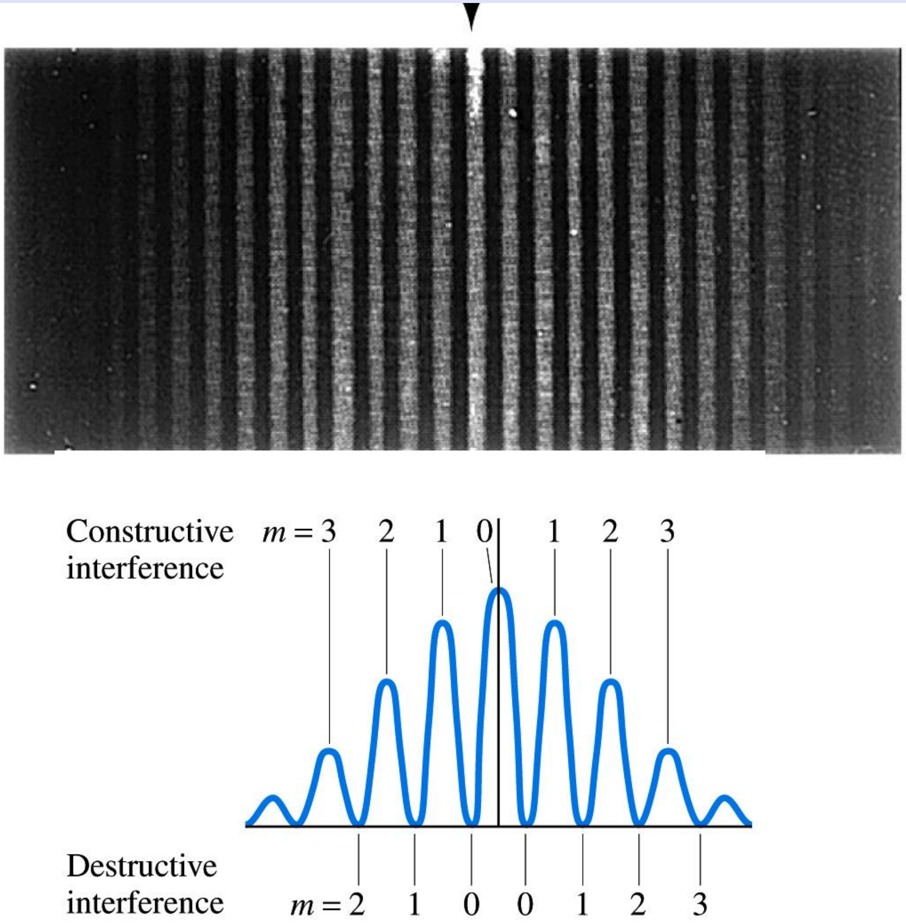
$$\boxed{\delta \approx d\sin\theta = m\lambda}$$

Minima occur when:

$$\delta = r_1 - r_2 = \left(m + \frac{1}{2}\right)\lambda$$

$$\boxed{\delta \approx d\sin\theta = \left(m + \frac{1}{2}\right)\lambda}$$

What the pattern looks like



Intensity Distribution, Electric Fields

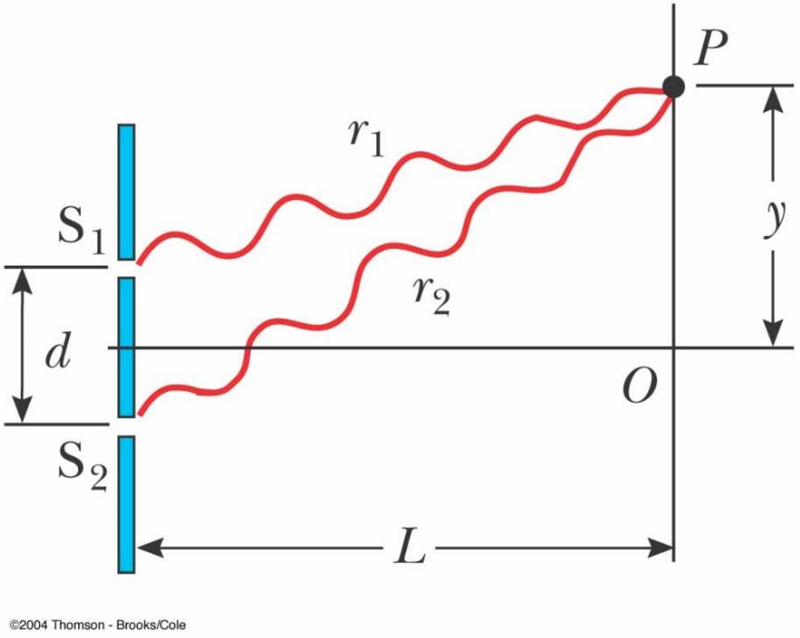
- The magnitude of each wave at point *P* can be found:

$$E_1 = E_o \sin \omega t$$

$$E_2 = E_o \sin (\omega t + \varphi)$$

Both waves have the same amplitude, E_o

$$\varphi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d \sin\theta$$



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Intensity Distribution, Resultant Field

- The magnitude of the resultant electric field comes from the superposition principle:

$$E_P = E_1 + E_2 = E_o[\sin \omega t + \sin (\omega t + \varphi)]$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

- This can also be expressed as: $E_P = 2E_o \cos\left(\frac{\varphi}{2}\right)\sin\left(\omega t + \frac{\varphi}{2}\right)$
  - E_P has the same frequency as the light at the slits
  - The magnitude of the field is multiplied by the factor $2\cos(\varphi/2)$

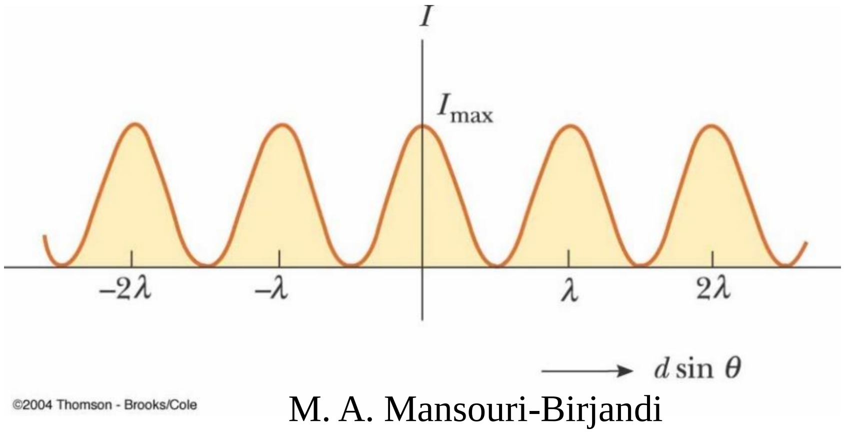
Intensity Distribution, Equation

- The expression for the intensity comes from the fact that

  the intensity of a wave is proportional to the square of the resultant electric field magnitude at that point

- Therefore, the intensity is:

$$I = I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right) \approx I_{\max}\cos^2\left(\frac{\pi d}{\lambda L}y\right)$$



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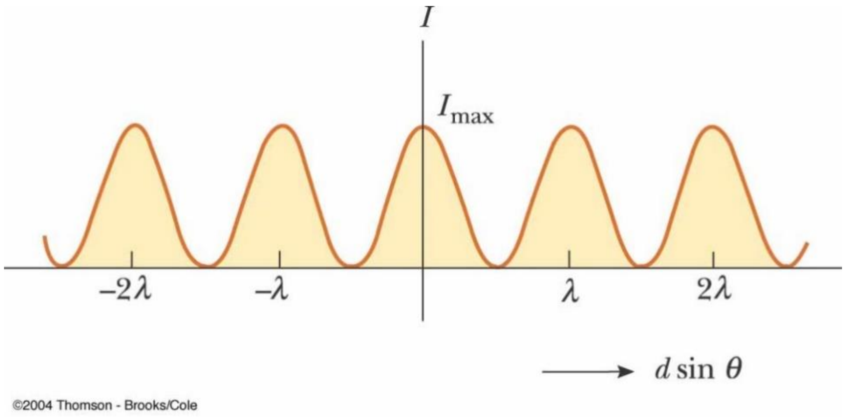
Resulting Interference Pattern

- The light from the two slits forms a visible pattern on a screen:
- The pattern consists of a series of bright and dark parallel bands called *fringes*
- *Constructive interference* occurs where a bright fringe occurs
- *Destructive interference* results in a dark fringe



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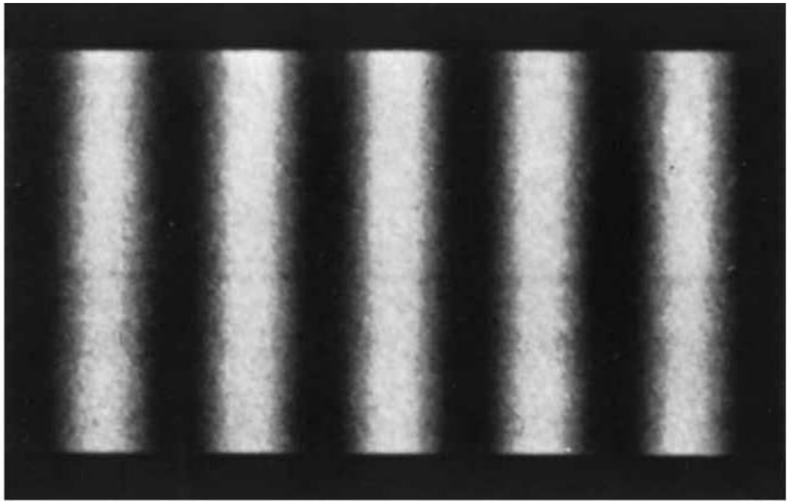

(b)

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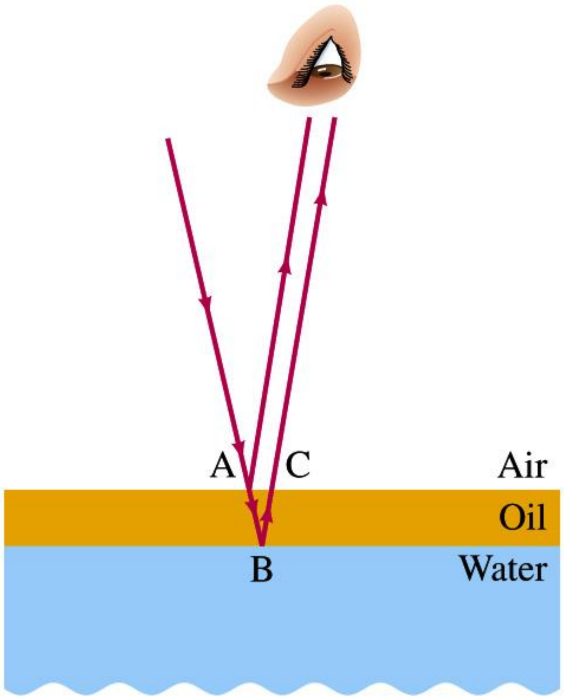
Thin Films



Constructive Interference (maxima)

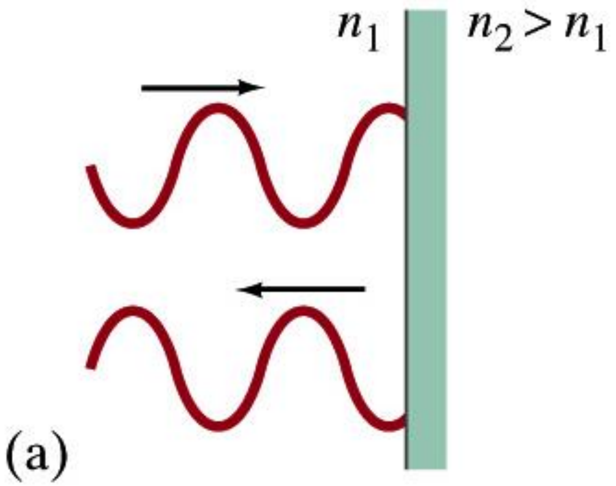
$$d_{ABC} = m\lambda_n$$

$$\lambda_n = \frac{\lambda_o}{n}$$

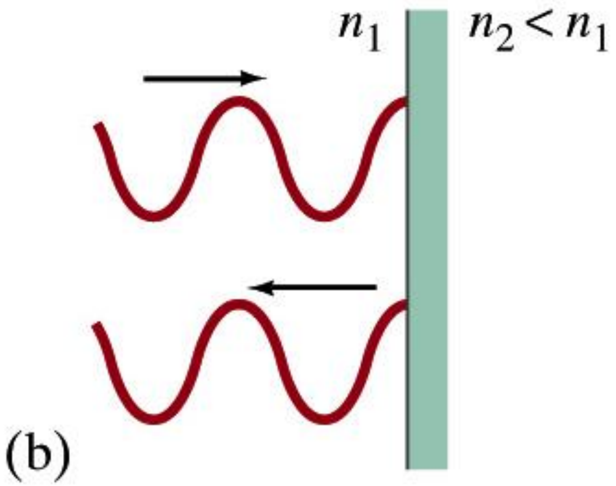
Destructive Interference (minima)

$$d_{ABC} = \left(m + \frac{1}{2}\right)\lambda_n$$

Phase shift on Reflection



External Reflection



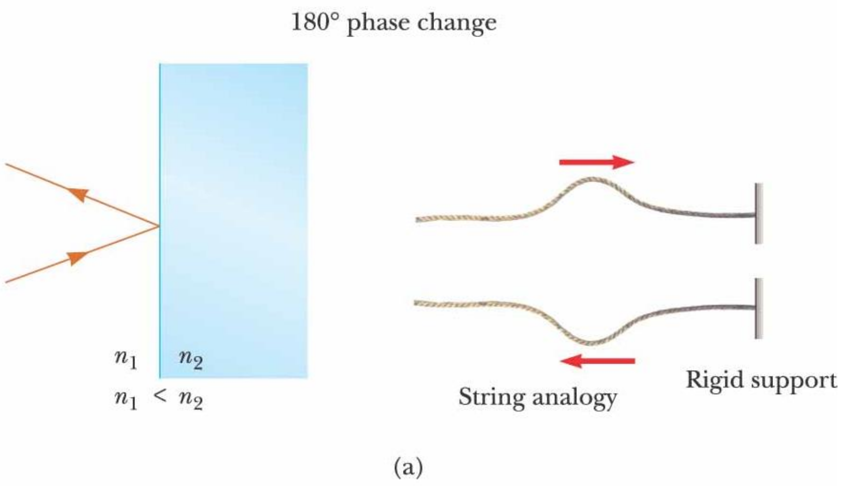
Internal Reflection

Now, If one reflection is internal and one reflection is external half wavelength path differences will result in constructive interference

Phase Changes due to Reflection

- An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling

- Analogous to a pulse on a string reflected from a rigid support



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Interference in Thin Films again

- Assume the light rays are traveling in air nearly normal to the two surfaces of the film

- Ray 1 undergoes a phase change of 180° with respect to the incident ray

- Ray 2, which is reflected from the lower surface, undergoes no phase change with respect to the incident wave

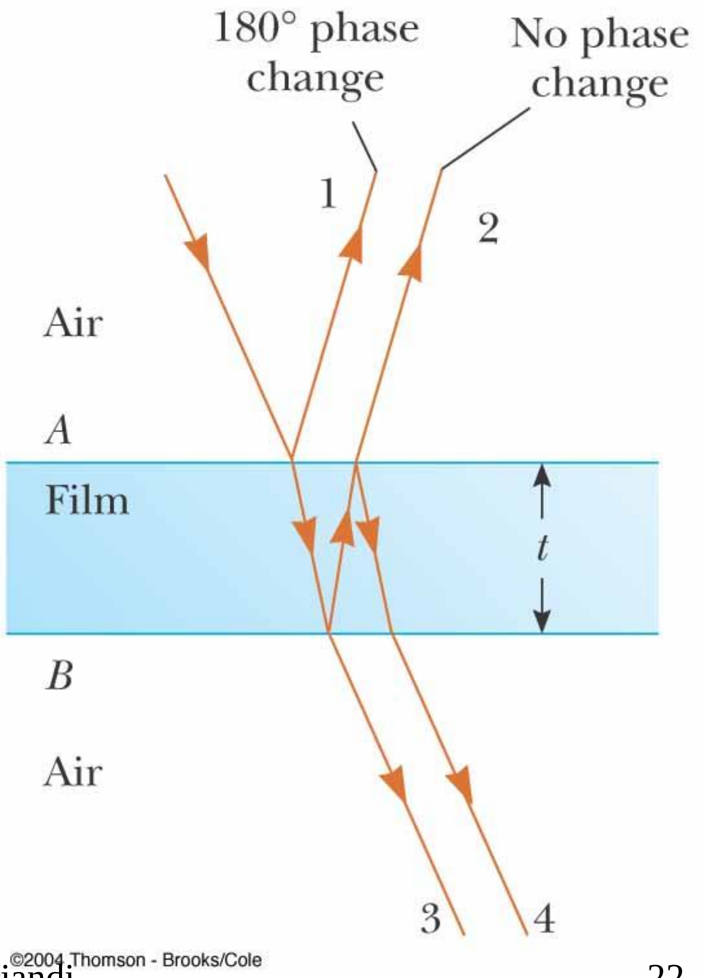
- For constructive interference

$$\delta=2t=(m+\tfrac{1}{2})\lambda_n \quad (m=0, 1, 2 \ldots)$$

- This takes into account both the difference in optical path length for the two rays and the 180° phase change

- For destructive interference

$$\delta=2t=m\lambda_n \qquad (m=0, 1, 2 \ldots)$$



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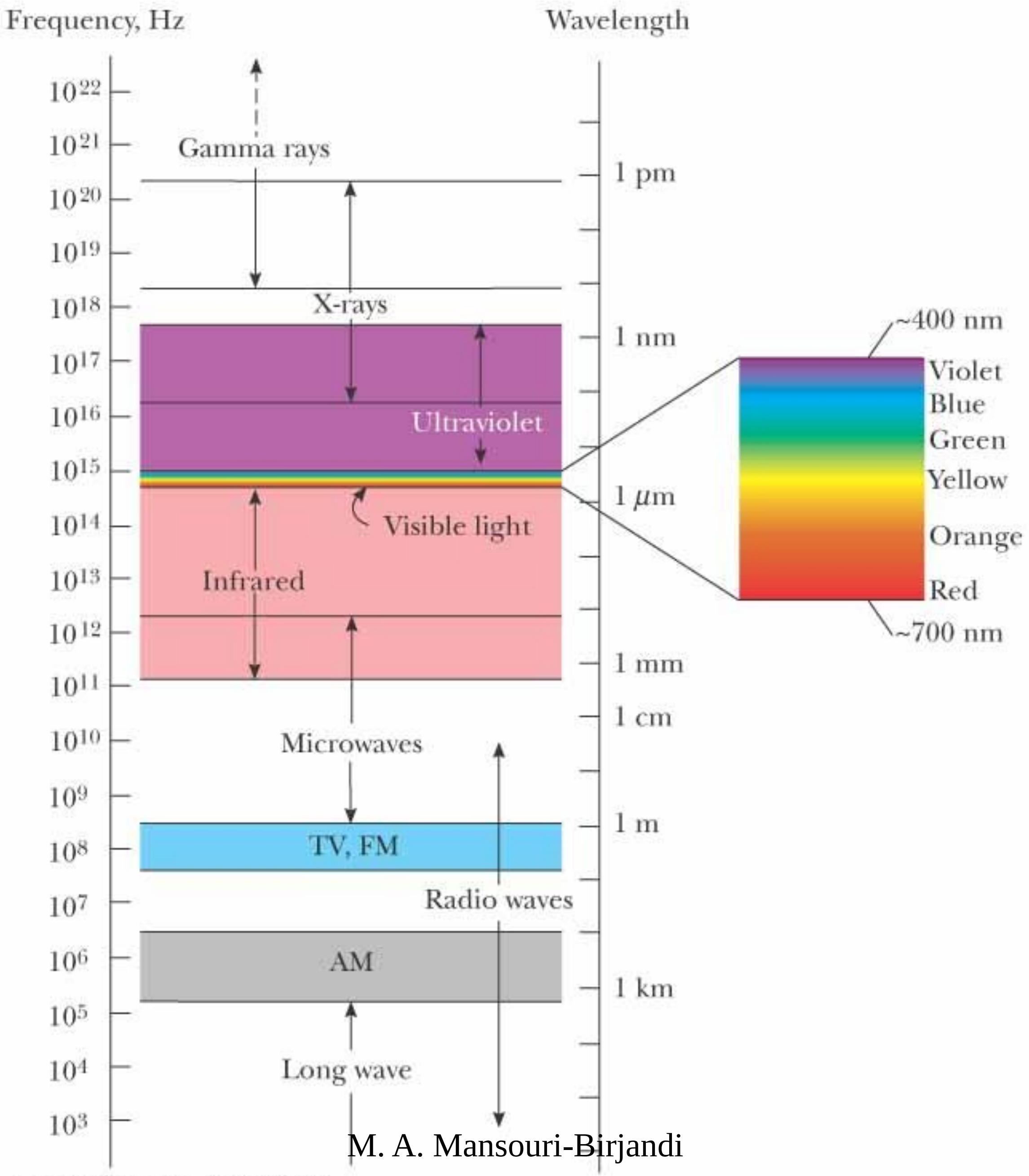

Frequency, Hz
Wavelength
10^{22}
10^{21}
10^{20}
10^{19}
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
Gamma rays
X-rays
Ultraviolet
Visible light
Infrared
Microwaves
TV, FM
Radio waves
AM
Long wave
1 pm
1 nm
1 μm
1 mm
1 cm
1 m
1 km
~400 nm
Violet
Blue
Green
Yellow
Orange
Red
~700 nm

Material Categorize

- از نظر وابستگی ضرایب اساسی شان (ε و μ) به **دامنه** میدانهای H, E:

۱- **خطی** (**Linear**)   ۲- **غیرخطی** (ضرایب ε و μ به صورت تابعی از H, E خواهند بود.)

- **از نظر وابستگی ضرایب اساسی شان (ε و μ) به موقعیت مختصاتی و مکانی:**

۱- همگن (Homogeneous)   ۲- غیرهمگن (ε و μ به صورت تابعی از مکان سه بعدی x,y,z خواهند بود.)

- **از نظر وابستگی ضرایب اساسی شان (ε و μ) به جهت اعمال میدانهای H, E:**

۱- همسانگرد (Isotropic)   ۲- ناهمسانگرد (anisotropic) (εو μ به صورت تانسوری خواهند بود.)

- **از نظر وابستگی ضرایب اساسی شان (ε و μ) به فرکانس:**

۱. غیرپاشنده (nondispersive)   ۲. پاشنده (Dispersive) (ε و μ تابعی از فرکانس میدان اعمالی خواهند بود.)

$$\boxed{\overline{D}(\overline{r}) = \varepsilon_0 \varepsilon(\overline{r}) \overline{E}(\overline{r})}$$

$$\boxed{\overline{B} = \mu_0 \overline{H}}$$

Electromagnetic

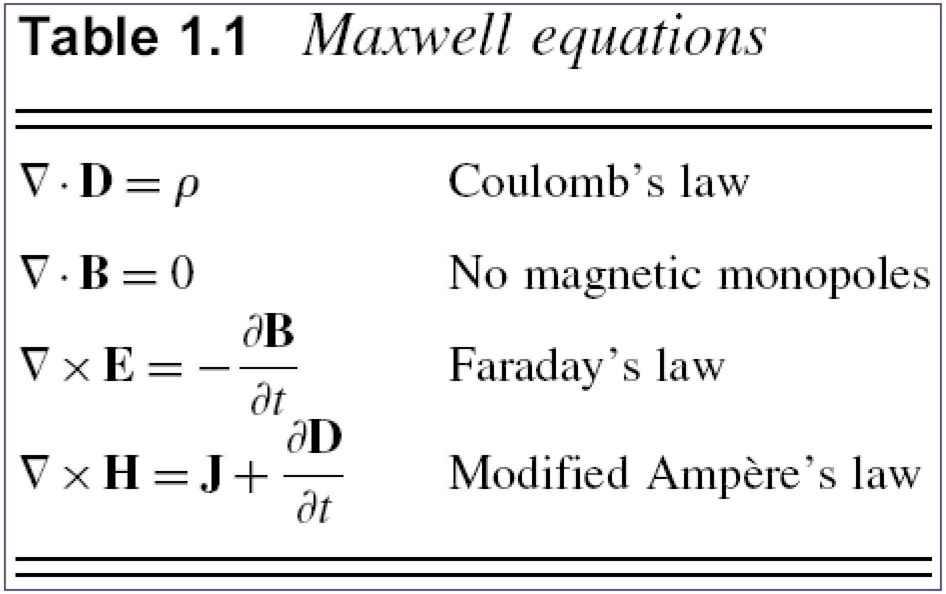


Table 1.1 *Maxwell equations*

$\nabla \cdot \mathbf{D} = \rho$	Coulomb's law
$\nabla \cdot \mathbf{B} = 0$	No magnetic monopoles
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	Modified Ampère's law

D :displacement vector field

E: electric field, or electric flux density

χ_e: electric susceptibility

P: electric polarization field

H and B: The magnetic field vector, or the magnetic flux density

$$\boldsymbol{D} = \varepsilon E = \varepsilon_0 \varepsilon_r \boldsymbol{E} = \varepsilon_0 (1 + \chi_e) \boldsymbol{E} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} = \mu_0 \mu_r \boldsymbol{H} = \mu_0 (1 + \chi_m) \boldsymbol{H}$$

$$= \mu_0 (\boldsymbol{H} + \boldsymbol{M})$$

μ: permeability,

μ_r: relative permeability,

χ_m: magnetic susceptibility,

$\boldsymbol{M}$: magnetization

Classical electromagnetism

Classical electrodynamics describes the spatial and temporal behavior of electric and magnetic fields.

Plane waves can be represented *spatially as:*

$$\sin(kx) = \frac{1}{2i}\left(e^{ikx} - e^{-ikx}\right) \qquad (74)$$

$$\cos(kx) = \frac{1}{2}\left(e^{ikx} + e^{-ikx}\right) \qquad (75)$$

$$e^{ikx} = \cos(kx) + i\sin(kx) \qquad (76)$$

Plane waves can be represented *temporally by:*

$$e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t) \qquad (77)$$

plane wave:

$$Ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \qquad (78)$$

A: amplitude of the wave,
K=2π/λ **:wave vector of magnitude,**
ω=2πf: angular frequency,
f=1/τ: frequency,
τ: periode

Wave equation

$$\nabla\times\left(\nabla\times y\right)=\nabla\left(\nabla.y\right)-\nabla^2 y$$

$$\nabla\times\left(\nabla\times E\right)=\nabla\left(\nabla.E\right)-\nabla^2 E, \qquad \nabla\bullet(E)=\nabla\bullet(D/\varepsilon)=0$$

$$\nabla\times(\nabla\times E)=-\nabla\times(\frac{\partial B}{\partial t})=-\nabla\times(\mu\frac{\partial H}{\partial t})\Rightarrow\nabla\left(\nabla.E\right)-\nabla^2 E=\mu\frac{\partial\left(\nabla\times H\right)}{\partial t}$$

$$\nabla^2 E=\mu\frac{\partial\left(\partial D/\partial t\right)}{\partial t}\Rightarrow$$

$$\nabla^2 E=\varepsilon\mu\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H=\varepsilon\mu\frac{\partial^2 H}{\partial t^2}$$

که همان معادله موج است

$$E\equiv E\left(x,y,z,t\right)$$
$$H\equiv H\left(x,y,z,t\right)$$

$$\nabla\Phi^2=\frac{1}{v_p^2}\frac{\partial^2\Phi}{\partial t^2}$$

$$v_p=\frac{1}{\sqrt{\mu_\varepsilon}}=\frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$$

Refractive Index

- The **propagation speed** of the electromagnetic waves in a **medium** v as function of the speed of light in **free space** c,

$$v \equiv \frac{c}{n}$$

- **n** represents the ***refractive index*** of the dielectric medium.

- The *refractive index* is related with the **optical constant** of the material medium and the dielectric permittivity and the magnetic permeability of the **free space** by:

$$n \equiv \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$

- In most of the materials (non-magnetic materials), and in particular in **dielectric media**, the magnetic **permeability** is very close to that of free space: $\mu \approx \mu_0$.

$$n \approx \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\varepsilon_r}$$

❋دسته بندی مدها (قطبش): انتشار در جهت z فرض شده است.

TE: Transverse Electric Mode

(**TE or n waves: Ey, Hx, Hz)**

$$\boxed{E_Z = 0, \quad H_Z \neq 0}$$

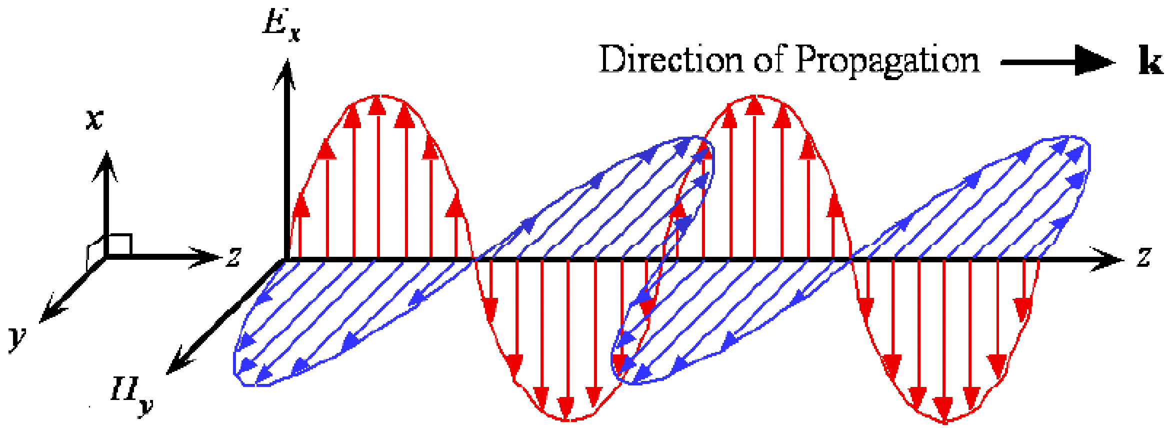
TM: Transverse Magnetic Mode

(TM or p waves: Ex, Ez, Hy)

$$\boxed{H_Z = 0, \quad E_Z \neq 0}$$

TEM: Transverse Electric and Magnetic Mode

$$\boxed{E_Z = 0, \quad H_Z = 0}$$



EM Waves at planar Dielectric Interfaces

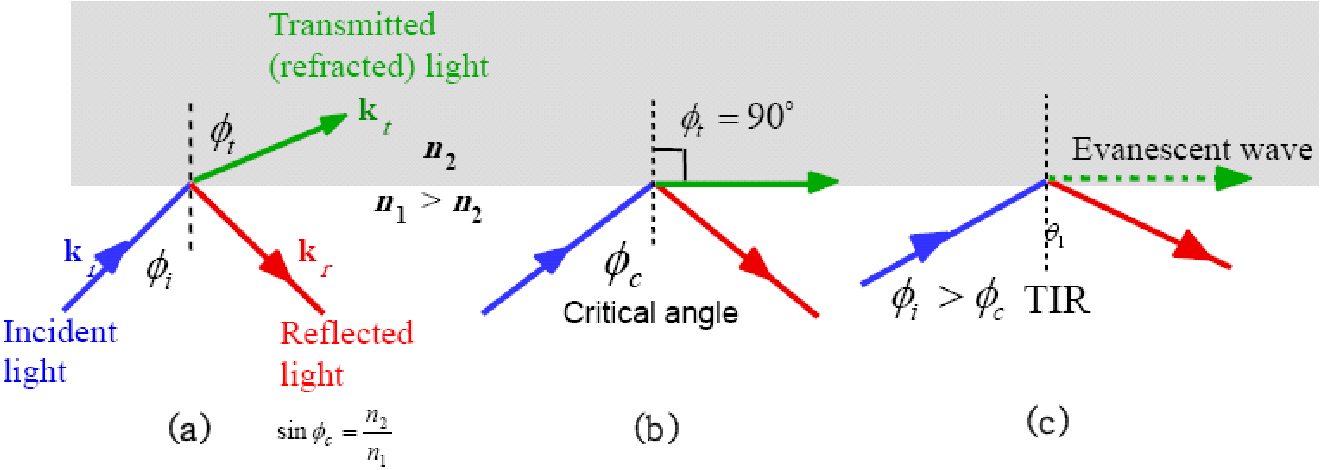
- The **relations** between the **incident**, **reflected** and **transmitted** waves are obtained by setting the adequate **boundary conditions** for the **fields** at **the planar interface**, which are derived directly from **Maxwell's equations.**

- $$\left.\begin{array}{l}\nabla.\mathbf{D}=0\\ \nabla.\mathbf{B}=0\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\left(\mathbf{D}^{\text{Normal}}\right)_{\mathbf{Medium1}}=\left(\mathbf{D}^{\text{Normal}}\right)_{\mathbf{Medium2}}\\ \left(\mathbf{B}^{\text{Normal}}\right)_{\mathbf{Medium1}}=\left(\mathbf{B}^{\text{Normal}}\right)_{\mathbf{Medium2}}\end{array}\right.$$

at interface

$$\left.\begin{array}{l}\nabla\times\mathbf{E}=-\dfrac{\partial\mathbf{B}}{\partial t}\\ \nabla\times\mathbf{H}=\mathbf{J}+\dfrac{\partial\mathbf{D}}{\partial t}\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\left(\mathbf{E}^{\text{Tangential}}\right)_{\mathbf{Medium1}}=\left(\mathbf{E}^{\text{Tangential}}\right)_{\mathbf{Medium2}}\\ \left(\mathbf{H}^{\text{Tangential}}\right)_{\mathbf{Medium1}}=\left(\mathbf{H}^{\text{Tangential}}\right)_{\mathbf{Medium2}}\end{array}\right.$$

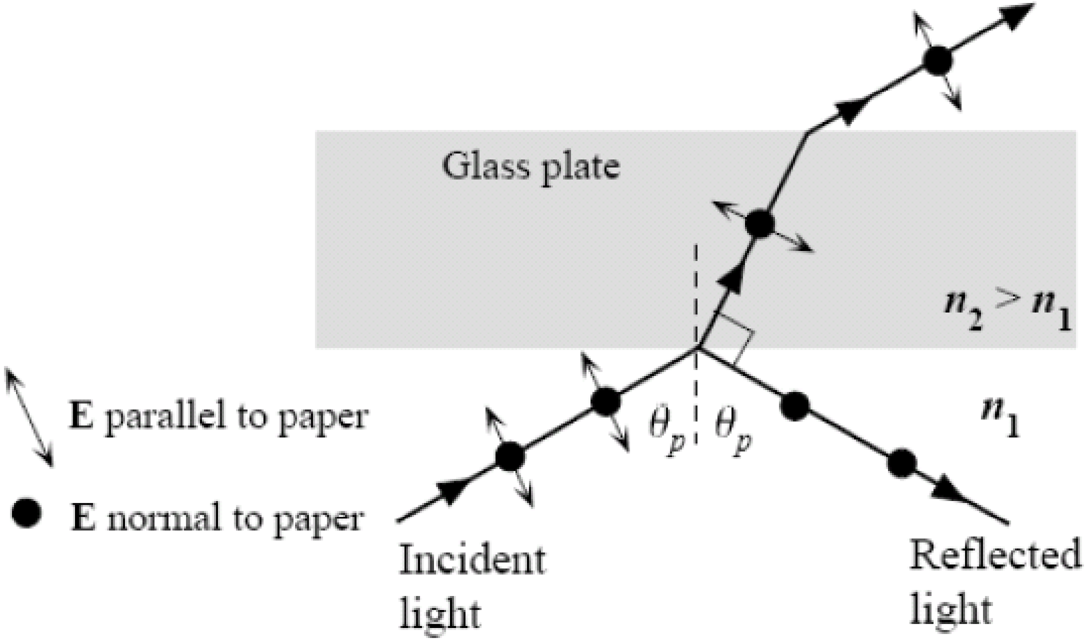
Total internal reflection, Critical angle



$$\sin\phi_c = \frac{n_2}{n_1}$$

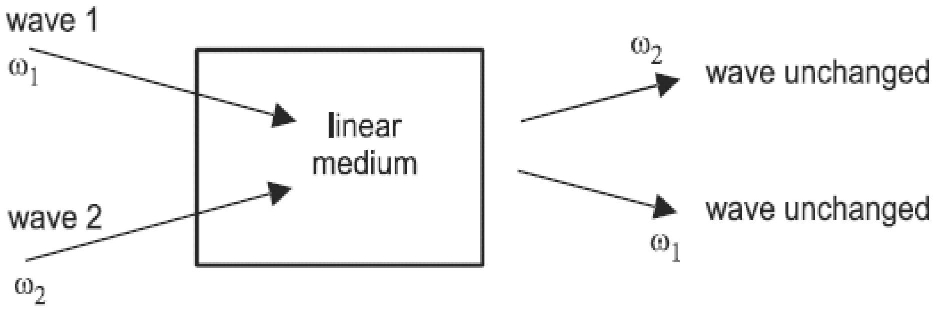
Brewster's angle or polarization angle (θ_p)

$$\tan\theta_p = \frac{n_2}{n_1}$$



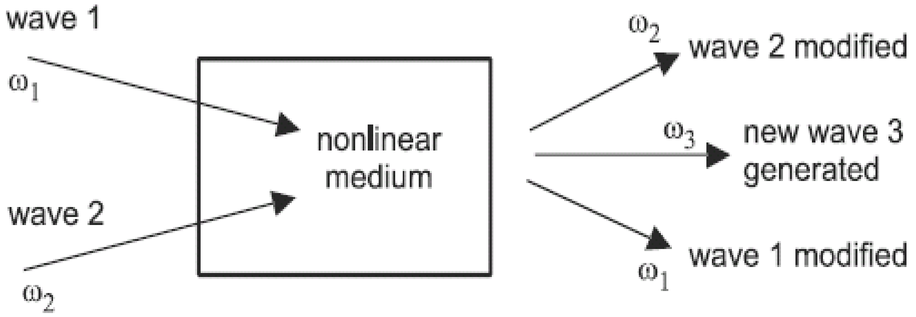
Nonlinear Optics

- A linear interaction: The presence of wave 2 has no affect on wave 1, and vice versa.



- A nonlinear interaction: Each wave can modify the properties of the other, changing, for example, the other wave’s amplitude or phase.

- Self-action: One of the waves can also modify its own amplitude or phase.



Birefringence (doubly refracting)

- Many important crystalline optical materials such as calcite ($CaCO_3$), quartz (SiO_2) and KDP (Potassium dihydrogen phosphate, KH_2PO_4) are anisotropic.

$$P = \varepsilon_0 . \chi . E$$

$$P = a_1 E + \frac{1}{2} a_2 E^2 + \frac{1}{6} a_3 E^3 + \ldots$$

$$a_1 = \varepsilon_0 . \chi^{(1)}$$

$$\chi \equiv \frac{P_x}{\varepsilon_0 E_x} \qquad \text{(electric susceptibility)}$$

$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} \mathbf{E}^2 + \chi^{(3)} \mathbf{E}^3 + \ldots \right) \equiv \mathbf{P}^{(1)} + \mathbf{P}^{(2)} + \mathbf{P}^{(3)} + \cdots$$

بلورهای دوشکستی: ضریب شکست، نه تنها به راستای انتشار امواج، بلکه به راستای قطبش بلور نیز بستگی دارد.

$$D = \varepsilon_0 . E + P = \varepsilon_0 . E + \varepsilon_0 . \chi . E = \varepsilon_0 (1 + \chi) E$$

$$D = \varepsilon_0 . E + P = \varepsilon_0 . E + \varepsilon_0 . \chi . E = \varepsilon_0 (1 + \chi) E$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_0 (1+\chi_x) & 0 & 0 \\ 0 & \varepsilon_0 (1+\chi_y) & 0 \\ 0 & 0 & \varepsilon_0 (1+\chi_z) \end{bmatrix} . \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$n_x = \sqrt{1+\chi_x} \qquad n_y = \sqrt{1+\chi_y} \qquad n_z = \sqrt{1+\chi_z}$$

$$\Delta(1/n^2) = r\mathcal{E} + P\mathcal{E}^2$$

$r.E$: *Pockels* effect, $P.E^2$: *Kerr* effect

r: **linear electro-optic** coefficient P: **quadratic** electro-optic coefficient