جلسه ۱۳

http://ecee.colorado.edu/~bart/book/book/title.htm

CHAPTER 2

Continuity equation

2.9. Continuity equation

2.9.1. Derivation

- The continuity equation describes a basic concept, namely that a change in carrier density over time is due to the difference between the incoming and outgoing flux of carriers plus the generation and minus the recombination.
- The flow of carriers and recombination and generation rates are illustrated with Figure 2.9.1.

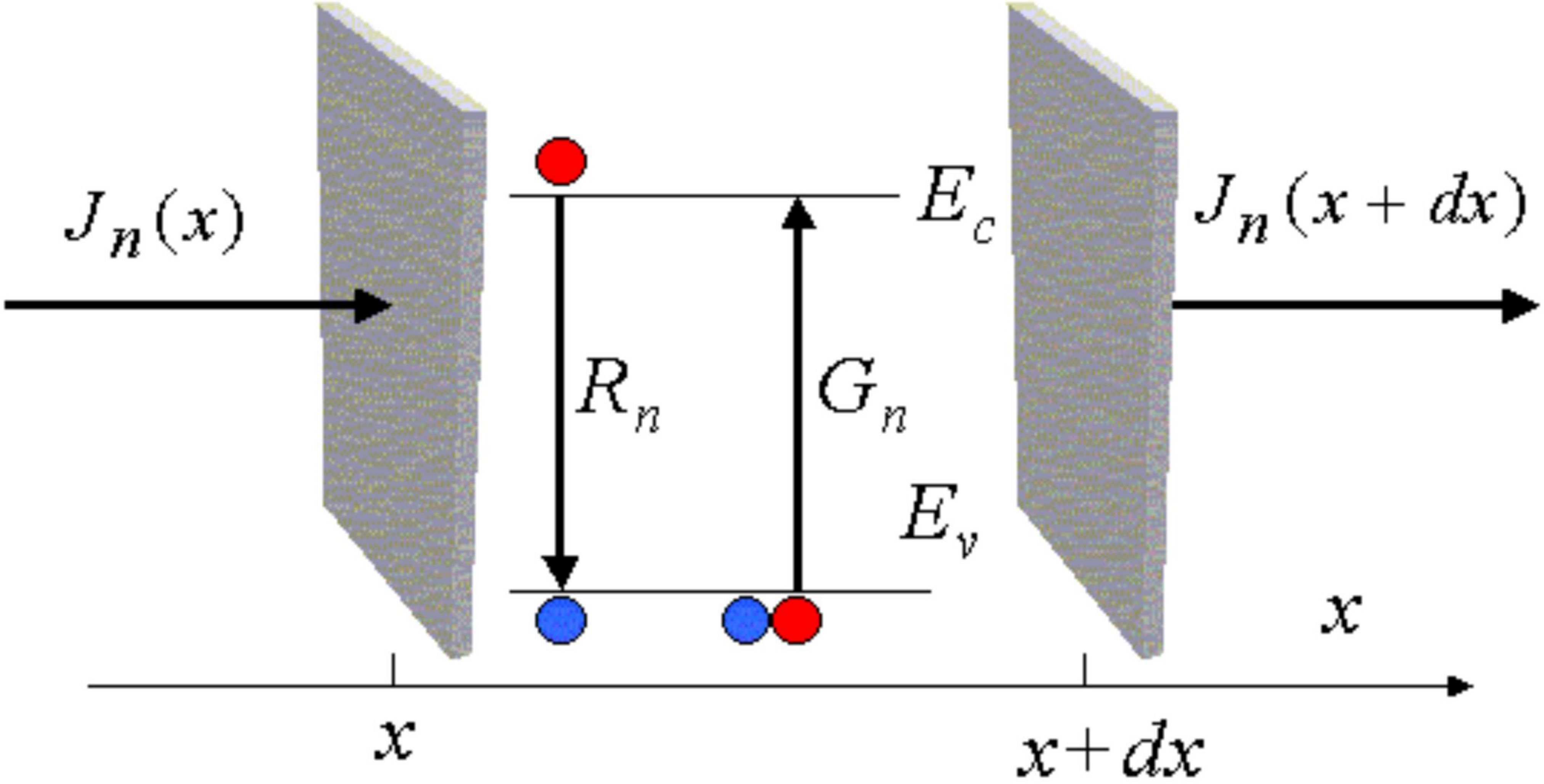

Figure 2.9.1 : Electron currents and possible recombination and generation processes

- The rate of change of the carriers between *x* and *x* + d*x* equals the difference between the incoming flux and the outgoing flux plus the generation and minus the recombination:

$$\frac{\partial n(x,t)}{\partial t} A\,dx = \left(\frac{J_n(x)}{-q} - \frac{J_n(x+dx)}{-q}\right) A + (G_n(x,t) - R_n(x,t))\,A\,dx \qquad (2.9.1)$$

- where $n(x,t)$ is the carrier density, A is the area, $G_n(x,t)$ is the generation rate and $R_n(x,t)$ is the recombination rate. Using a Taylor series expansion,

$$J_n(x+dx) = J_n(x) + \frac{d\,J_n(x)}{dx} dx \qquad (2.9.2)$$

❑This equation can be formulated as a function of the derivative of the current:

$$\frac{\partial n(x,t)}{\partial t} = \frac{1}{q}\frac{\partial J_n(x,t)}{\partial x} + G_n(x,t) - R_n(x,t) \qquad (2.9.3)$$

and similarly for holes one finds:

$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q}\frac{\partial J_p(x,t)}{\partial x} + G_p(x,t) - R_p(x,t) \qquad (2.9.4)$$

A generalization in three dimensions yields the following continuity equations for electrons and

holes:

$$\frac{\partial n(x,y,z,t)}{\partial t} = \frac{1}{q}\vec{\nabla}\,\vec{J}_n(x,y,z,t) + G_n(x,y,z,t) - R_n(x,y,z,t) \qquad (2.9.7)$$

$$\frac{\partial p(x,y,z,t)}{\partial t} = -\frac{1}{q}\vec{\nabla}\,\vec{J}_p(x,y,z,t) + G_p(x,y,z,t) - R_p(x,y,z,t) \qquad (2.9.8)$$

CHAPTER 3

p–n Junction

- In the preceding chapters we considered the carrier concentrations and transport phenomena in homogeneous semiconductor materials.
- In this chapter we discuss the behavior of single-crystal semiconductor material containing both p- and n-type regions that form a p–n junction.
- A p–n junction serves an important role both in modern electronic applications and in understanding other semiconductor devices.
- It is used extensively in rectification, switching, and other operations in electronic circuits.

- It is a key building block for the bipolar transistor and thyristor (Chapter 4), as well as for metal-oxide semiconductor field-effect transistors (MOSFETs) (Chapters 5 and 6).
- Given proper biasing conditions or when exposed to light, the p–n junction also functions as either a microwave (Chapter 8) or photonic device (Chapters 9 and 10).
- We also consider a related device—the heterojunction, which is a junction formed between two dissimilar semiconductors.
- It has many unique features that are not readily available from the conventional p–n junction.

- The heterojunction is an important building block for heterojunction bipolar transistors (Chapter 4), modulation doped field-effect transistors (Chapter 7), quantum-effect devices (Chapter 8), and photonic devices (Chapters 9 and 10).
- Specifically, we cover the following topics:
  - The band diagram of a p–n junction at thermal equilibrium.
  - The behavior of the junction depletion layer under voltage biases.
  - The current transport in a p–n junction and the influence of the generation and recombination processes.

- The charge storage in a p–n junction and its influence on the transient behavior.
- The avalanche multiplication in a p–n junction and its impact on the maximum reverse voltage
- The heterojunction and its basic characteristics.

3.1 THERMAL EQUILIBRIUM CONDITION

- The most important characteristic of p–n junctions is that they rectify: that is, they allow current to flow easily in only one direction.

- Figure 1 shows the current-voltage characteristics of a typical silicon p–n junction.

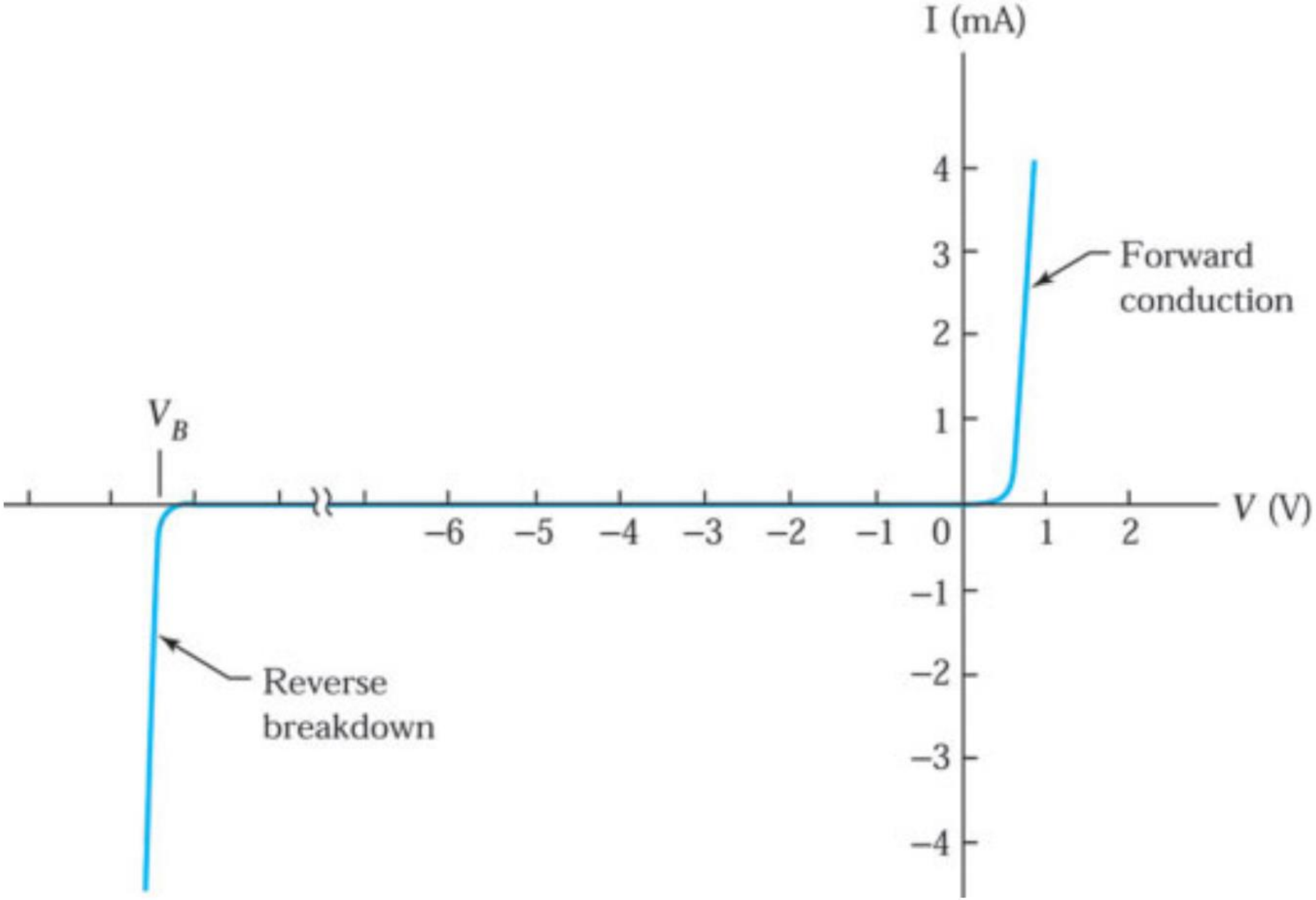

Fig. 1 Current-voltage characteristics of a typical silicon *p–n* junction.

- When we apply “forward bias” to the junction (i.e., positive voltage on the p-side), the current increases rapidly as the voltage increases.
- However, when we apply a “reverse bias,” virtually no current flows initially.
- As the reverse bias is increased the current remains very small until a critical voltage is reached, at which point the current suddenly increases.
- This sudden increase in current is referred to as the junction breakdown.
- The applied forward voltage is usually less than 1V, but the reverse critical voltage, or breakdown voltage, can vary from just a few volts to many thousands of volts, depending on the doping concentration and other device parameters.

3.1.1 Band Diagram

- In Fig. 2a, we see two regions of p- and n-type semiconductor materials that are uniformly doped and physically separated before the junction is formed.
- Note that the Fermi level E_F is near the valence band edge in the p-type material and near the conduction band edge in the n-type material.
- While p-type material contains a large concentration of holes with few electrons, the opposite is true for n-type material.

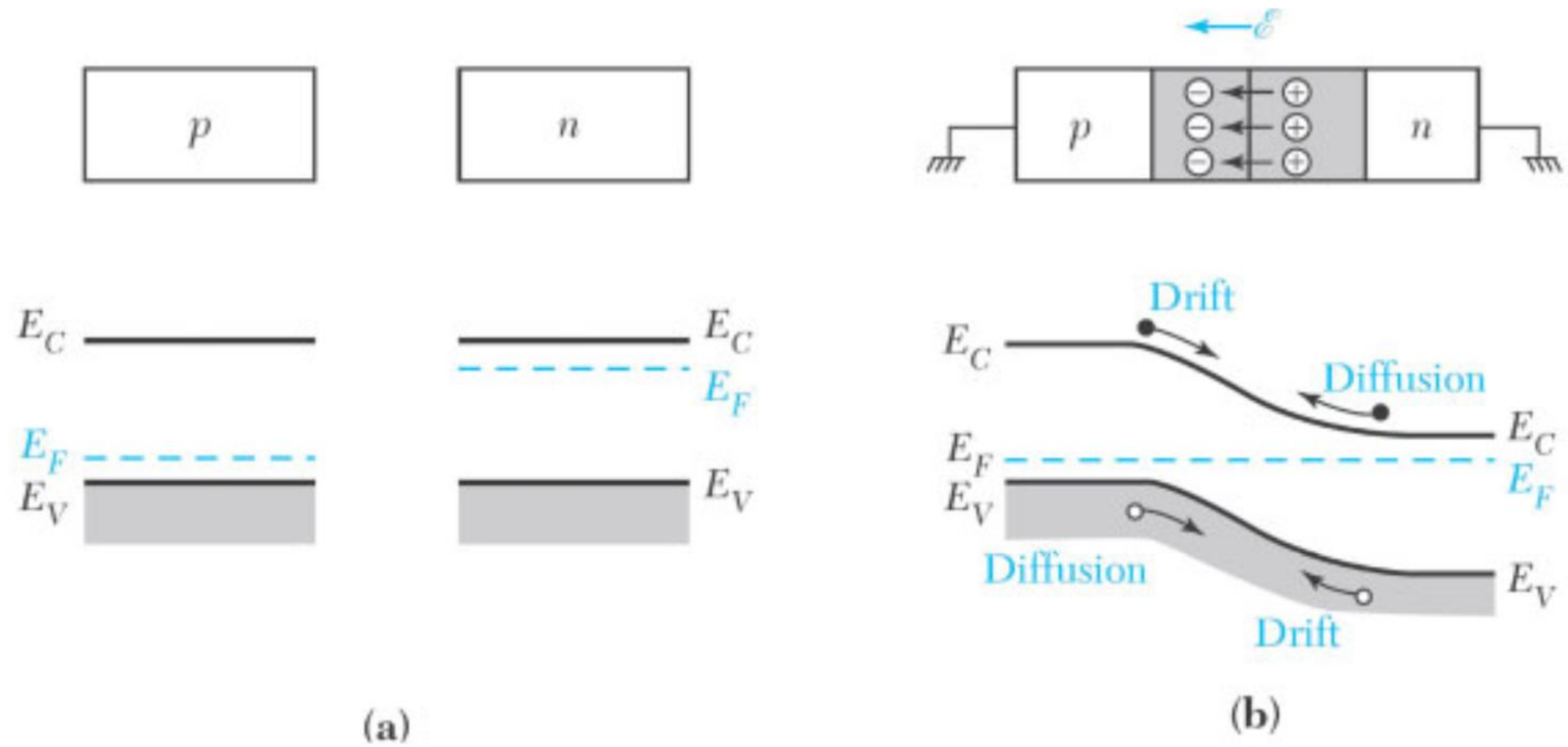

Fig. 2 **(*a*) Uniformly doped *p*-type and *n*-type semiconductors before the junction is formed. (*b*) The electric field in the depletion region and the energy band diagram of a *p*–*n* junction in thermal equilibrium.**

- When the *p*- and *n*-type semiconductors are jointed together, the large carrier concentration gradients at the junction cause carrier diffusion.

- Holes from the *p*-side diffuse into the *n*-side, and electrons from the *n*-side diffuse into the *p*-side.

- As holes continue to leave the *p*-side, some of the negative acceptor ions (N_A^-) near the junction are left uncompensated because the acceptors are fixed in the semiconductor lattice, whereas the holes are mobile.

- Similarly, some of the positive donor ions (N_{D+}) near the junction are left uncompensated as the electrons leave the *n*-side.

- Consequently, a negative space charge forms near the *p*-side of the junction and a positive space charge forms near the *n*-side.

- This space charge region creates an electric field that is directed from the positive charge toward the negative charge, as indicated in the upper illustration of Fig. 2*b*.

- The lower illustration of Fig. 2*b* shows that the hole diffusion current flows from left to right, whereas the hole drift current due to the electric field flows from right to left.

- The electron diffusion current also flows from right to left, whereas the electron drift current flows in the opposite direction.

Animations:
https://www.pveducation.org/pvcdrom/pn-junctions/formation-of-a-pn-junction
https://www.pveducation.org/pvcdrom/pn-junctions/p-n-junction-diodes

3.1.2 Equilibrium Fermi Levels

- At thermal equilibrium, i.e. the steady-state condition at a given temperature with no external excitations, the individual electron and hole currents flowing across the junctions are identically zero.

- Thus, for each type of carrier the drift current due to the electric field must exactly cancel the diffusion current due to the concentration gradient.

From Eq. 32 in Chapter 2,

$$\begin{aligned} J_p &= J_p(\text{drift}) + J_p(\text{diffusion}) \\ &= q\mu_p p\mathcal{E} - qD_p \frac{dp}{dx} \\ &= q\mu_p p\left(\frac{1}{q}\frac{dE_i}{dx}\right) - kT\mu_p \frac{dp}{dx} = 0, \end{aligned} \qquad (1)$$

About final derivation:

(first why $\mathcal{E} = \frac{1}{q}\frac{dE_i}{dx}$)

Consider conduction in a homogeneous semiconductor material.

- Figure 4*a* shows an *n*-type semiconductor and its band diagram at thermal equilibrium.

- Figure 4*b* shows the corresponding band diagram when a positive biasing voltage is applied to the right-hand terminal.

- When an electric field E is applied to a semiconductor, each electron will experience a force – qE from the field. The force is equal to the negative gradient of potential energy; that is,

$$-q\mathscr{E} = -(\text{gradient of electron potential energy}) = -\frac{dE_C}{dx} \ . \quad (7)$$

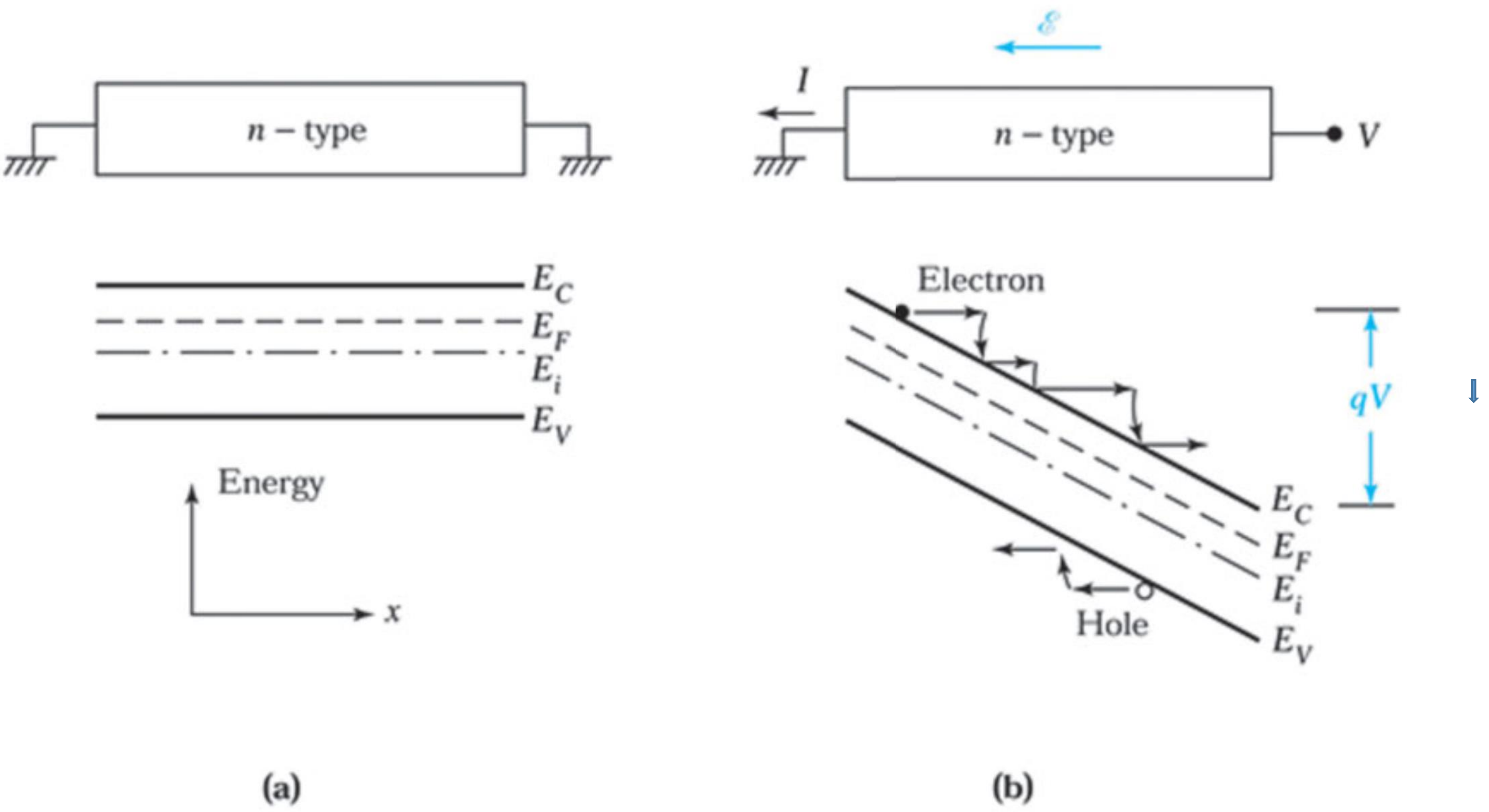

Fig. 4 **Conduction process in an n-type semiconductor (a) at thermal equilibrium and (b) under a biasing condition.**

- Recall that in Chapter 1, the bottom of the conduction band E_c corresponds to the potential energy of an electron.

- Since we are interested in the gradient of the potential energy, we can use any part of the band diagram that is parallel to E_C (e.g., E_F, E_i, or E_V, as shown in Fig. 4*b*).

- It is convenient to use the intrinsic Fermi level E_i because we shall use E_i when we consider *p–n* junctions.

- Therefore, from Eq. 7 we have

$$\mathcal{E} = \frac{1}{q}\frac{dE_C}{dx} = \frac{1}{q}\frac{dE_i}{dx}. \qquad (8)$$

- We can define a related quantity ψ as the *electrostatic potential* whose negative gradient equals the electric field:

$$\mathcal{E} \equiv -\frac{d\psi}{dx}. \qquad (9)$$

Comparison of Eqs. 8 and 9 gives

$$\psi = -\frac{E_i}{q}, \tag{10}$$

- which provides a relationship between the electrostatic potential and the potential energy of an electron.

- For the homogeneous semiconductor shown in Fig. 4*b*, the potential energy and E_i, decrease linearly with distance; thus, the electric field is a constant in the negative x-direction.

- Its magnitude is the applied voltage divided by the sample length.

- The electrons in the conduction band move to the right side, as shown in Fig. 4b.
- The kinetic energy corresponds to the distance from the band edge (i.e., E_C for electrons).
- When an electron undergoes a collision, it loses some or all of its kinetic energy to the lattice and drops toward its thermal equilibrium position.
- After the electron has lost some or all its kinetic energy, it will again begin to move toward the right and the same process will be repeated many times.
- Conduction by holes can be visualized in a similar manner but in the opposite direction.

(second why $D_n = \left(\frac{kT}{q}\right)\mu_n$)

We can write

$$\frac{1}{2}m_n\upsilon_{th}^2 = \frac{1}{2}kT . \tag{28}$$

$$D_n = \upsilon_{th}l = \upsilon_{th}(\upsilon_{th}\tau_c) = \upsilon_{th}^2\left(\frac{\mu_n m_n}{q}\right) = \left(\frac{kT}{m_n}\right)\left(\frac{\mu_n m_n}{q}\right), \tag{29}$$

$$\boxed{D_n = \left(\frac{kT}{q}\right)\mu_n .} \tag{30}$$

Equation 30 is known as the Einstein relation. It relates the two important constants (diffusivity and mobility) that characterize carrier transport by diffusion and by drift in a semiconductor. The Einstein relation also applies between D_p and μ_p.